NONLINEAR ROBUST CONTROL AND ITS APPLICATION TO MAGNETIC SUSPENSION TECHNOLOGY[1]

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SUMMARY

Nonlinear robust control is a development of the modern robust approach to the stability of linear control systems. For nonlinear systems, this approach was used in the theory of absolute stability. The absolute stability methods are bounded as a rule by the class of stable SISO-objects. However, magnetic suspension technology deals with unstable and MIMO-objects in the general case.

Design of stabilizing systems for magnetic suspension may effectively be carried out on the basis of a proposed approach to nonlinear robust control synthesis. The proposed approach is based on the criterion of the Maximum Region of Attraction for stabilizing equilibrium in the phase space of a closed loop system. Actually the same criterion is used in absolute stability theory whose methods allow relation of the topological identity, the phase space structure of the system with the nonlinear control, to the phase space structure of the linear system.

The analytical solution of the synthesis problem in a form of a nonlinear robust control law allows the determination of the dependence of the optimal regulator's structure and parameters from the objects. The theoretical background of using regulators has been added to the base of the proposed synthesizing approach and new nonlinear robust control laws for stabilizing magnetic suspension systems have been found.

One of the new control laws is the control of angular motion of a shaft in magnetic bearings with gyroscopic effects. Finding this law required the development of the suggested approach from unstable to stable objects having conservative stability.

INTRODUCTION

Taking into account the realistic restrictions of the control action makes a mathematical model of a stabilizing system nonlinear. It results in an attractive region concept for a stabilizing equilibrium in the phase space of a closed loop system. In a nonlinear system, unstable periodic motions are the main cause of inadmissible reduction of a region of attraction, as shown in [1].

In order to avoid these unstable periodic motions, the original approach to synthesizing control laws for unstable objects is suggested in [2]. In this approach, the control law is synthesized from the criterion of attaining the Maximal Region of Attraction (MRA-criterion) of stabilizing equilibrium in the phase space of a closed loop system.

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The synthesizing results give the theoretical background for many regulators used in magnetic suspension technology [3]. As shown in [4] for the simplest magnetic suspension, a control synthesized from MRA-criterion satisfied a number of conventional quality performance criteria simultaneously.

The shaft magnetic suspension attracts researchers [5] by its practical significance and as a more complex object having a control system which stabilizes the shaft position in three translational degrees of freedom and two angular ones. Synthesis of nonlinear robust control of a shaft magnetic suspension has been done in [6] using a series of simplifying suppositions. One of these suppositions is the neglect of the gyroscopic effect. As shown in [7], increasing the shaft rotation speed leads to stability loss of the magnetic suspension stabilizing system. In order to account for the gyroscopic effect one may accept the simplified supposition about consideration of only angular degrees of freedom of the shaft with respect to its mass center for synthesis of restricted control of the shaft magnetic suspension with accounting for the gyroscopic effect.

PROBLEM STATEMENT

Under the accepted supposition, the shaft in the Active Magnetic Bearings (AMB) may be considered as a solid with the fixed point as shown in Fig. 1. The fixed point O of the shaft coincides with its mass center. The origin of the fixed coordinate system O, ξ, η, ζ is placed at this point. The mobile coordinate system O, ξ', η', ζ' is connected with the principal axes of the shaft and axis O, ζ' is directed along the axis of its rotation. The position of this axis is given by angles θ_1, θ_2 in the fixed coordinate system. The control forces from AMB $F_{1\text{-}4}$ are found in two planes where the AMB are placed as shown in Fig. 1. Let these planes be placed symmetrically with respect to point 0 at the distance $\pm l/2$.

