# FUZZY LOGIC CONTROL OF MAGNETIC BEARINGS FOR SUSPENSION OF VIBRATION DUE TO SUDDEN IMBALANCE

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## 1. Introduction

In recent years, the use of magnetic bearings in rotating machinery has received much interest due to its potential advantages. The non-contact magnetic bearing exhibits much better performance in high speed rotating machinery. While it is very important to suppress the vibration of the rotating body due to imbalance, active magnetic bearings offer a way to support the rotating shaft as well as to control the vibration. Sudden imbalance in rotating machinery usually results from partial or full blade loss or from sudden shedding of accumulated deposits. The resulting transient may cause severe internal rubs between stator and rotor. Damage from this action may have catastrophic consequences in terms of cost. Active magnetic bearings (AMB) provide the capability for adaptive control of shaft vibration via supervisory systems which detect the upset conditions or changes in operating parameters. Most AMB's utilize a linear PID controller with power amplifiers, magnetic actuators and inductive, eddy current or optical sensors. These electronic components act linearly up to a saturation limit. The linear controller usually does not include logic for maintaining accurate control with non-linear system behavior.

In this work, fuzzy logic control is constructed by designing a rule base to implement a non-linear control strategy. The antecedent and consequent of each rule operate on the positions of input and output variables in predefined membership functions. These membership functions possess qualitative descriptions which generalize the notion of assigning a single degree to a specific response severity or corrective action level. Fuzzy logic controllers for active magnetic bearings are synthesized and designed for suppression of sudden imbalance vibration. The rule base is constructed to provide nonlinear resistance with respect to the position of the rotor and the control current of the electromagnetic iron coil.

The main objective of this paper is to develop robust controllers for maintaining magnetic bearing control periods of sudden high force such as blade loss, flywheel partial burst, component loss, etc., to demonstrate superiority of nonlinear fuzzy logic control over linear control. Simplification of the nonlinear fuzzy logic algorithm for practical implementation on a DSP digital controller or on an analog controller is also performed. Section 2 briefly describes the basic theoretical background of the non-linear property of active magnetic bearings as well as our experimental result as a supplement to the understanding of the features of the heteropolar magnetic bearings under bipolar operating conditions. In Section 3 the linear quadratic regulator is implemented to the

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linearized model of the system. Then fuzzy logic controllers based on Takagi-Sugeno are synthesized and designed with respect to different partitions of the input universe of discourse. Simulation results for each type of controllers are provided and their performance specifications can be compared accordingly. The effectiveness of the fuzzy logic controller is evaluated against the performance of the optimal linear controller. Finally in Section 4 conclusions are drawn and discussions with respect to the limitations and further study are presented.

## 2. Theory and Experiment

The electromagnetic force $f_c$ acting on the rotor made up of ferromagnetic materials ( $\mu_r >> 1$) at the bearings is non-linear in both the displacement $x$ and the active control current $i_c$ , i.e. neglecting fringing and metal path reluctance;

$$f_c = k[\frac{(i_0 + i_c)^2}{(s_0 - x)^2} - \frac{(i_0 - i_c)^2}{(s_0 + x)^2}] \tag{1}$$

where $i_0$ is the bias current and $s_0$ is the air gap of the magnetic bearing, and

$$k = 0.25\mu_0 n^2 A_a$$

is the magnetic force constant, in which

$$\mu_0 = 4\pi \times 10^{-7} \qquad (N/A^2)$$

is the permeability of vacuum, $n$ is the number of turns of the coil, and $A_a$ is the area of the cross section of the magnetic iron.

Fig. 1 shows the nonlinear property of the magnetic force with respect to the control current and the rotor displacement from its original position.

