INVESTIGATION OF RESONANCE CHARACTERISTICS OF A IDLE INFLEXIBLE ROTOR UNDER CONDITIONS OF AXIAL ELECTROMAGNETIC SUSPENSION AT A PLANE MOTION

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Abstract
Service conditions of workable in OKBM turbomachine with a vertical shaft are such that active electromagnetic bearings have to be used there. One of main problems of a control system electromagnetic a bearing is to ensure of stability of the system (rotor - control system). In given work conditions of occurrence and limiting of fluctuations of a rotor (a pendulum-type) in a nonlinear formulation are analysed. At the given stage of studies the problem is limited by frameworks of idle, inflexible rotor assuming absence of energy dissipation.

1 Introduction
Unique turbomachine with a vertical shaft of about 100 tons and about 29 m length is currently developed by OKBM [1,2]. Service conditions of the turbomachine are such that active electromagnetic bearings have to be used there. There are no close analogues of such bearing system, therefore special analytical and experimental activities need to be done during the system design. Some issues of stability are considered in given work.

2 Statement of a problem
Contact-frees suspension of a rotor in a magnetic field can only be done by changing a value of a field intensity with the aid of a bearing control system in conformity with the received information on a position of a rotor in the space. One of main problems of a control system electromagnetic a bearing is to ensure of stability of the system (rotor - control system).

In given work conditions of occurrence of parametrical fluctuations of a rotor (a pendulum-type), caused by effect of an axial bearing control system are analysed. As far as the opportunity of such fluctuations necessitates enhancement of the requirements to a control system of radial bearings, it seems to be important to find out conditions lowering a resonance response. At the given stage of studies the problem is limited by frameworks of idle, inflexible rotor assuming absence of energy dissipation.

3 Model
Movement of a rigid body may be determined, knowing movement of its centre of mass and rotation relatively to a centre of mass [3]. The system of equations has a form:

$$\begin{cases} M\dfrac{d^2x}{dt^2} = (F - Mg)\cos\alpha \\ \\ J\dfrac{d^2\alpha}{dt^2} = -FL\sin\alpha, \end{cases} \quad (1)$$

where
M – is a mass of a rotor, kg;
x – is a centre of mass co-ordinate, m;
α - is an angle of a rotor deviation in xz plane from a vertical line;
g – is a acceleration of free fall, kg m/s^2;
J - is a moment of inertia of a rotor relatively y axis, kg m^2;
L – is a distance from a centre of mass to a point of a control system impact to a rotor, m;
t – is a time, s.

Fig.1 shows the principle scheme.

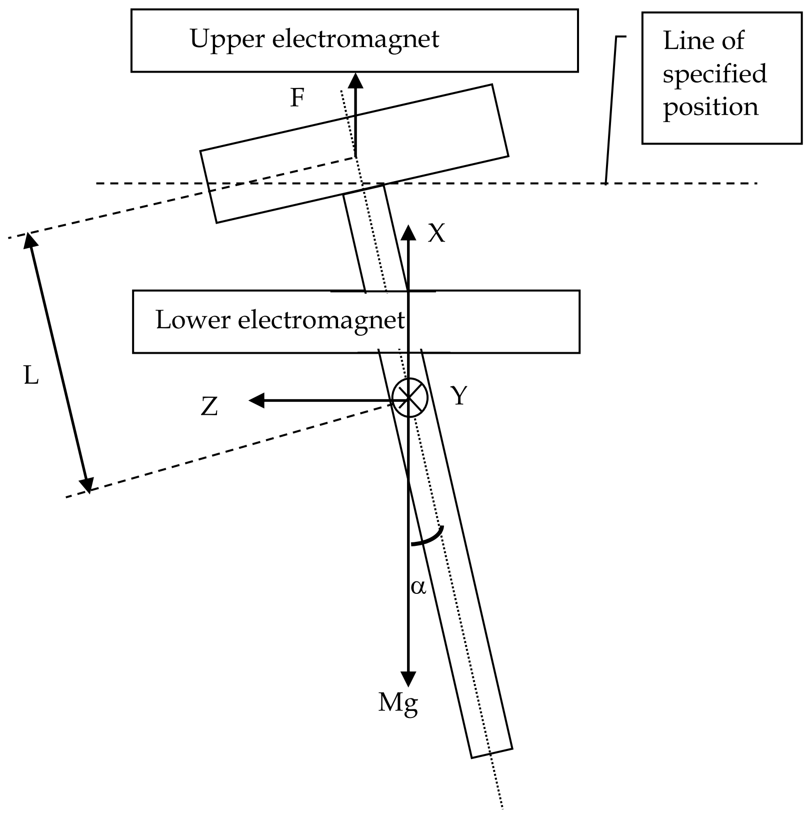

Fig 1: Principle scheme

The force F, in a common case, depends on a current and spatial position of a rotor (in this case it depends on axial co-ordinate) in a permanent magnetic field

