ITERATIVE CONTROL-RELEVANT IDENTIFICATION AND CONTROLLER ENHANCEMENT OF A MIMO MAGNETIC BEARING SYSTEM

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SUMMARY

The magnetic bearing systems are intrinsically unstable, and need the feedback control of electromagnetic forces with measured displacements. So the controller design plays an important role in constructing high performance magnetic bearing system. In case of magnetic bearing systems, the order of identified model can be high because of unknown dynamics included in closed loop systems - such as sensor dynamics, actuator dynamics - and non-linearity of magnetic bearings. "Identification for control" – joint optimization of system identification and controller design - is proposed to get the limited-order model which is suited for the design of high-performance controller. We applied the joint identification/controller design scheme to MIMO rigid rotor system supported by magnetic bearings. First, we designed controller of a nonlinear simulation model of MIMO magnetic bearing system with this scheme and proved its feasibility. Then, we performed experiments on MIMO rigid rotor system supported by magnetic bearings, and the results shows that the performance of the closed-loop system is gradually improved during the iteration.

INTRODUCTION

Active magnetic bearing (AMB) systems have been widely used for their unique advantages such as: non-contact, lubricant-free operation, the possibility of high rotational speed, and the controllability of the bearing characteristics. Since AMB systems always require the feedback control of magnetic force for stable levitation of a rotor, the characteristics of AMB system is mainly determined by the feedback controller. Therefore, the design of controller plays a significant role in building high performance AMB system.

Since nominal models of AMB systems are generally constructed ignoring the non-linearity of magnetic force, dynamics of sensors and actuators, and the spill-over of a rotor, the identification of AMB systems is essential to enhance the performance of whole system. So far, identifications of AMB systems have been performed to get experimental values of specific physical parameters, which were current stiffness and displacement stiffness of AMB or poles of open-loop plant. However, the estimation of system parameters in the sense of controller design law is required to achieve better performance.

In these days, the control-relevant identification has become more and more important area of control researchers. An idea has been raised that the model identification and controller design is not performed independently. This has **led** to the iterative design of model-based controller. While conventional identification methods put emphasis upon getting accurate model, the objective of the control-relevant identification is to get the nominal model that is suitable for the design of a high-performance controller. Various control-relevant identification schemes [1-2] were proposed recently and achieved good results in many industrial applications [3-4].

This paper presents the iterative identification and controller design of a MIMO rigid rotor AMB system. A theoretical model of the system is derived, and the iterative scheme based on LQG criterion is suggested. We verified the feasibility of this scheme through simulations, and performed experiments on a MIMO rigid rotor AMB system. The performance of the closed-loop system is enhanced successively during iterative designs.

MATERIAL AND METHOD

Active Magnetic Bearing

The magnetic force equation near the operating point is approximated by

$$F = k_i \cdot i + k_z \cdot z \tag{1}$$

$$k_i = \frac{\mu_0 N_c^2 A}{g_0^2} i_b, \quad k_z = \frac{\mu_0 N_c^2 A}{g_0^3} i_b^2 \tag{2}$$

where displacement is given by z, input current by i, permittivity in vacuum by μ_0, the number of coil turns by N_c, the area of actuator pole by A, nominal gap by g_0, and bias current by i_b, respectively.

A configuration of rigid rotor AMB model is shown in Figure 1 and the theoretical model of a rigid rotor

AMB system using magnetic force equation (1) is described as follows

$$M_b \ddot{z}_b + C_b \dot{z}_b + K_b z_b = f_b \tag{3}$$

$$M_b = \begin{bmatrix} (ml_2^2 + J_x)/(l_1+l_2)^2 & (ml_1l_2 - J_x)/(l_1+l_2)^2 & 0 & 0 \\ (ml_1l_2 - J_x)/(l_1+l_2)^2 & (ml_1^2 + J_x)/(l_1+l_2)^2 & 0 & 0 \\ 0 & 0 & (ml_2^2 + J_y)/(l_1+l_2)^2 & (ml_1l_2 - J_y)/(l_1+l_2)^2 \\ 0 & 0 & (ml_1l_2 - J_y)/(l_1+l_2)^2 & (ml_1^2 + J_y)/(l_1+l_2)^2 \end{bmatrix}$$