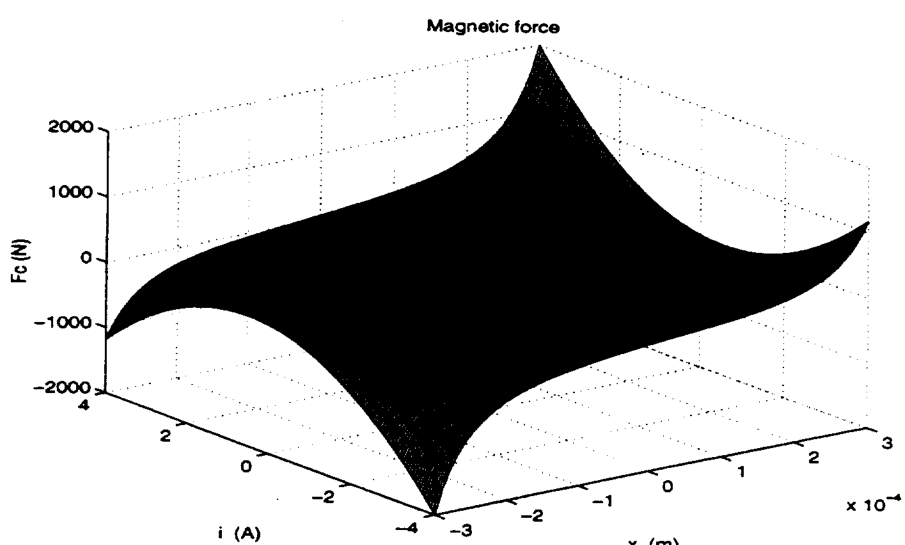


Fig. 1 Surface view of magnetic force

It can be seen from this figure that . as $i_c$ and $x$ increase, the control force $f_c$ increases very sharply and goes to infinity when $x$ reaches the air gap clearance.

Sudden imbalance in rotating machinery occurs in the case of blade loss, flywheel partial burst, component loss, etc., resulting in high vibrating force. The imbalance force $f_u$ can be expressed as follows:

$$f_u = me\omega^2 \sin(\omega t)$$

where $\omega$ is the spin rate of the shaft, $e$ is the offset of the center of gravity of the rotor due to imbalance, and $m$ is the equivalent mass.

The rigid rotor model of the complete system is now written as

$$m\ddot{x} = k[\frac{(i_0 + i_c)^2}{(s_0 - x)^2} - \frac{(i_0 - i_c)^2}{(s_0 + x)^2}] + me\omega^2 \sin(\omega t) \quad (2)$$

The objective of the controller is to generate appropriate control current $i_c$ fed to the coil of the electromagnetic iron to control the vibration. Basically there are two kinds of control schemes: linear control which is based on a linear approximation of the system, and non-linear control, which in our research, utilizes the fuzzy logic theory to stabilize the AMB system. Comparison of linear control and fuzzy logic control will be carried out and the superiority of the non-linear fuzzy logic control over linear control is demonstrated.

To fully understand the behavior of the active magnetic bearing, an experiment was carried out to study the relations of the flux density $B$ vs. control current $i_c$ for a heteropolar magnetic bearing. The objective of this experiment is to investigate the property of the magnetic bearing under bipolar working conditions. To show this, both positive current and negative current are fed to coils respectively and the flux density, which is measured through a Hall probe, is recorded. The experimental setup is shown in Fig. 2.

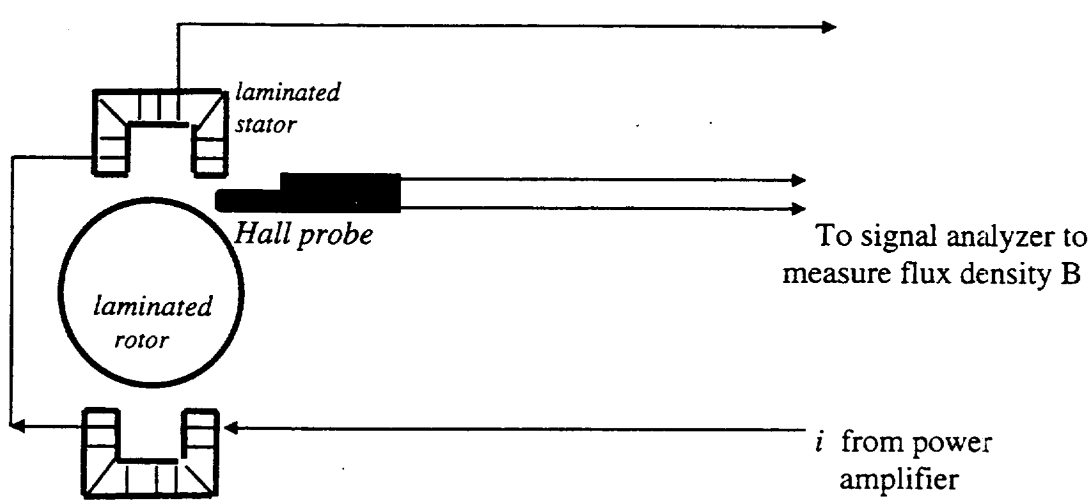


Fig. 2 Schematic diagram of the experimental setup

In this experiment, an operational amplifier (Kepco 72 v, 6A) generates a current $i$ fed into the coils of the heteropolar magnetic bearing. A Hall probe is used to sense the flux density $B$. The following table records the test result:

| *Current i (A)* | -3.1 | -3.0 | -2.5 | -2.2 | -1.6 | -1.0 | -0.76 | -0.5 | -0.26 | -0.11 |
|---|---|---|---|---|---|---|---|---|---|---|
| *Flux density (Tesla)* | 1.17 | 1.17 | 1.12 | 1.02 | 0.75 | 0.46 | 0.33 | 0.22 | 0.11 | 0.046 |

| *Current i (A)* | 0.257 | 0.51 | 0.75 | 1.0 | 1.58 | 2.41 | 3.62 | 3.72 |
|---|---|---|---|---|---|---|---|---|
| *Flux density (Tesla)* | -0.123 | -0.23 | -0.34 | -0.47 | -0.75 | -1.02 | -1.24 | -1.24 |

FIg. 3 shows the relation between $B$ and $i$.

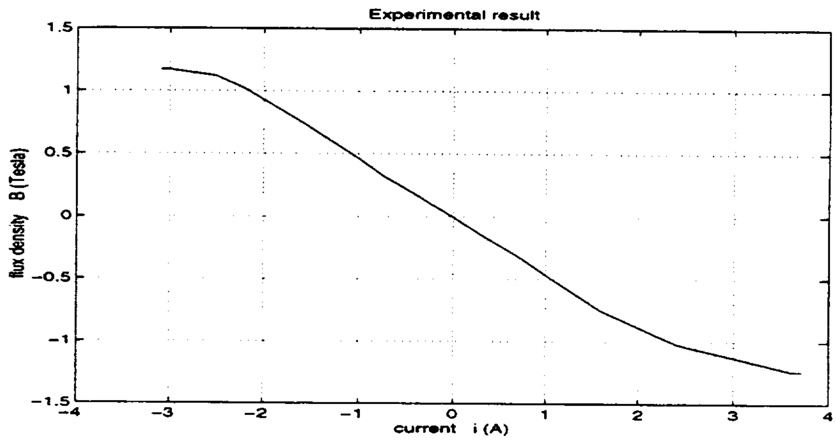


Fig. 3 Experimental result of flux-current relation

This result exhibits that for a heteropolar magnetic bearing, the reverse of the current polarity will have the same effect on the flux density , only with an opposite direction. We will use this result in our controller where negative input current $i = i_0 + i_c$ are allowed.

## 3. Controller Design and Simulation

### 3.1. Linear optimal controller

To apply a linear quadratic regulator to an active magnetic bearing system, first linearize the system around a chosen operating point. The system equation is rewritten as:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = k/m\left[\frac{(i_0+i_c)^2}{(s_0-x)^2} - \frac{(i_0-i_c)^2}{(s_0+x)^2}\right] \stackrel{\Delta}{=} f(x_1, x_2, i_c) \qquad (3)$$

where we denote $x_1 = x$.

For the linearization of the system, the following partial derivatives are derived:

$$\frac{\partial f}{\partial x_1} = 2k/m[\frac{(i_0+i_c)^2}{(s_0-x)^3} - \frac{(i_0-i_c)^2}{(s_0+x)^3}]$$

$$\frac{\partial f}{\partial x_2} = 0 \tag{4}$$

$$\frac{\partial f}{\partial i_c} = 2k/m[\frac{(i_0+i_c)}{(s_0-x)^2} - \frac{(i_0-i_c)}{(s_0+x)^2}$$

Therefore

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{\partial f}{\partial x_1} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{\partial f}{\partial i_c} \end{bmatrix} i_c + \begin{bmatrix} 0 \\ f(x_1, x_2, i_c)|_* - \frac{\partial f}{\partial x_1}|_* x_1^* - \frac{\partial f}{\partial i_c}|_* i_c^* \end{bmatrix} \tag{5}$$

The above equation is in control canonical form. We can further write it in a more compact form:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u + \mathbf{c} \tag{6}$$

where $u$ denotes the control current $i_c$.

The general cost function $J$ is

$$J = \int_0^\infty (\mathbf{x}^T\mathbf{Q}\mathbf{x} + \mathbf{u}^T\mathbf{R}\mathbf{u})dt \tag{7}$$

where **Q** and **R** are weight matrices.

Solve the algebraic Riccati equation for **P**:

$$\mathbf{PA} + \mathbf{A}^T\mathbf{P} - \mathbf{PBR}^{-1}\mathbf{B}^T\mathbf{P} + \mathbf{Q} = \mathbf{0} \tag{8}$$

and the state feedback gain matrix is

$$\mathbf{K} = \mathbf{R}^{-1}\mathbf{B}^T\mathbf{P} \tag{9}$$

Therefore, the closed loop system is

$$\dot{\mathbf{x}} = (\mathbf{A} - \mathbf{BK})\mathbf{x} \tag{10}$$

***Simulation:***

As an example, the parameters used in the simulation are as follows:

$\omega$=3000 (rpm), m=1.0 (kg), $s_o$=0.4 (mm), n=150, $A_a$=200 ($mm^2$), $i_o$=5 (A).