In the case of small angles, the shaft dynamics are given by following equations

$$J\frac{d^2\theta_1}{dt^2} + J_\zeta\Omega\frac{d\theta_2}{dt} - \frac{al^2}{4}\theta_1 = -\frac{l}{2}(F_1 - F_3)$$

$$J\frac{d^2\theta_2}{dt^2} - J_\zeta\Omega\frac{d\theta_1}{dt} - \frac{al^2}{4}\theta_2 = -\frac{l}{2}(F_2 - F_4)$$

where $J_\xi = J_\eta = J, J_\zeta$ are the moments of inertia for the axial symmetrical shaft with respect to principal axes, Ω is the angular speed of the shaft, supposed as constant, m is the mass of the shaft, a is the negative stiffness of the AMB, l is the distance between planes of top and bottom AMB. It is supposed also that control actions are restricted $|F_{\xi,\eta}| \leq F^+$.

By introduction of the time scale $t_m = \sqrt{\frac{4J}{al^2}}$ the mathematical model of the control object may be presented in the standard dimensionless form

$$\frac{d\mathbf{x}}{dt} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \tag{1}$$

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -H \\ 0 & 0 & 0 & 1 \\ 0 & H & 1 & 0 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 0 & 0 \\ -1 & 0 \\ 0 & 0 \\ 0 & -1 \end{pmatrix}.$$

$$|u_{1,2}| \le u^+,$$

where the vector of phase coordinates has the following components:

$$x_1 = \theta_1, \quad x_2 = t_m\frac{d\theta_1}{dt}, \quad x_3 = \theta_2, \quad x_4 = t_m\frac{d\theta_2}{dt}.$$

$H = \frac{J_\zeta \Omega t_m}{J}$ is a parameter describing the ratio of the inertia moments of the shaft and the rotation speed, $u_1 = \frac{F_\eta l_2}{mgl_1}$, $u_2 = \frac{F_\xi l_2}{mgl_1}$ are the control actions restricted by the magnitude $u^+ = \frac{F^+ l_2}{mgl_1}$ Using the symmetry of equations (1) allows us to reduce its order in half by the complex variables

$$z_1 = x_1 + ix_3; \quad z_2 = x_2 + ix_4; \quad v = u_1 + iu_2, \tag{2}$$

where $(i = \sqrt{-1})$. As a result the mathematical model of the object may be presented either in the standard form of the complex phase space

$$\frac{d\mathbf{z}}{dt} = \mathbf{A}_c\mathbf{z} + \mathbf{B}_c v, \tag{3}$$

$$\mathbf{A}_c = \begin{pmatrix} 0 & 1 \\ 1 & iH \end{pmatrix}, \quad \mathbf{B}_c = \begin{pmatrix} 0 \\ -1 \end{pmatrix},$$

or as a differential equation of second order

$$\frac{d^2 z_1}{dt^2} - iH\frac{dz_1}{dt} - z_1 = -v \tag{4}$$

with respect to the complex variable z_1. These equations contain the complex function of admissible controls v, whose components $u_1 = Re v$, $u_2 = Im v$ are supposed as the simplest piece-wise linear functions as shown in Fig. 2

$$\begin{aligned} &\mathrm{u} = \beta\sigma, |\beta\sigma| \le u^+, u_{1,2} = u^+, \beta\sigma \ge u^+, \\ &u_{1,2} = -u^+, -u^+ \ge \beta\sigma, \quad \sigma = \mathrm{C(H)x}, \end{aligned} \tag{5}$$

where β is some scalar and C(H) is some feedback control matrix which depends on H. The problem consists in synthesizing this function as some dependence from the state space variables of the object which provides a stabilization of the axis shaft vertical position under the maximal admissible initial departures in the phase coordinates for given control restrictions or under the minimal admissible control restrictions for given angular initial departures of the shaft. In this case the quality control performance will be some optimal structure of the phase space for the closed loop system.