$$C_b = \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \\ 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \end{bmatrix} J_z\Omega,\quad K_b = \begin{bmatrix} -K_{x1} & 0 & 0 & 0 \\ 0 & -K_{x2} & 0 & 0 \\ 0 & 0 & -K_{y1} & 0 \\ 0 & 0 & 0 & -K_{y2} \end{bmatrix},\quad \begin{aligned} z_b &= [x_1\ \ x_2\ \ y_1\ \ y_2]^T \\ f_b &= [K_{ix1}i_{x1}\ \ K_{ix2}i_{x2}\ \ K_{iy1}i_{y1}\ \ K_{iy2}i_{y2}]^T \end{aligned}$$

where m denotes rotor mass, J_p, J_x, J_y polar and diametrical moment of inertia, K_b displacement stiffness of AMB, Ω rotating speed, f_b bearing force and z_b displacement at bearing location .

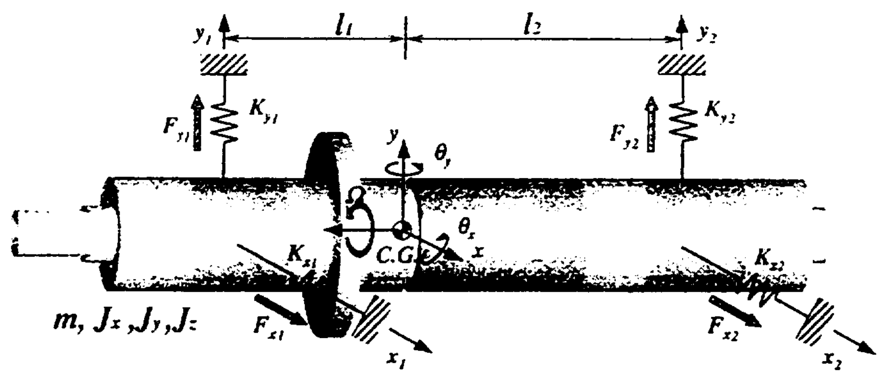

Figure 1. The model of a rigid rotor AMB system

Ignoring the gyroscopic effects, theoretical model above has 4 rigid body modes and these modes give 4 double poles in real-axis, which are symmetric to imaginary-axis.

Control-relevant identification

The basic principle of the control-relevant identification problem is described by the following triangle inequalities, as considered by Schrama [5]:

$$\|J(G_0,C)\| \leq \|J(\hat{G},C)\| + \|J(G_0,C) - J(\hat{G},C)\| \tag{4}$$

$$\|J(G_0,C)\| = J^{ach} : \text{the achieved performance} \tag{4-a}$$

$$\|J(\hat{G},C)\| = J^{des} : \text{the desired performance} \tag{4-b}$$

$$\|J(G_0,C) - J(\hat{G},C)\| = J^{id} : \text{the identification criterion} \tag{4-c}$$

where G_0 means true plant. $\hat{G}$ estimated model of true plant. and C the controller.

Basically. the objective of all controller design is the minimization of cost function (4-a). which is the left part of equation (4). But. that is unrealizable since the exact description of a true plant G_0 is not available. The aim of the control-relevant identification is the minimization of the right part of equation (4). which means the upper bound of the achieved performance cost (4-a). It can be done by the minimization of both (4-b) and (4-c), but the simultaneous minimization of equations (4-b) and (4-c) is impossible by ordinary identification and control-design method. Therefore. separate optimizations over $\hat{G}$ and over C are performed in an iterative way. The model and controller of i^{th} iteration are obtained by

$$\hat{G}_{i+1}(\theta) = \arg\min_{\theta} \| J(G_0,C_i) - J(\hat{G}(\theta),C_i) \|, \quad C_{i+1} = \arg\min_{C} \| J(G_{i+1}(\theta),C) \| \tag{5}$$

where θ denotes the parameter vector that is estimated in the identification procedure and i means the iteration step.

The iterative identification and control design procedure described above will be modified for LQ criterion. The state-space model ofanLQG controller is described as

$$\dot{\hat{x}} = A_c\hat{x} + Hy, \quad u = -K\hat{x} \tag{6}$$

where $\hat{x}$ is estimated state. y system output, H Kalman filter gain. K LQ gain and A_c augmented system matrix. individually.