Choose the operating point at $i_c^* = 0, x_1^* = 0$, then

$$A = \begin{bmatrix} 0 & 1 \\ 1561.95 & 0 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 124.96 \end{bmatrix}$$

Choose the weight matrices as

$$Q = \begin{bmatrix} 20 & 0 \\ 0 & 0.1 \end{bmatrix}, R = 15$$

then the optimal feedback gains are

$$k_p = 20.05,\ k_d = 0.64$$

With the LQR applied, the system converges until the offset $e$ reaches $86E\text{-}4(m)$. When $e \geq 86E\text{-}4(m)$, the system diverges. Fig. 4 shows the simulation result.

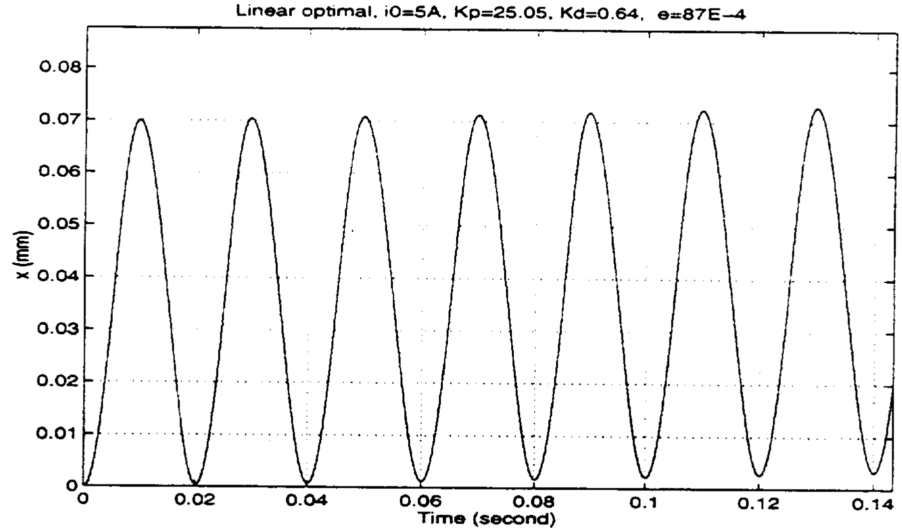


Fig. 4 Response of linear optimal control when offset $e=87E\text{-}4$

## 3.2 Fuzzy logic control

### (1) Theoretical background

The fuzzy logic controller design is based on piece-wise linearization of the system. Pole placement method is applied to each subsystem to obtain the feedback gains. It has been proved that, if the poles of each subsystem is placed at the same location, the global non-linear system will be stable.

Generally, a SISO system can be described as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\cdots\ \cdots \\ \dot{x}_n &= f(x_1, x_2, \ldots, x_n, F) \end{aligned} \tag{11}$$

where $F$ is the control force.

We introduce $l$ operating points around which the system (11) is linearized as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ & \cdots & & \cdots & \\ -a_1^i & -a_2^i & -a_3^i & \cdots & -a_n^i \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} u^i + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ d^i \end{bmatrix}, \quad i = 1,2,\ldots,l \tag{12}$$

or in a more compact form

$$\dot{\mathbf{x}} = \mathbf{A}^{i}_{op}\mathbf{x} + \mathbf{b}u^{i} + \mathbf{d}^{i} \ , \ i = 1,2,\ldots,l$$

where the parameters $a$ and $d$, appropriately indexed, are obtained as follows:

$$\begin{aligned} f(x_1,x_2,\ldots,x_n,F) &\approx f(x_1^{i*},x_2^{i*},\ldots,x_n^{i*}) + \left.\frac{\partial f}{\partial x_1}\right|_* (x_1 - x_1^{i*}) + \left.\frac{\partial f}{\partial x_2}\right|_* (x_2 - x_2^{i*}) + \ldots \\ &+ \left.\frac{\partial f}{\partial x_n}\right|_* (x_n - x_n^{i*}) + \left.\frac{\partial f}{\partial F}\right|_* \Delta F^i \\ &= \left.\frac{\partial f}{\partial x_1}\right|_* x_1 + \left.\frac{\partial f}{\partial x_2}\right|_* x_2 + \ldots + \left.\frac{\partial f}{\partial x_n}\right|_* x_n + \left.\frac{\partial f}{\partial F}\right|_* \Delta F^i \\ &- \left(\left.\frac{\partial f}{\partial x_1}\right|_* x_1^{i*} + \left.\frac{\partial f}{\partial x_2}\right|_* x_2^{i*} + \ldots + \left.\frac{\partial f}{\partial x_n}\right|_* x_n^{i*}\right) + f(x_1^{i*},x_2^{i*},\ldots,x_n^{i*}) \\ &\triangleq [-a_1^i,-a_2^i,\ldots,-a_n^i][x_1,x_2,\ldots x_n]^T + u^i + d^i \quad ,i = 1,2,\ldots l \end{aligned}$$

and * denotes values of the relevant variables at the given operating points.

Therefore, for $i=1,2,\ldots,l$, we have

$$\begin{aligned} d^i &= -\left(\left.\frac{\partial f}{\partial x_1}\right|_* x_1^{i*} + \left.\frac{\partial f}{\partial x_2}\right|_* x_2^{i*} + \ldots + \left.\frac{\partial f}{\partial x_n}\right|_* x_n^{i*}\right) + f(x_1^{i*},x_2^{i*},\ldots,x_n^{i*}) \\ a_k^i &= \left.\frac{-\partial f}{\partial x_k}\right|_* \\ u^i &= \left.\frac{\partial f}{\partial F}\right|_* \Delta F^i, \ \ k = 1,2,\ldots,n \\ \Delta F^i &= F^i - F^{i*} \ , F^{i*} = 0 \end{aligned} \tag{13}$$

Here $F^i$ is the actual control architecture input to the plant. The last two expressions in (3) lead to $F^i = u^i \Big/ \left.\frac{\partial f}{\partial F}\right|_*$ where $u^i$ will be drawn by the fuzzy inference process described below.

Based on the piece-wise linear model (12), state feedback control is employed to stabilize the given non-linear system:

$$u^i = -k_1^i x_1 - k_2^i x_2 - \ldots - k_n^i x_n + k_0^{i*} \ , \ i = 1,2,\ldots,l \tag{14}$$

Motivated by the Takagi-Sugeno model, the fuzzy rule base takes the following form:

Rule i ($i=1,2,\ldots,l$):

*If* $x_1$ *is* $S_1^i$ *and* $x_2$ *is* $S_2^i$ *and...* $x_n$ *is* $S_n^i$ *and* $u^i$ *is* $U^i$, *then*

$$\dot{\mathbf{x}} = \mathbf{A}_{op}^i \mathbf{x} + \mathbf{b}u^i + \mathbf{d}^i$$

*and the control for the next step is*

$$u^i = -k_1^i x_1 - k_2^i x_2 - \ldots - k_n^i x_n + k_0^{i*}$$

where $S_j^i$ and $U^i$ are fuzzy sets centered at the $i$th operating point for variable $x_j$ and control $u$.

With the implementation of the state feedback control law (14), the resulting closed loop system can be written, in terms of the Takagi-Sugeno architecture, as

Rule i ($i=1,2,\ldots,l$):

*If* $x_1$ *is* $S_1^i$ *and* $x_2$ *is* $S_2^i$ *and...* $x_n$ *is* $S_n^i$ *and* $u^i$ *is* $U^i$, *then*

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ & \cdots & & \cdots & \\ -(a_1^i + k_1^i) & -(a_2^i + k_2^i) & -(a_3^i + k_3^i) & \cdots & -(a_n^i + k_n^i) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ k_0^{i*} + d^i \end{bmatrix}$$

It can be seen that one of the design objectives is to use $k_0^{i*}$ to cancel the linearized constant $d^i$, that is

$$k_0^{i*} = -d^i = \left( \left.\frac{\partial f}{\partial x_1}\right|_* x_1^{i*} + \left.\frac{\partial f}{\partial x_2}\right|_* x_2^{i*} + \ldots + \left.\frac{\partial f}{\partial x_n}\right|_* x_n^{i*} \right) - f(x_1^{i*}, x_2^{i*}, \ldots, x_n^{i*})$$

Then we have for the $i$th subsystem the control canonical form:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ & \cdots & & \cdots & \\ -(a_1^i + k_1^i) & -(a_2^i + k_2^i) & -(a_3^i + k_3^i) & \cdots & -(a_n^i + k_n^i) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}. \tag{15}$$