LINEAR ANALYSIS OF UNCONTROLLED OBJECT

Synthesis of a control law from the MRA-criterion is carried out based on the eigen-motions for an uncontrolled object. These motions are given by the eigenvalues of matrix A_c, or by the roots of a characteristic equation for an uncontrolled object (3), (4)

$$\chi(\lambda) = \lambda^2 - iH\lambda - 1 = 0$$

The solution of this quadratic equation has the following analytical expression

$$\lambda_{1,2} = \frac{1}{2}(iH \pm \sqrt{4 - H^2}) \tag{6}$$

The dependence of the roots (6) from the parameter H is illustrated by the upper half complex plane in Fig. 3. The dependence of the complex conjugate roots to (6) and noted by sign (*) is shown in the bottom half of this plane. The whole Fig. 3 shows the dependence of the matrix A eigenvalues from the parameter H for object (1). The following specific points in this dependence conforming to the behavior of the roots may be stated. The first point conforms to the shaft without rotation when $H = 0$ and

$$\lambda_1 = \lambda_1^* = 1, \quad \lambda_2 = \lambda_2^* = -1.$$

In this case, there are two positive eigenvalues of the matrix A that relate to instability of the nonrotational shaft due to a negative stiffness of AMB. The second specific point conforms to the beginning of the shaft gyroscopic stabilization when $H = 2$ and

$$\lambda_1 = \lambda_2 = i; \quad \lambda_1^* = \lambda_2^* = -i.$$

In this case, all eigenvalues of the matrix A are imaginary ones and the shaft has conservative stability. The portions of the dependence $\lambda(H)$ which characterizes the stability of the control object may also be deduced. The first portion conforms to the range (0,2) of parameter H. This portion is characteristic of the object instability as far as the roots (6) in this case

$$\lambda_1 = r + i\frac{H}{2}, \quad \lambda_1^* = r - i\frac{H}{2} \tag{7}$$

have the positive real part $r > 0$. The second portion conforms to the range $H \geq 2$. It is characteristic of the object's conservative stability. In this case, all eigenvalues of the matrix A are imaginary ones

$$\lambda_1 = i\omega_P, \quad \lambda_1^* = -i\omega_P, \quad \lambda_2 = i\omega_N, \quad \lambda_2^* = -i\omega_N, \tag{8}$$

where ω_P and ω_N denote the precessional and nutational frequencies of the high rotational speed shaft. For $H > 1$ these frequencies are approximately equal to $\omega_P = 1/H$, $\omega_N = H$. By this means, the studied problem deals with the object with stability without control dependencies from one parameter H. Parameter H characterizes the shaft rotational speed which may be changed from zero to some final value H_m. It is proposed that the point $H = 2$ lies within the range of the admissible values of this parameter. In this case, the changing range of parameter H is divided in two parts on indication of the object stability. The object is unstable when $0 \leq H < 2$, and it has conservative stability when $H \geq 2$.

SYNTHESIZING A NONLINEAR ROBUST CONTROL OF THE UNSTABLE SHAFT

The optimum structure of the closed loop system phase space is the nonlinear robust control performance providing the fulfillment of the MRA-criterion. Synthesis of a nonlinear robust control for an unstable shaft is carried out using the approach suggested in [2]. With respect to this approach the equations (3) are transformed to canonical form

$$\frac{dy}{dt} = \Lambda y - v$$

where Λ is the Jordan form of the matrix A_c. This transformation may be carried out by means of a linear transformation of variables

$$y = Dx, \quad D = \begin{pmatrix} 1/\lambda_1 & 1 \\ 1/\lambda_2 & 1 \end{pmatrix}$$

Next is separated the unstable part of the object conforming to the matrix A_c eigenvalue having the positive real part

$$\frac{dy_1}{dt} = \lambda_1 y_1 - v \tag{9}$$

where $y_1 = \frac{x_1}{\lambda_1} + x_1$ is the unstable variable of the object. Nonlinear robust control v is synthesized in a class of the simplest piece-wise linear functions (5) satisfying the following conditions [2]:

a) dependence of control upon unstable variable only; b) asymptotic stability of the system's unstable part under this control. If the control dependence from the feedback signal in a linear area control function is

$$v = \beta\sigma, \quad \sigma = \lambda_1 y_1, \quad \beta > 1 \tag{10}$$

then these conditions are satisfied. The transition from variables y to variable x in this dependence by the use of (2), (7) defines the feedback matrix in (5)

$$\mathbf{C}_1(\mathbf{H}) = \begin{pmatrix} 1 & r(H) & 0 & -\frac{H}{2} \\ 0 & \frac{H}{2} & 1 & r(H) \end{pmatrix} \tag{11}$$

for nonlinear robust control of the unstable shaft when $0 \leq H < 2$. The results of the changing roots within the parameter range $0 \leq H < 2$ is shown in Fig. 3 by dashline for different values of parameter β.