The control design procedure for LQ criterion minimizes a following cost function.

$$J_{LQG} = \lim_{N \to \infty} \frac{1}{N} \sum_{t=0}^{N-1} E\left\{\hat{x}^T Q \hat{x} + u^T R u\right\} \tag{7}$$

The identification procedure with LQG criterion minimizes

$$J^{id} = \lim_{N \to \infty} \frac{1}{N} \sum_{t=0}^{N-1} \{W_z(z)(y(t) - \hat{y}(t,\theta))\}^T \{W_z(z)(y(t) - \hat{y}(t,\theta))\} \tag{8}$$

, which is equivalent to equation (9) in frequency domain

$$J^{id} = \left\| W(j\omega)(G_0(j\omega) - \hat{G}(j\omega,\theta)) \right\|_2^2 \tag{9}$$

$W(s)$ is a control-relevant weighting function in frequency domain and is calculated as

$$W(s) = W_{CR}(s)(I + \hat{G}(s,\theta)C(s))^{-1} \tag{10}$$

where $W_{CR}{}^T(s)W_{CR}(s) = H^T(sI - A_c)^{-T}(Q + K^T R K)(sI - A_c)^{-1} H$ can be calculated by Schur decomposition.

A flow chart of the iterative identification and control design procedure is described in Figure 2.

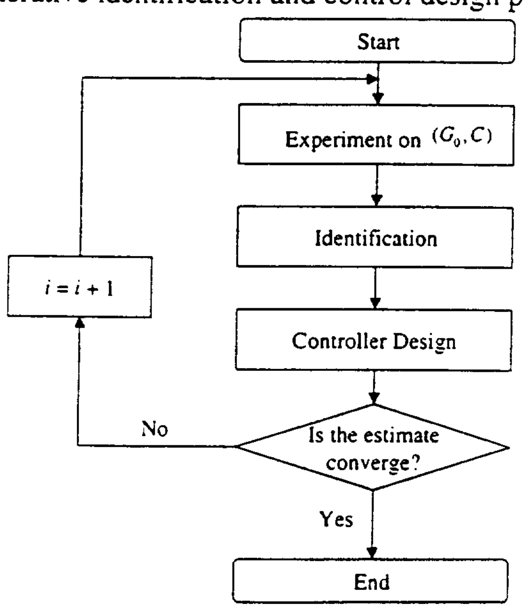

Figure 2. A flow chart of the iterative identification and control design procedure

Frequency domain identification

The averaged frequency spectra of input and output signals are used to get the frequency response function (FRF) of a plant. Experiments are done with 8 different excitation signals. The frequency spectra of input and output signals are $U^{(i)}(j\omega) \in C^{nu\times 1}$, $Y^{(i)}(j\omega) \in C^{ny\times 1}, i = 1,2,\cdots 8$ and nu, ny the number of input and output, respectively. The entire input/output data are defined as

$$Y(j\omega) = [Y^{(1)}(j\omega)\, Y^{(2)}(j\omega) \cdots Y^{(8)}(j\omega)\,]^T \in C^{ny\times 8} \tag{11}$$

$$U(j\omega) = [U^{(1)}(j\omega)\, U^{(2)}(j\omega) \cdots U^{(8)}(j\omega)\,]^T \in C^{nu\times 8} \tag{12}$$

Then non-parametric transfer function matrix $G(j\omega) \in C^{ny\times nu}$ can be obtained by [6]

$$G(j\omega) = Y(j\omega)U(j\omega)^{+} \tag{13}$$

where superscript + denotes the Moore-Penrose pseudo-inverse.

Frequency-domain curve fitting algorithm using polynomial matrix fractional descriptions (MFD) is used for the identification of MIMO system. The transfer function $\hat{G}(\omega,\theta)$ is described via left MFD.

$$\hat{G}(\omega,\theta) = A(\omega,\theta)^{-1} B(\omega,\theta) \tag{14}$$

A and B denote polynomial matrices as follows.