Using the weighted sum to defuzzify the system, we have the closed loop system described as:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \vdots\\ \dot{x}_n\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & \cdots & 0\\ 0 & 0 & 1 & \cdots & 0\\ & \cdots & & \cdots & \\ \dfrac{-\sum_{i=1}^{l}(w^i a_1^i+w^i k_1^i)}{\sum_{i=1}^{l} w^i} & \dfrac{-\sum_{i=1}^{l}(w^i a_2^i+w^i k_2^i)}{\sum_{i=1}^{l} w^i} & \dfrac{-\sum_{i=1}^{l}(w^i a_3^i+w^i k_3^i)}{\sum_{i=1}^{l} w^i} & \cdots & \dfrac{-\sum_{i=1}^{l}(w^i a_n^i+w^i k_n^i)}{\sum_{i=1}^{l} w^i}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ \vdots\\ x_n\end{bmatrix}. \tag{16}$$

where $w^i$ is the grade of membership of the input **x** with respect to the *i*th rule.

Now using triangular shaped membership functions with the following property:

$$\sum_{i=1}^{l} w^i = 1 \ , \tag{17}$$

Eq. (16) is simplified to

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \vdots\\ \dot{x}_n\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & \cdots & 0\\ 0 & 0 & 1 & \cdots & 0\\ & \cdots & & \cdots & \\ -\sum_{i=1}^{l}(w^i a_1^i+w^i k_1^i) & -\sum_{i=1}^{l}(w^i a_2^i+w^i k_2^i) & -\sum_{i=1}^{l}(w^i a_3^i+w^i k_3^i) & \cdots & -\sum_{i=1}^{l}(w^i a_n^i+w^i k_n^i)\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ \vdots\\ x_n\end{bmatrix}. \tag{18}$$

This system model is in *control canonical form.* Therefore it is easy to apply linear control design techniques to stabilize the system and to obtain satisfactory performance.

Pole placement method is recommended to perform the design of the piece-wise linear subsystems. By choosing suitable feedback gains $k_j^i$ for $i=1,2,...,l$ and $j=1,2,...,n$ such that all linear subsystems (15) have the *same set of eigenvalues*, a globally stable fuzzy controller can be obtained. In this design, the closed loop system behaves *uniformly* across the entire operating range.

**(2) Fuzzy logic controller design and simulation**

We choose 5 operating points for the displacement and the control current respectively, which results in a fuzzy logic controller of Takagi-Sugeno architecture with 25 rules.

The input membership functions are of triangular and trapezoidal shapes centered at the 5 operating points of each input universe of discourse. Fig. 5 depicts the input membership functions.

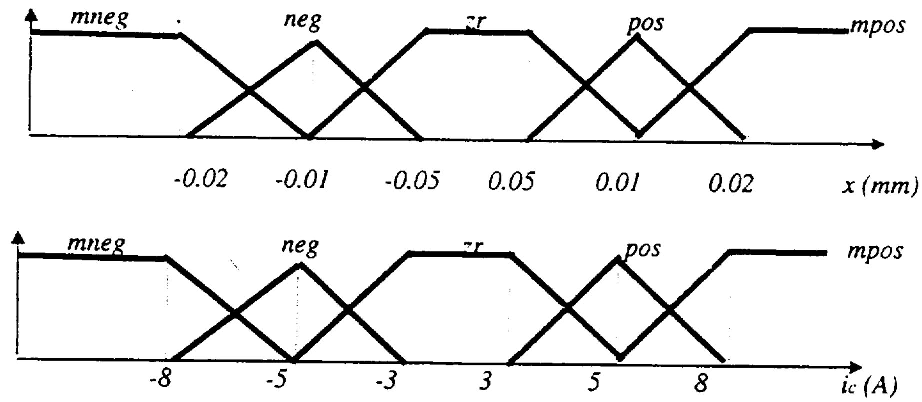


Fig. 5 Membership functions of input spaces

The output membership functions are formulated in terms of pole placement method for a piece-wisely linearized system where the closed loop poles are placed at $P_1=(60+6j)$, $P_2=(60-6j)$.

The rules base is formulated as:

*Rule i : If x is $X_i$ and $i_c$ is $U_i$*
*Then the control current is $-k_{pi}x - k_{di}\dot{x} + k_{0i}$*
*(i=1,2,...,25)*

where $X_i$ and $U_i$ are membership functions in the $x$ and $i_c$ input spaces respectively. The control coefficients $k_{pi}$ and $k_{di}$ are obtained via pole placement method for each linear subsystem.