$$F = \frac{f(I_U)}{(x_0 - x)^2} - \frac{f(I_L)}{(x + x_0)^2} \ , \qquad (2)$$

where
x_0 – is an approximation coefficient, m;
I_U and I_L – are control currents of upper and lower electromagnets (in a common case they depend on x), A;
f – is a function, expressing a force from control currents (Fig 2):

$$f(I_U) = aI_U + bI_U^2 \ , \tag{3}$$

$$f(I_L) = aI_L + bI_L^2 \ , \tag{4}$$

a and b - coefficients of approximation of a square-law dependence, kg $m^3/(s^2$ A) and kg $m^3/(s^2$ $A^2)$, respectively.

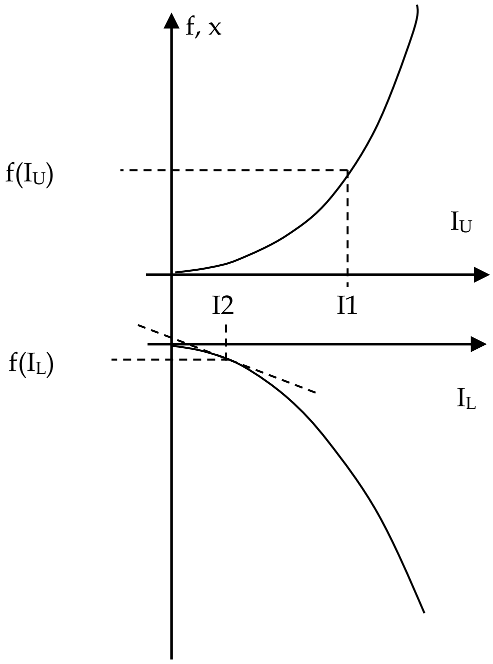

Fig 2: Functions, expressing a force from control currents

Control currents may be presented by the following dependencies

$$I_U = -sx - \gamma \frac{dx}{dt} + I1, \tag{5}$$

$$I_L = sx + \gamma \frac{dx}{dt} + I2 \ , \tag{6}$$

where
s – is a coefficient of proportional regulation, A/m;
γ - is a coefficient of differential regulation, A s/m;
I1 – is a steady bias current in an upper electromagnet, A;
I2 - is a steady bias current in an lower electromagnet, A.

Simplifying the dependence 2, linearizating relatively to x the function of a force linkage with a spatial position of a rotor in permanent magnetic field, assuming for the given operation that $I \neq \varphi(x)$ and then substituting 5 and 6, we receive a dependence for control force on x and $\frac{dx}{dt}$:

$$F \cong \varphi(x, \frac{dx}{dt}, x^2, x\frac{dx}{dt}, x^3, x\left(\frac{dx}{dt}\right)^2, x^2\frac{dx}{dt}) \ . \tag{7}$$

Consider the equation of fluctuations on α, assuming x as an amplitude of an external force, in order to identify steady. If they will be found, out may possible to assert, that there are forced fluctuations of an angle α, exist in the system 1, initiated by axial force from a control system. At that, we assume, that because of non-perfect control system (sluggishness, dead zones and etc.) the rotor fluctuates along x axis near a state of balance. Decomposing these movements in a Fourier series. They may be presented in a form of a superposition of periodic fluctuations within a certain spectrum of frequencies.

Then, each of harmonics may be considered independently as an external force. As the first step we conduct the analysis of an opportunity of existence of fluctuations under effect of external force at own frequency of a pendulum.

Omitting a number of transformations and applying standard methods we receive approximate, so-called "shorted" equation in Van der Pol variables relative to an amplitude of pendulum fluctuations:

$$j\dot{B} = B(\Omega - \frac{1}{\Omega}) + \frac{E^2}{4\Omega}(\eta + j\rho)B^* \ , \qquad (8)$$

where
the points over variables designate differentiation on a dimensionless time

$$\tau = t\omega ; \qquad (9)$$

$$\omega = \sqrt{\frac{LMg}{J}} ; \qquad (10)$$

B – is a slowly varying complex function

$$\alpha(\tau) \sim B(\tau)exp(j\tau) ; \qquad (11)$$

E – is an amplitude of axial fluctuations

$$x = \frac{1}{2}E(e^{jt} + e^{-jt}) ; \qquad (12)$$

$$\Omega = 1 + \varepsilon , \qquad (13)$$

ε - is a deviation from natural frequency (tuning away), $\varepsilon << 1$;

$$\eta = \frac{4bs}{Mgx_0}(I2 - I1) \ ; \qquad (14) \quad ;$$

$$\rho = \frac{4b\gamma\sqrt{L}}{\sqrt{MgJ}x_0}(I2 - I1) \ ; \qquad (15) \quad .$$