$$B(\xi,\theta) = \sum_{i=d}^{d+o_b-1} B_i\, p(\xi)^i, \quad B_i \in R^{ny\times nu}, \quad A(\xi,\theta) = I_{ny\times ny} + \xi \sum_{i=d}^{o_a} A_i\, p(\xi)^i, \quad A_i \in R^{ny\times ny} \tag{15}$$

where polynomial matrices are written in terms of a (scalar) polynomial basis

$$p(\xi) \equiv p_0 + p_1\xi + p_2\xi^2 + \cdots + p_n\xi^n \tag{16}$$

The parameters are estimated via minimizing 2-norm of each element of weighted error matrix

$$\hat{\theta} = \arg \min_{\theta\in\Re} \sum \left\| E^{ij}(\theta) \right\|_F^2 \tag{17}$$

The $(i,j)^{th}$ component of weighted error matrix is parameterized as follows.

$$E^{ij}(\omega,\theta) = \left[W_o(\omega)A(\omega,\theta)^{-1}\right]^{il}\left[G^{lm}(\omega) - \bar{A}^{lq}(\omega,\theta)G^{qm}(\omega) - B^{lm}(\omega,\theta)\right]W_i^{mj}(\omega) \quad (18)$$

where $A(\xi,\theta) = I_{n\times n} - \bar{A}$, $\bar{A}^{ij}(\omega_k) = \phi(\omega_k)\cdot\left[A_0^{ij} \quad A_1^{ij} \quad \cdots \quad A_{n_a}^{ij}\right]^T$, $B^{ij}(\omega_k) = \psi(\omega_k)\cdot\left[B_0^{ij} \quad B_1^{ij} \quad \cdots \quad B_{n_b}^{ij}\right]^T$

$\psi(\omega_k) = \left[p^0(\xi(\omega_k)) \quad p^1(\xi(\omega_k)) \quad \cdots \quad p^{n_b}(\xi(\omega_k))\right], \phi(\omega_k) = -\xi(\omega_k)\left[p^0(\xi(\omega_k)) \quad p^1(\xi(\omega_k)) \quad \cdots \quad p^{n_a}(\xi(\omega_k))\right]$

The equation (17) above are converted to general least squares form.

$$\hat{\theta} = \arg\min_{\theta\in\mathfrak{R}} \sum \| Y_{wt} - P_{wt}\cdot\theta \|_F^2 \quad (19)$$

Y_{wt} is denoted by

$$Y_{wt}(\omega_k) = \begin{bmatrix} \{Y_w^{1\bullet}(\omega_k)\}^T \\ \vdots \\ \{Y_w^{p\bullet}(\omega_k)\}^T \end{bmatrix}, \text{ where } Y_w^{ij} = \left[W_{ot}^{i1} \quad W_{ot}^{i2} \quad \cdots \quad W_{ot}^{in}\right]\cdot\begin{bmatrix} G^{11} & G^{12} & \cdots & G^{1n} \\ G^{21} & G^{22} & \cdots & G^{2n} \\ \vdots & \vdots & \ddots & \vdots \\ G^{p1} & G^{p2} & \cdots & G^{pn} \end{bmatrix}\cdot\begin{bmatrix} W_i^{1j} \\ W_i^{2j} \\ \vdots \\ W_i^{nj} \end{bmatrix} \quad (20)$$

and P_{wt} is

$$P_{wt}(\omega_k) = \begin{bmatrix} \{P_w^{1\bullet}(\omega_k)\}^T \\ \vdots \\ \{P_w^{p\bullet}(\omega_k)\}^T \end{bmatrix} = W_{ot} \otimes \left[(W_i)^T \otimes \psi \quad \{G\cdot W_i\}^T \otimes \phi\right] \quad (21)$$

where $W_{ot}(\omega_k) = W_o(\omega_k)A(\xi,\theta)^{-1}$, $\{P_w^{i\bullet}(\omega_k)\}^T = W_{ot}^{i\bullet} \otimes \left[(W_i)^T \otimes \psi \quad \{G\cdot W_i\}^T \otimes \phi\right]$, $\otimes$ is Kronecker product.

First, SK algorithm based on QR factorization [7-8] estimates the approximate value of system parameter. Then, using this approximate estimate as the initial value, nonlinear least square problem is solved with MATLAB optimization toolbox function [9].