Fig.6 shows the simulation results with maximum unbalance offset $e=137E\text{-}4\ (m)$

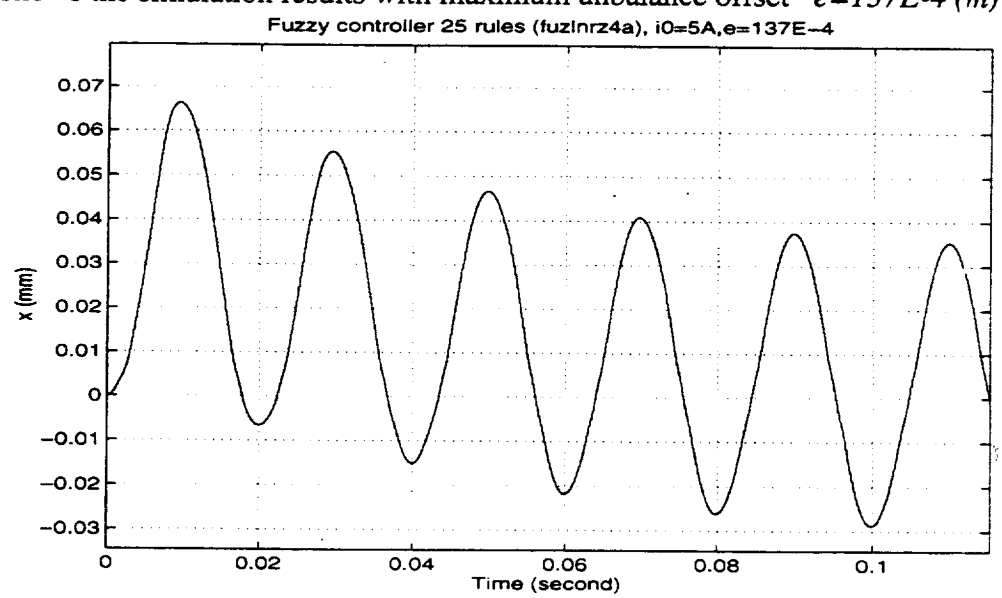


Fig. 6 Response in the presence of imbalance (25 rules)

To make the fuzzy controller capable of being implemented in a real time DSP controller, further simplification of the rule base is made to reduce the total number of rues from 25 to only 9, with little sacrifice of the maximum allowable offset from *e=137E-4 (m)* to *e=130E-4 (m)*. The membership functions in this case are shown in Fig. 7.

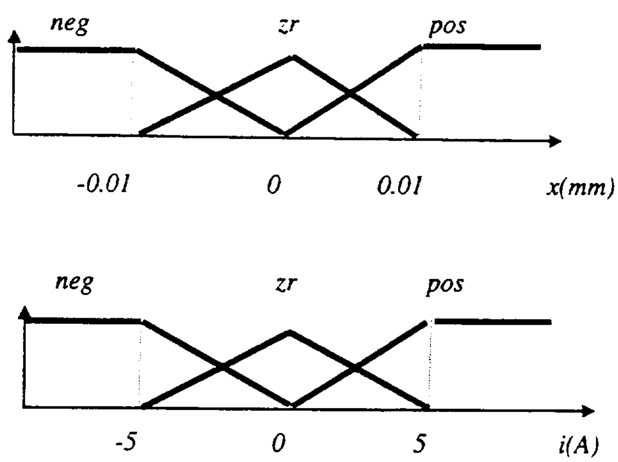


Fig. 7. Membership functions of input spaces

The output membership functions are formulated in a similar way as before and the poles of each linear subsystem are placed at the same locations: $P_1=(60+6j)$, $P_2=(60-6j)$

Fig. 8 shows that the system converges at a maximum unbalance offset of $e=130E\text{-}4$ (*m*)

