IDENTIFICATION AND CONTROL OF A MAGNETICALLY LEVITATED TABLE

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SUMMARY

A new vibration isolation system is developed. A table is levitated by an active magnetic bearing and isolated from any external vibration. Subspace identification methods are used to find a state-space model for this system from input-output data. A PID controller is designed to levitate the table, and a random signal is used to excite the system in addition to the controller inputs. This paper points out some problems when subspace methods are used for identification of closed-loop systems. Although three subspace methods are applied to the present problem, only one of them works well. Two subspace methods are not able to find a model with order of a reasonable value. We present one of the reasons why those subspace methods do not work

INTRODUCTION

Vibration isolation is an important topic in a lot of engineering fields such as manufacturing a semiconductor or micro-scale observation using an electron microscope. Vibration-proof rubber, coil spring, or air bearings are used for passive vibration isolation. However, those passive methods are not yet able to achieve a working environment required by the preceding engineering fields. As an alternative to the passive methods, magnetic bearings are used for active vibration isolation[1-3]. A table is levitated by an active magnetic bearing and isolated from any external vibration. This system is expected to achieve good vibration isolation when compared with the preceding passive methods. Traditional controllers for magnetic bearings are designed to regulate the relative displacement between a levitated object and a base; those bearings are fixed on the base. In order to isolate the table from external disturbance, we have to redesign a controller to regulate the absolute displacement of the levitated table in an inertial coordinate system. Moreover, it would be necessary to obtain a precise model of the controlled system in order to design a good controller. One of the excellent ways to get a precise model would be the one based on the input and output data of the controlled system. From this philosophy, many methods have been developed for the system identification based on the input and output data. In particular, subspace identification methods[4] have been proven to be a valuable alternative to classical prediction-error methods. Unlike the classical methods they do not suffer from such problems as a priori parameterizations, initial estimates and nonlinear optimizations. Subspace methods are used to find a state-space model for the present system from input-output data. Since this system is essentially unstable, we first design a PID controller to levitate the table and then add a random signal to the controller inputs in order to excite the system for a frequency region of interest. So, the system identification is made owing to closed-loop data of the inputs and the outputs.

This paper points out some problems when subspace methods are used for identification of closed-loop systems. Although three subspace methods are applied to the present problem, only one of them works well. Subspace methods determine the state-space dimension owing to the singular value decomposition of a matrix; the state dimension is the number of the singular values different from zero. The singular values could be different between the methods. In other words, the state dimension might be different between the subspace methods. In particular, two subspace methods are not able to find a model with order of a reasonable value. We present one of the reasons why those subspace methods do not work.

HARDWARE DESCRIPTION

A table (a rectangular plate) is levitated by electromagnets in order to isolate it from external disturbance such as floor vibration. Here the system configuration is shown in figure 1. The system contains a table to be isolated from any disturbance, three electromagnets, and a base; the base supports those actuators and table and fixed on the floor of our laboratory. The table motion is vertically limited to 0.35 mm, that is, gap width of the electromagnetic actuator. The electromagnetic actuators are able to generate an attractive force upward and downward in order to levitate the table. An eddy-current-type displacement sensor is built in each electromagnetic actuator to measure the relative displacement of the table to the floor, neglecting mechanical flexibility of the base. Moreover, three acceleration sensors are located on the table to measure the table motion in an inertial coordinate system.

Here we present the control system for the preceding sensors and actuators. A digital controller is implemented on a digital signal processor (DSP) built in a PC. All the sensor outputs are amplified, passed through an anti-aliasing filter, and put into the DSP through an analog-to-digital converter (16 bit resolution). The controller input is calculated by the digital controllers and put out of the PC through a digital-to-analogue converter (12 bit resolution). Moreover, the input is passed through a smoothing filter and a Pulse-Width-Modulation (PWM) circuit and converted into an electric current driving the electromagnetic actuator.

IDENTIFICATION SCHEME

Subspace methods are used to find a state-space model for the present system from input-output data. The control system for the levitated table is essentially unstable if it is in open loop, although the clearance of the magnetic bearings would limit the table motion. Since we assume a state-space model should describe the system, input and output data are necessary for the identification such that the table moves not contacting the clearance limits. Thus, we first design a PID controller to levitate the table in that way and then add a random signal to the controller inputs to excite the system for a frequency region of interest. In other words, the system identification is made owing to closed loop data of the inputs and the outputs.