SYNTHESIZING A NONLINEAR ROBUST CONTROL UNDER SHAFT GYROSCOPIC STABILIZATION

In case of sufficiently great angular momentum of the shaft ($H \geq 2$), i.e. when the gyroscopic stability condition is fulfilled, the control's main goal is provision of the asymptotical attenuation for undamping precessional and nutational modes of the appropriate frequencies ω_P, ω_N. The best control performance may be achieved also by the use of MRA-criterion under the synthesis problem solution. Only in contrast to the unstable object having the bounded region of controllability in this case the maximal region of attraction is coincident with the whole phase space. In this case a robust indication of synthesis of control with respect to an uncertainty of its nonlinear characteristic will be the absolute stability [8] of a closed loop system. For synthesis of control providing the system absolute stability in some class of nonlinear functions it is comfortable to use the object mathematical model (4) and to carry out the variable substitution to a rotational coordinate system

$$z_1 = w\,exp(i\omega t) \tag{12}$$

where ω is a new complex variable which change in time takes place with some cycle frequency ω. In new variables, equation (4) has a view of an undamped oscillator also

$$\frac{d^2w}{dt^2} + i(2\omega - H)\frac{dw}{dt} + k(\omega)w = v\,exp(-i\omega t) \tag{13}$$

At the same time its natural frequency

$$k(\omega) = H\omega - \omega^2 - 1 > 0 \tag{14}$$

in the contract to (4) may be positive explicitly. Really, inequality (14) is fulfilled under the gyroscopic stabilization condition if the cycle frequency in (12) is chosen within the range $[\omega_P, \omega_N]$ determined by the cycle frequencies of the shaft precession and nutation. It is illustrated by dependence $k(\omega)$ shown in Fig. 4 under condition $H > 2$. The values of the natural frequencies ω_P, ω_N are given by the intersecting points of the curve $k(\omega)$ with the abscissa axis. The choice of the cycle frequency value in the variable substitution (12) is realized in the maximum point of this dependence $\omega = \omega_m = \frac{H}{2}$ when $k(\frac{H}{2}) = \frac{H^2}{4} - 1$. For achievement of the system (13) absolute stability, i.e. for obtaining transient asymptotical stability and of the equilibrium state uniqueness (placed in the origin), control action is chosen as a monotone function from only a variable velocity

$$v\,exp(-i\omega t) = u(\frac{dw}{dt}). \tag{15}$$

For the simplest piece-wise linear functions (5), i.e. the linear functions with restrictions on the level u^+, it is not difficult to show that the absolute stability condition for the closed loop system (13), (15) is fulfilled. The coefficient β value must be sufficiently great with the aim of transient velocity increasing and of its aperiodicity. Therefore the choice of this coefficient value is accomplished from an aperiodicity transient condition

$$\beta \geq \frac{1}{2}\sqrt{k(\frac{H_m}{2})} \tag{16}$$

in the closed loop system for any values of parameter H from the range $2 < H \leq H_m$. To obtain the control as seen in (5) the inverse variable substitution

$$\frac{dw}{dt} = (\frac{dz_1}{dt} - i\omega z_1)exp(-i\omega t)$$

and transition from complex variable z_1 to variables x is fulfilled. As a result the feedback matrix

$$C_2(H) = \begin{pmatrix} 0 & 1 & \frac{H}{2} & 0 \\ -\frac{H}{2} & 0 & 0 & 1 \end{pmatrix} \tag{17}$$

for the absolute stability system of the high rotational speed shaft control is determined.