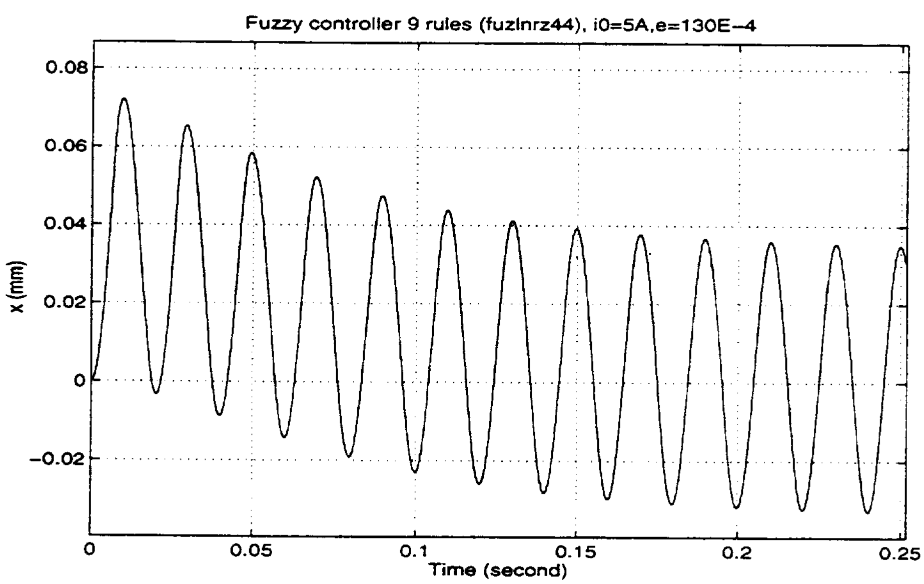


Fig. 8 Response in the presence of unbalance (9 rules)

## 4. Conclusion

This paper develops the use of fuzzy logic controller of active magnetic bearings for suppression of sudden imbalance generated vibration. Fuzzy logic controllers in terms of Takagi-Sugeno architecture are studied and designed. Their superiority over the best linear controller, i.e. linear quadratic regulators, is demonstrated. Moreover, the simplification of fuzzy logic controller by reducing the number of membership functions is performed, which makes the implementation on real time DSP control practical. Results show significant improvement in maximum controllable imbalance, resulting shaft excursions versus optimal linear control. In summary the largest stable unbalance eccentricity is 87e-4 for the LQR based control and 137e-4 for the non-linear fuzzy logic controller.

The work in this paper is based on rigid rotor model with lumped mass and adjustable mass eccentricity to represent the imbalance. Further study for an active magnetic bearing system which includes flexible shaft, non-collocated sensor and actuator, dynamics of the electronic processing circuit and the eddy current effect will be covered in our later research in terms of finite element studies.

**Acknowledgment**
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## References

[1] C. Kim, A. B. Palazzolo *et al.* Eddy current effects on the design of rotor-magnetic bearing systems, *Transactions of the ASME,* Vol. 117, June 1995.
[2] P.Tang, A. B. Palazzolo *et al.* An electromechanical simulation method for the active vibration control of a magnetic bearing supported rotor. *Proceedings of the 1993 International Gas Turbine and Aeroengine Congress and Exposition,* Cincinnati, Ohio, May 24-27, 1993
[3] Shuliang Lei and R. Langari. An approach to synthesis and approximation of stable fuzzy logic controllers, *Fifth IEEE International Conference on Fuzzy Systems,* New Orleans, Sept. 8-11, 1996.
[4] K. Tanaka and M. Sugeno. Stability analysis and design of fuzzy control systems, *Fuzzy Sets and Systems,* 45 (1992) 135-156 North-Holland.
[5] K. Tanaka and M. Sano. A robust stabilization problem of fuzzy control systems and its application to backing up control of a truck-trailer, *IEEE Transactions on Fuzzy Systems,* Vol. 2, No. 2 May 1994.

[6] T. Takagi and M. Sugeno. Fuzzy identification of systems and its application to modeling and control, *IEEE Trans. Syst. Man Cyber.* Vol. SMC-15, No. 1, Jan/Feb, 1985.
[7] A. El Hajjaji and A. Rachid. Analytic formulation of linguistic rules for fuzzy controller, *Fuzzy Sets and Systems,* 73 (1995) 219-225.
[8] R. Langari and Shuliang Lei. Hierarchical fuzzy control of a double inverted pendulum, *Technical Report,* Center of fuzzy logic, robotics and intelligent systems research, Texas A&M University, Collage Station, TX 77843.
[9] Gerhard Schwitzer *et al. Active magnetic bearings,* Hochschulverlag AG an der ETH Zurich,1994
[10] J. Doyle, B. Francis and Tannenbaum. *Feedback control theory*, Macmillan, 1991.
[11] D. Filev. Polynomial approach to the synthesis of stable fuzzy systems, *1996 Biennial Conference of the North American Fuzzy Information Processing Society - NAFIPS,* June 1996.
[12] M. Gopal. *Modern Control System Theory*, John Wiley & Sons, 1993.
[13] M. Anderson. Lyapunov optimal feedback control of a nonlinear inverted pendulum, *Tran. AMSE, Vol. 111*, p. 554, Dec. 1989.
[14] J. Cruz, Jr. Feedback in systems, *Feedback systems,* McGraw-Hill Co., New York, 1972
[15] Terano. *Fuzzy Systems Theory and Its Applications*, Academic Press, 1987.