Changing to actual variables and assuming a non-trivial decision we receive a characteristic equation for definition λ (Re{B}, Im{B} ~ exp{λτ}):

$$\lambda^2 = \frac{E^4}{16\Omega^2}(\rho^2+\eta^2)-(\Omega-\frac{1}{\Omega})^2 \qquad (16)$$

If a tuning away is sufficiently small,

$$-\frac{1}{16}E^2\sqrt{\rho^2+\eta^2} < \varepsilon < \frac{1}{16}E^2\sqrt{\rho^2+\eta^2} \qquad (17)$$

amplitude α will build up, hence in a system a parametric instability would be realized (it was assumed at deriving of the inequalities that $\Omega^2 \cong 1+2\varepsilon$). The given inequalities (17) define a zone of a resonance, the borders of which are given in fig. 3.

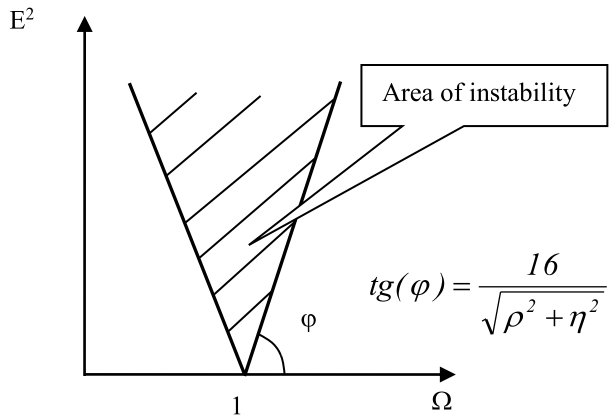

Fig 3: Area of parametrical instability under impact at natural frequency

Consider influence of the second harmonic (that is condition of realization of a main parametrical resonance) on an opportunity to sustain fluctuations on α

$$x = \frac{1}{2}E(e^{2jt}+e^{-2jt}). \qquad (18)$$

Using similar techniques and neglecting small terms, containing E^2 and E^3 we receive the following characteristic equation:

$$\lambda^2 = \frac{E^2}{\Omega^2}(\mu^2+\frac{1}{4}k^2)-(\Omega-\frac{1}{\Omega})^2 , \qquad (19)$$

where

$$\mu = \frac{2\gamma\sqrt{L}}{\sqrt{MgJ}x_0}\left[a + b(I2 + I1)\right]; \tag{20}$$

$$k = \frac{2}{Mgx_0^2}\left[a(I2 + I1) + b(I2^2 + I1^2)\right] - \frac{2s}{Mgx_0}\left[a + b(I2 + I1)\right], \tag{21}$$

The borders of a main parametrical resonance are indicated in fig. 4.

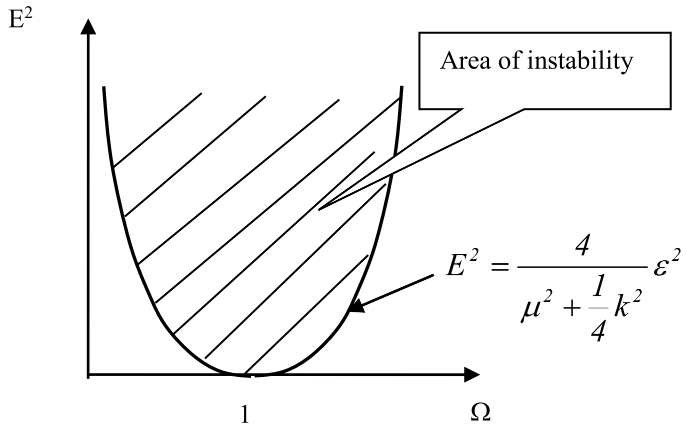

Fig. 4: Area of parametrical instability under impact at twofold frequency

Since the external impact amplitudes range close to zero a derivative $\frac{dE^2}{d\Omega}$ is close to a zero, excitation of fluctuations at frequency close to natural one, at a main parametric resonance realizes in a wider frequency range then in case of external force impact at a natural frequency of a pendulum.

4 Summary

4.1 Thus, an opportunity of parametrical instability occurrence at a natural and double frequencies of external impact is confirmed.

4.2 The following solutions lead to narrowing of both areas of instability:

- reduction in distance between a centre of mass and place of control forces application (L),
- increase in sluggishness of a rotor (mass and moment of inertia);
- increase in bias currents in both the electromagnets and, as result, reduction in a difference between them;
- decrease in efficiency of a feedback on derivative $\frac{dx}{dt}$ (decreasing γ).

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