PHASE SPACE STRUCTURE OF SYNTHESIZED SYSTEM

The phase space structure of the synthesized system is optional from the MRA-criteria for nonlinear robust control. The view of these maximal regions of attraction for synthesized systems depends on the parameter H value. The phase space structure is considered for three specific values of this parameter: $H = 0$; $0 < H < 2$; $H \geq 2$. The first two cases conform to the unstable object. In these cases the phase plane of the systems unstable parts gives an indication of the whole four-dimensional phase space structure of the closed loop system [2]. In the case of nonrotational shaft (when $H = 0$ and there are two coinciding positive roots $\lambda_1 > 0$) the structure of the unstable variables y_1^+, y_2^+ is shown in Fig. 5a. In the case of a slowly rotating shaft (when $0 < H < 2$ and there are two complex conjugate roots λ_1, λ_1^* with positive real parts) the structure of phase plane for the real and imaginary parts of the unstable variable y_1 is shown in Fig. 5b. The bounded maximal region of attraction for stabilizing equilibrium is the specific feature of the phase space structure in these cases. The third case of the high rotational speed shaft conforms to its conservative stability. In this case the closed loop system under the proposed control embodies absolute stability. The phase space structure of that system is topologically equivalent to the structure of the four dimensional stable linear system.

CONCLUSION

The analytical expression of the nonlinear robust control law adapted to shaft rotational speed is obtained. This expression coincides with one obtained earlier for the nonrotational shaft and shows the differences arising from accounting for gyroscopic effects. These specifications consist of the appearance of the cross feedbacks in the control for each angle and radical distinctions of the control law from and relating to the gyroscopic stabilization. The control law is suddenly changed in the point $H = 2$ that takes into account sudden changing of the object stability property. Obtaining this solution became possible due to use of the unified MRA-criterion and made it possible to obtain the nonlinear robust control. The obtained control robustness resides in the maximal region of attraction under the nonlinear control function uncertainty given by the Hurwitz angle. By this means the robust approach widely used in the stability theory of linear systems at present has been successfully developed for nonlinear systems. For the objects having a conservative stability the nonlinear robust control is the development of absolute stability theory.

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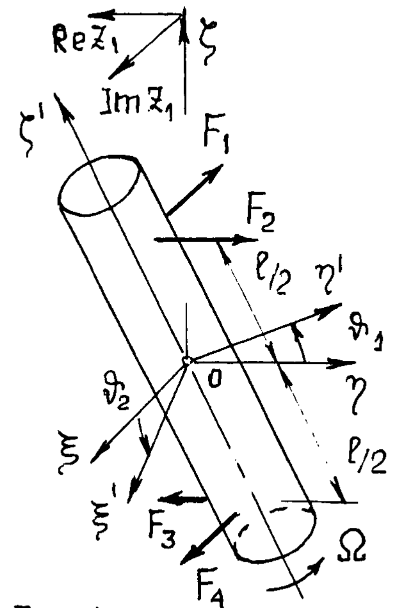