RESULT

Simulations

Eight different excitation signals are generated using multi-sine of random phases. In general, the amplitude of an excitation signal should be as large as possible to ensure sufficient signal-to-noise ratio(SNR). Except safety or economic reasons, there are two important limitations of input amplitude on AMB – linearity region and slew rate of magnetic actuator. The slew rate of magnetic bearing rotor system can be described by (22)

$$\frac{di}{dt} = \frac{2g_0}{\mu_0 N_c A}\left[V_S - (i_b + i)(R_c + R_f + R_{FET})\right] \tag{22}$$

where V_S is supply voltage, R_c coil resistance, R_f the resistor connected to FET, and R_{FET} FET resistance.

A nonlinear AMB rotor model is constructed by using MATLAB Simulink and the proposed control-relevant identification scheme is applied through the flow chart of Figure 2. Estimated model parameter converges as iterations are repeated. Although **identified model** differs from theoretical model, it can be considered as the approximated model that is suitable in the control-design point of view. The eigenvalues of identified plant are given in Table 1 and the values of cost function J^{uch} and J^{id} at each iteration are shown in Table 2.

Table 1. Eigenvalue of identified system matrix $\hat{A}$ (Simulation)

	1st eigenvalue	2nd eigenvalue	3rd eigenvalue	4th eigenvalue
Nominal	627.68	-627.58	493.05	-493.05
Step1	649.27	-737.29	550.50	-608.09
Step2	648.83	-791.93	552.66	-633.86
Step3	648.83	-791.93	552.63	-633.87

Table 2. The cost function value (Simulation)

	Step1	Step2	Step3
J^{uch}	4.69	4.58	4.58
J^{id}	0.63	0.58	0.58

The parameters of identified plant converge in 3^{rd} step, and the cost function value of the achieved closed-loop performance J^{ach} decreases. This results from the reduction of the identification cost function, J^{id}.

The number of iteration is only two, which is relatively small in general control-relevant identification scheme. This is because control-relevant schemes are effective in case that high-order plant should be approximated to low-order model for the design of reduced-order controller. The simulated plant can be sufficiently described as the model of 2nd order, so the decrease of cost function is not drastic. But the enhancement of performance still can be shown.