The configuration of the inputs and the outputs for this system is shown in figure 2. If the open-loop plant is given by the state-space realization: $P(z) = C(zI - A)^{-1}B + D$, the system is written in a state-space form:

$$x_{k+1} = Ax_k + Bu_k, \qquad y_k = Cx_k + Du_k \tag{1}$$

$$u_k = v_k - w_k \qquad \text{with } v_k = K(z)y_k \tag{2}$$

Here, x_k is an n-dimensional state-vector, the matrix A is $n \times n$, B is $n \times m$, C is $l \times n$ and D is $l \times m$; $K(z)$ is a PID controller, and w_k is a random signal. Now the controller input u_k is completely measurable, and we don't account for measurement noise, process noise, model mismatch, and so on. In other words, the present system is completely deterministic. The problem is now to find the system matrices, A, B, C, and D, from the input and output data, u_k's and y_k's. Subspace identification methods are applied to this problem, although it is known that these methods don't work in closed-loop stochastic system. Although many subspace methods have been proposed[4], we have applied three methods (Intersection Method[5], N4SID[6], and Balanced-basis method[6,7]) to the present problem, and only one of the three works well. Since this successful method has received little attention as far as the authors know, we would like to present a part of the algorithms in order to point out the differences between the three subspace methods.

Subspace methods start with introducing a block Hankel matrix such as

$$U_{0|i-1} = \begin{pmatrix} u_0 & u_1 & \cdots & u_{j-1} \\ u_1 & u_2 & \cdots & u_j \\ \cdots & \cdots & \cdots & \cdots \\ u_{i-1} & u_i & \cdots & u_{i+j-2} \end{pmatrix} \tag{3}$$

where the subscript of U denotes the subscripts of the first and last element of the first column. A block Hankel matrix formed with the output is defined as $Y_{0|i-1}$ in the same way. Intersection methods continues to introduce the singular value decomposition of the following matrix:

$$H = \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \begin{pmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{pmatrix} \begin{pmatrix} \Sigma_1 & 0 \\ 0 & 0 \end{pmatrix} V^T \text{ with } H_1 = \begin{pmatrix} Y_{0|i-1} \\ U_{0|i-1} \end{pmatrix} \text{ and } H_2 = \begin{pmatrix} Y_{i|2i-1} \\ U_{i|2i-1} \end{pmatrix} \tag{4}$$

It is easily found that the following relation holds for the intersection of the row space of the matrices H_1 and H_2:

$$U_{12}^T H_1 = -U_{22}^T H_2 \tag{5}$$

If we introduce the state-vector sequence $X_2 = \begin{pmatrix} x_i & x_{i+1} & \cdots & x_{i+j-1} \end{pmatrix}$, it is proved in reference 5 that the row space of X_2 is equal to the row space of $U_{12}^T H_1$ or $U_{22}^T H_2$. Therefore, a sequence of state vectors can be determined by the intersection of the Hankel matrices up to within a similarity transformation. Moreover, the sequence of the state vectors is used to find the system matrices[5], although the state dimension and basis have been determined already.

On the other hand, the other two methods, N4SID and Balanced-Basis method, continues to the optimal projection of a Hankel matrix as follows:

$$Z_i = Y_{i|2i-1} \Bigg/ \begin{pmatrix} U_{0|i-1} \\ U_{i|2i-1} \\ Y_{0|i-1} \end{pmatrix} \equiv L_i^1 U_{0|i-1} + L_i^2 U_{i|2i-1} + L_i^3 Y_{0|i-1} \tag{6}$$

where $A/B = AB^T \left(BB^T\right)^{-1} B$, that is, the optimal projection of the row space of A onto the row space of B. Equation (6) gives the projection of the future outputs onto the past and future inputs and the past outputs. The extended observability matrix is defined as

$$\Gamma_i^T = \left(C^T \quad (CA)^T \quad \cdots \quad \left(CA^{i-1}\right)^T\right) \tag{7}$$

It is shown in Ref that the column space of Γ_i coincides with the column space of the following matrices:

$$T_1 = L_i^1 U_{0|i-1} + L_i^3 Y_{0|i-1}, \qquad T_2 = L_i^1 + L_i^3 L_i^2 \tag{8}$$

where N4SID and Balanced-basis method take T_1 □and□ T_2, respectively, as a matrix with the same column space as that of Γ_i. Therefore, we can put $\Gamma_i = U_1 \Sigma_1^{1/2}$ when the singular value decomposition of T_1 or T_2 is given by

$$T_k = \begin{pmatrix} U_1 & U_2 \end{pmatrix} \begin{pmatrix} \Sigma_1 & 0 \\ 0 & 0 \end{pmatrix} V^T \tag{9}$$

Moreover, we can observe $\Gamma_{i-1} = \underline{U_1} \Sigma_1^{1/2}$ owing to the definition of equation (7), where $\underline{U_1}$ is defined as U_1 without the last l rows. If the relation, $\text{rank}(\Gamma_i) = \text{rank}(\Gamma_{i-1})$, holds, the two subspace methods work. Here the state-vector dimension is the number of the singular values different from zero, whereas the state basis has been fixed by selecting a matrix with the same column space of Γ_i. Furthermore, Γ_i is used to find the system matrices as shown in reference 6.