Fig. 1

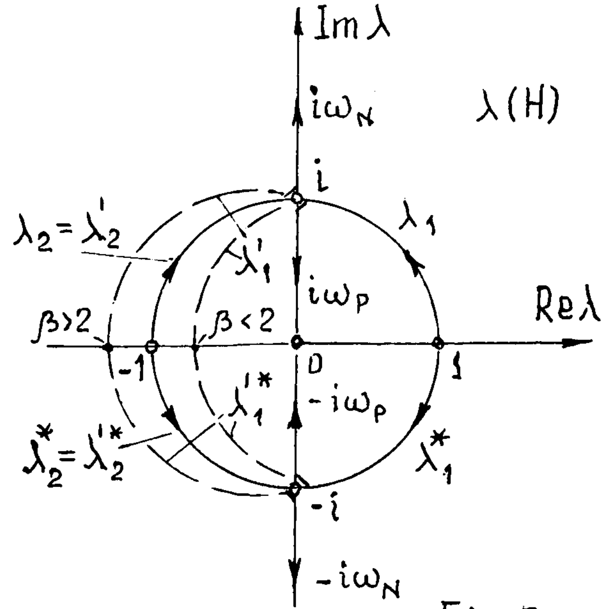

Fig. 3

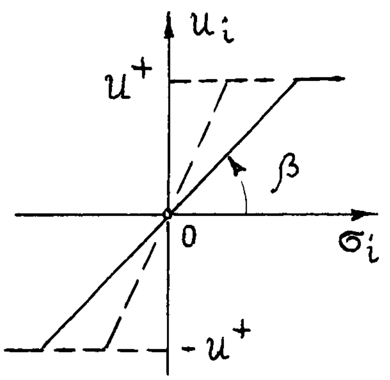

Fig. 2

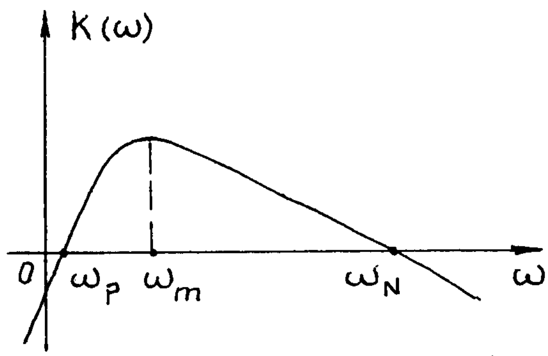

Fig. 4

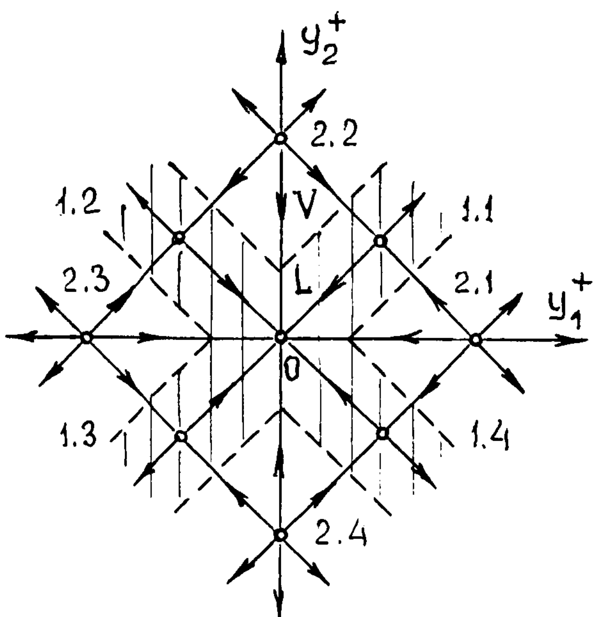

Fig. 5 a

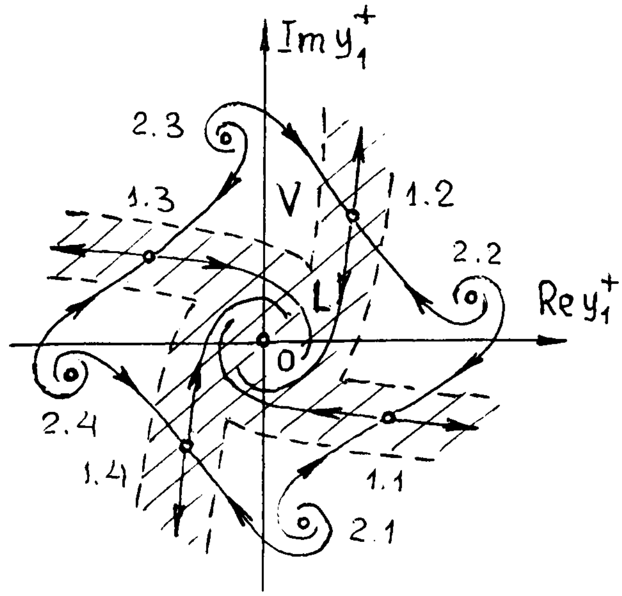

Fig. 5 в

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