Experiment

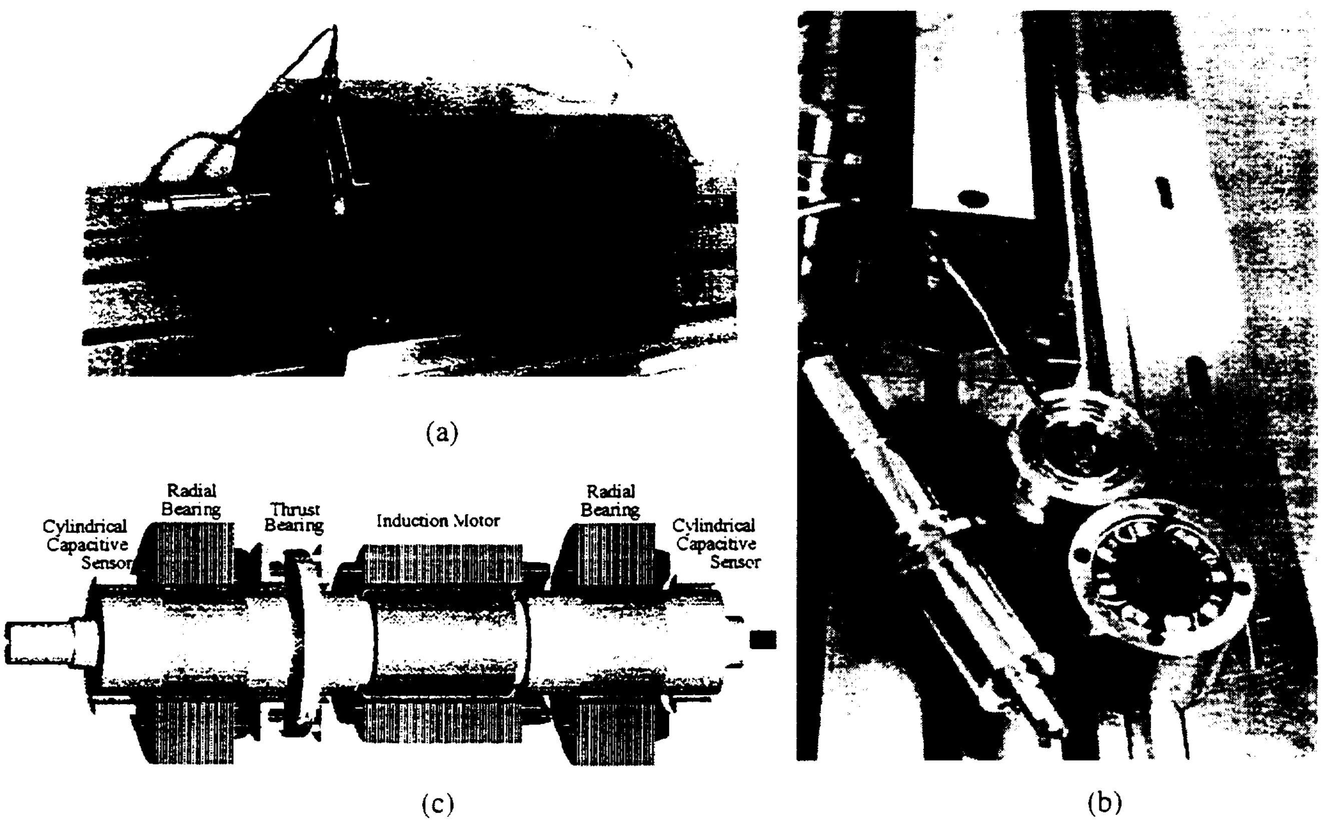

Figure 3. Rigid rotor AMB system (a) assembly (b) elements (c) schematics

A rigid rotor AMB experimental setup consists of two AMB units with built-in cylindrical capacitive sensor, thrust bearing, rigid rotor, and induction motor, as shown in Figure 3 [10]. The AMB unit is designed **for ease of assembly like a commercial bearing. The cylindrical capacitive sensor and** electro-magnetic actuator are embedded in the housing of the AMB unit as shown in Figure 3(b). In case of wiring, small connectors are used for the convenience of assembling and machining. The rotor has outer radius of 39*mm* at the AMB unit and the maximum rotational speed is 60000 rpm. The water jacket is embedded around 3kW induction motor. TMS320C44 40MHz DSP board with 8 ch. 16 bit, 200kHz AD/DA is used.

Experiments are performed through the same procedure as simulation. The eigenvalues of the identified system matrix are shown in Table 3, and cost function values are shown in Table 4.

Table 3. Eigenvalue of identified system matrix $\hat{A}$ (Experiment)

	1st eigenvalue	2nd eigenvalue	3rd eigenvalue	4th eigenvalue
Nominal	627.58	-627.58	493.05	-493.05
Step1	629.12	-618.82	559.66	-565.99
Step2	612.84	-588.46	533.88	-544.18
Step5	613.82	-588.07	541.99	-533.62
Step8	616.58	-587.55	535.76	-537.47
Step10	615.90	-592.75	534.38	-533.95

Table 4. The cost function value (Experiment)

	Nominal	Step1	Step2	Step5	Step8	Step10
J^{ach}	9.33	7.28	7.26	7.24	7.24	7.22
J^{id}	1.25	0.813	0.591	0.544	0.551	0.539

Measured FRF and identified transfer functions are shown in Figure 4. Because of unknown dynamics of rotor system and other factors, nominal transfer function is not matched well with measured FRF of system. As shown in Figure 4, the identified model (step 7) with the control-relevant scheme does not look better than the model estimated without control-relevant weighting (step 1). But the decrease of the achieved cost function J^{ach} (Table 4) proves that the identified model is appropriate for high-performance controller design. Figure 5 shows the relative error of the closed-loop transfer function, $T(s)$ between real and identified plant. The relative error decreases as the iteration steps increase. The experimental results confirm the simulation results that the iterative scheme can enhance system performance.