In theory the difference between the two subspace methods is the state-space basis, which constitutes a similar transformation connecting the two state space models. In practice, the state dimension n could be different between the two methods because it is determined by counting the singular values larger than a threshold value; those singular values might be different between the two methods. In order to clarify this point, let us consider a problem reducing a model of order n to a model of order r by truncating the singular values σ_k's of T_1. The following relation then holds for N4SID[7]:

$$\left\| \left(P(z) - \hat{P}(z)\right) S_u(z) \right\|_\infty \le 2 \sum_{k=r+1}^{n} \sigma_k (1 + \alpha) \qquad (\alpha > 0) \tag{10}$$

where $\hat{P}(z)$ is the reduced model of order r; $S_u(z)$ is the spectral factor of $U(z)$. Equation (10) shows that the error of the reduced model would be small where the frequency content of the input is large. In other words, a lot of input energy in a certain frequency region leads to an accurate model in that region. The similar relation to equation (10) holds for Balanced-Basis method[7]:

$$\left\|\left(P(z) - \hat{P}(z)\right)\right\|_{\infty} \leq 2\sum_{k=r-1}^{n}\sigma_k \tag{11}$$

Here, σ_k is a singular value of T_2. This reduction cannot depend on a spectrum of input energy. Judging from equations (10) and (11), the state-space dimension n might be different between the two methods unless the input is white noise.

EXPERIMENTAL RESULT

Although three subspace methods are introduced in the preceding chapter, those subspace methods are used to identify the control system for the table levitated by the electromagnetic actuators. Those subspace methods are implemented in MATLAB, whereas all the programs for controlling the table system are written in C language. An M-sequence (maximum length null sequence) signal of 15th order is used to excite the table as a random input w_k. This M-sequence is generated by a PC and keeps its power spectral density constant up to a given frequency. The input and output data for the identification test are sampled 15000 times into the PC through an analog-to-digital converter; the sampling rate is 3000 Hz.

Although we have applied the three subspace methods to the present system, we show the calculated results here. The column number of the Hankel matrices is 14800 through all the calculation for system identification. When using the Intersection method, we are not able to find the intersection up to order of 50. This number of the model order is the upper limit in our PC. Since the smallest singular values of equation (4) is 6.9 even at $i = 50$, the matrices U_{12} and U_{22} don't exist, that is, there is no intersection of those two Hankel matrices. Therefore, the Intersection method does not work well in the present case. When we take N4SID to identify the present system, we encountered a problem similar to the problem we have already observed. The matrix T_1 is always of full rank with respect to the column up to 90; the smallest singular value is 3.1×10^{-2} at $i = 90$. Since Γ_i is then of full rank, $\mathrm{rank}(\Gamma_i)$ cannot be equal to $\mathrm{rank}(\Gamma_{i-1})$ due to the definition, which means the state dimension n is not constant. Therefore, N4SID subspace method does not work, either. Finally, Balanced-Basis method is used to identify this table system and works well. The smallest singular value of T_2 is 5.9×10^{-5} at $i = 42$. By truncating the singular values larger than 1.0×10^{-4}, we get a state-space model of order 40. The transfer function $P(z)$ is computed owing to the identified system matrices (Figure 3).

Let us consider the reason why N4SID does not work well whereas Balanced-Basis method works. As shown in equations. (10) and (11), the two methods would yield the same system matrices within up to a similar transformation if the input is white noise. However, it would be difficult to cause the input to be white noise in closed-loop settings because the input absolutely contains the frequency content to stabilize the system, in the present case, v_k. It would be possible to solve the problem by passing the input sequence through a filter if the spectral factor S_u is invertible.

CONCLUSION

A new vibration isolation system is developed. A table is levitated by an active magnetic bearing and isolated from any external vibration. Subspace identification methods are used to find a state-space model for this system from input-output data. A PID controller is designed to levitate the table, and a random signal is used to excite the system in addition to the controller inputs. This paper points out some problems when subspace methods are used for identification of close-loop systems. Although three subspace methods are applied to the present problem, only one of them works well. Two subspace methods are not able to find a model with order of a reasonable value. We present one of the reasons why those subspace methods do not work

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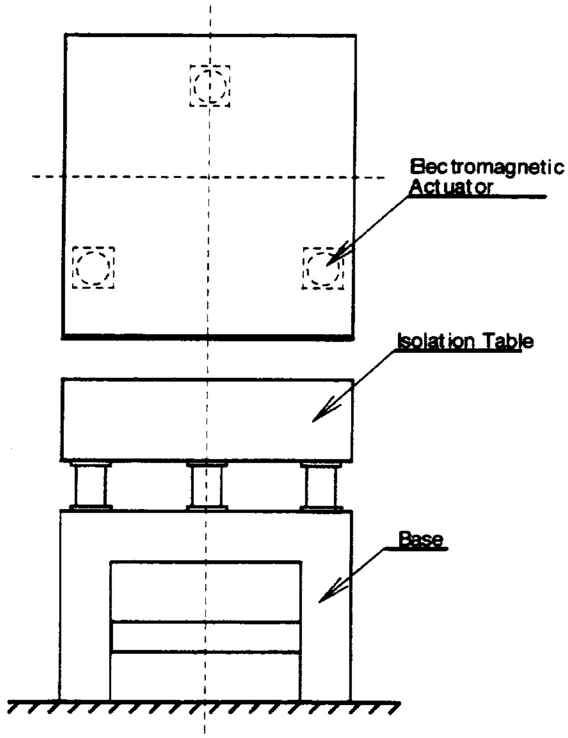

Figure 1. Vibration isolation system using electromagnetic actuators

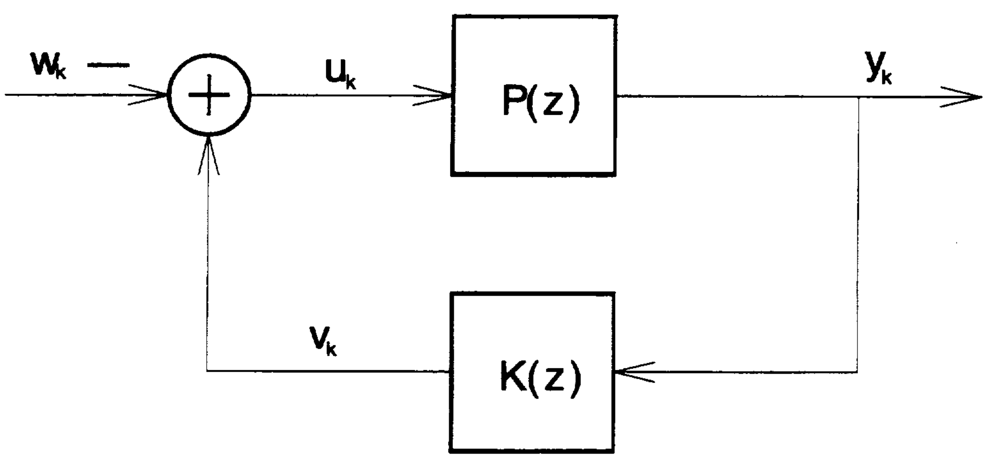

Figure 2. Block diagram for identification tests

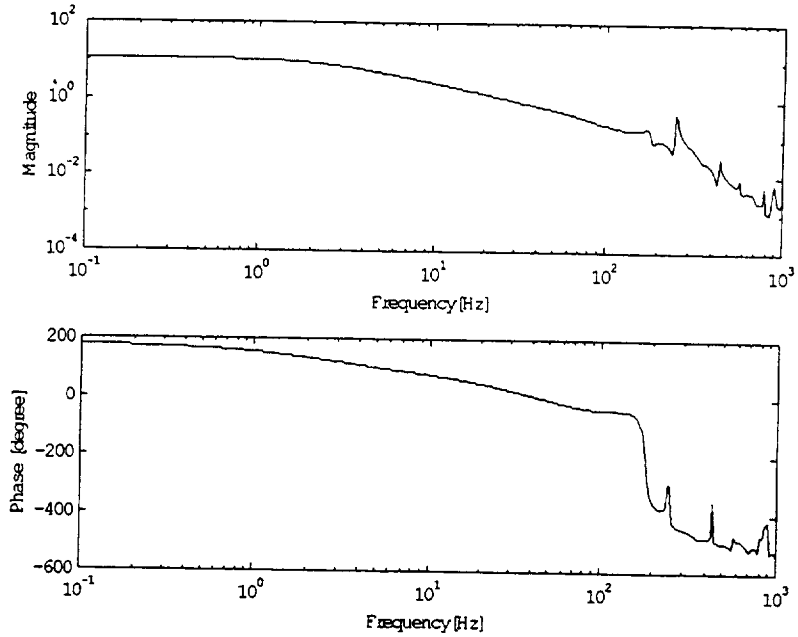

Figure 3 Calculated transfer function using the estimated state-space model