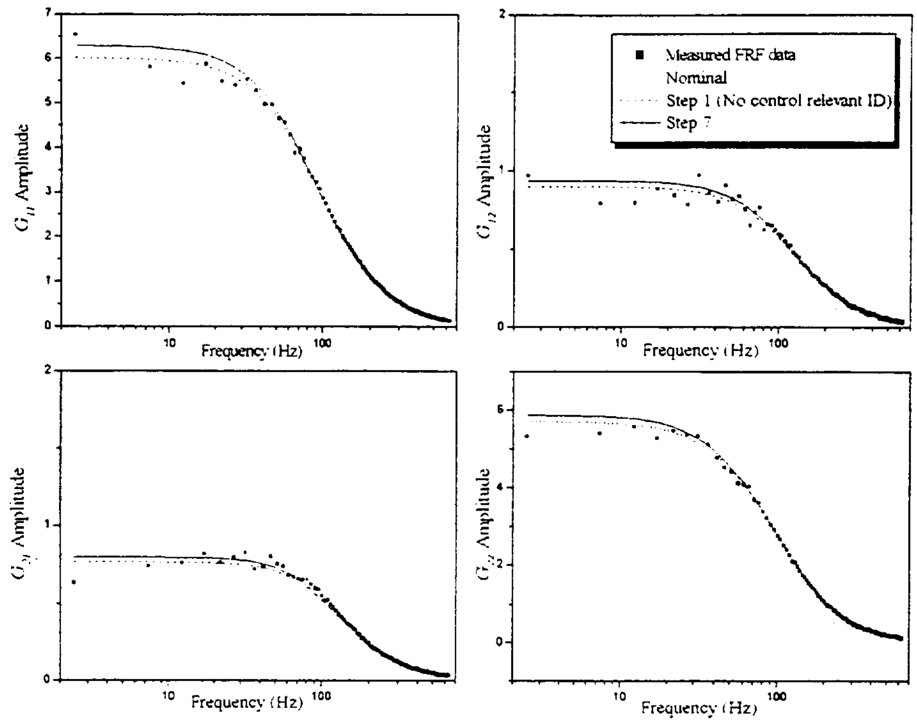

Figure 4. Measured FRF and identified models

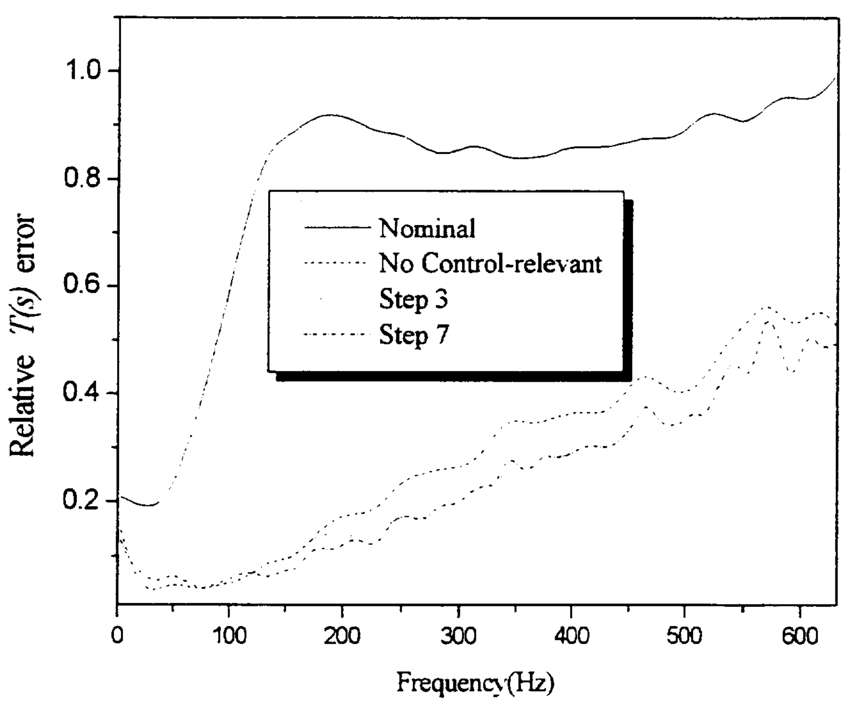

Figure 5. Relative errors of closed-loop transfer function

CONCLUSION

We applied the joint identification/controller design scheme to MIMO rigid rotor system supported by magnetic bearings and the performance of the closed-loop system is improved sequentially during the iteration. Although the rigid rotor AMB system is not significantly high-order plant, the control-relevant identification method gives better performance. In case of high-order system, such as flexible rotor supported by AMB, the improvement of performance may be more conspicuous.

For the future work, the control-relevant identification with other robust confrol design scheme – H_∞ or μ control synthesis – will be developed. Also a considerable amount of work is needed to apply this scheme to a flexible rotor AMB system.

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