HIGH POWER ELECTRICAL MACHINES FOR SPACE APPLICATIONS ELECTROMAGNETIC LAUNCHERS (QUALITY FUNCTIONS AND GOODNESS FACTORS)

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ABSTRACT

In order to drastically increase the power and energy densities of the pulsed electrical machines for space applications electromagnetic launchers a "system approach" of the entire electromechanical and electrodynamical chain must be adapted. A unified treatment and optimization of both rotating power supplies and accelerators leads to a set of goodness functions and quality factors exploring the relative merits of the system. Such quality indicators must be based on a rigorous theoretical foundation from both electrodynamical and electromechanical points of view.

For electromechanical aspects, the general theory of electrical machines, together with Kron's tensorial formulations for transient and pulsed conversion lead to a quality indication as a "goodness factor" or "goodness function" generalized from steady state to pulsed electrical machines and, further, to systems of machines (rotating power supply, self-excitation system, accelerator).

By using Maxwell's equations for moving media and the notion of flux derivative, three different formulations for electromagnetic forces in electrical machines are introduced and interrelated leading to an assessment for the electromagnetic recoil and for the distribution of forces in air core and iron core advanced machines and electromagnetic accelerators.

BACKGROUND

For many years, the Electromagnetic Launchers (EMLs) have been proposed as alternative means for space launching. After more than one decade of experimentation with different electromagnetic guns and launchers (under U. S. Army and DARPA sponsorship), the issue of the size of the electric power supplies for such devices has emerged as one of the most critical which may determine if any practical space or military use is to be expected in the foreseeable future.

The power supplies used for significant electromagnetic gun experiments have been capacitors, inductors, or high-voltage electrical machines. The homopolar machine is a very low-voltage electrical machine and does not have the capability of delivering the high level of power required by the electromagnetic launcher. The inductor charged relatively slowly by the homopolar and the opening explosive switch represent, jointly, the power supply [1].

It is always understood that the power supply has also, like the ones mentioned above, the capability of storing the necessary energy and a common way of characterizing the power supplies is by the specific stored energy, while strongly implying that sufficient means (such as high output voltage and low internal impedance) exist in order to deliver such energy in an extremely short period of time.

The variables of state, uniquely defining the stored energy , are the electric field intensity *E(v/m)* for capacitor films, the magnetic flux density *B(T)* for inductors, and the peripheral velocity *u(m/sec)* for the flywheel part of electrical machines, for the power supplies considered here in which the energy is stored in electric field, magnetic field, and as kinetic energy, respectively.

Taking into account the demonstrated state-of-the-art technology only, we arrive at the energy density levels listed in the Table 1. Table 1 shows that the balance is heavily tilted in favor of high-voltage, electrical machines, with kinetic energy store. A strong implication is that the power supply has the means of transferring very rapidly such energy to the load: high voltage output and small internal impedance.

Table 1. Performance energy density levels for different media of storage.

Energy storage medium	**Energy density J/m^3**	**Demonstrated levels of performance J/m^3**
1. Electric energy density	$w_E \frac{\varepsilon E^2}{2}$	$9.1\times10^6 \begin{cases} E = 1.6\times10^4 \frac{v}{mil} \\ \varepsilon = 5\varepsilon_0 \end{cases}$
2. Magnetic energy density	$w_M = \frac{B^2}{2\mu_0}$	$1.0\times10^8 \begin{cases} B = 16\ Tesla \\ \mu = \mu_0 \end{cases}$
3. Kinetic energy density	$w_K = \rho\frac{v^2}{2}$	$1.2\times10^9 \begin{cases} u = 1,150 \frac{m}{sec} \\ \rho = 1,800 \frac{kg}{m^3} \end{cases}$

The present paper tries to advance the idea that directions for drastic improvements in space application of EML and their power supplies must come from a unified treatment and optimization of both introducing a system of quality (or goodness) factors and functions applicable to the entire "electromechanical chain" (power supply and launcher), in a system approach.

Maxwell's Equations for Media in Motion

The expression for Maxwell's equations customarily used do not contain explicitly the motion of the media so important for machines and accelerators. As an example, the very important ones, Ampere's Law and Faraday's Law, respectively, are:

$$\oint_{c_2} \overline{H} \bullet d\overline{l} = \iint_{s_c} \overline{J} \bullet \overline{ds} + \frac{d}{dt}\iint_{s_{c_2}} \overline{D} \bullet ds \qquad \oint_{c_1} \overline{E} \bullet d\overline{l} = -\frac{d}{dt}\iint_{s_{c_1}} \overline{B} \bullet \overline{ds} \qquad (1)\ \&\ (2)$$

In the moving media, both the curve on which the circulation is taken *(c)* and the surfaces through which the flux is considered *(s_c)* are moving and by using the known formula for flux derivative, the two Maxwell's equations above become:

$$\oint_{C} \overline{H} \bullet \overline{dl} = \iint_{S_c} [\overline{J} \bullet \overline{ds} + \frac{\partial \overline{D}}{\partial t} \bullet \overline{ds}] + \iint_{S_c} \overline{u} \bullet div\ \overline{D}\ \overline{ds} + \iint_{S_c} curl\ (\overline{D} \times \overline{u})\overline{ds} \qquad (1a)$$

$$\oint_{C} \overline{E} \bullet \overline{dl} = -\iint_{S_C} \frac{\partial \overline{B}}{\partial t} \bullet \overline{ds} - \iint_{S_C} \overline{u} div\ \overline{B} \bullet \overline{ds} - \iint_{S_C} curl\ (\overline{B} \times \overline{u})\overline{ds} \qquad (2a)$$

or in differential form:

$$curl\ \overline{H} = \overline{J} + \frac{\partial \overline{D}}{\partial t} + \overline{u}\ \ div\ \ \overline{D} + \ curl\ \ (\overline{D} \times \overline{u}) \qquad (1b)$$

$$curl\ \overline{E} = -\frac{\partial \overline{B}}{\partial t} - \overline{u} div \overline{B} - curl\ \ (\overline{B} \times \overline{u}) \qquad (2b)$$

In the objects of our considerations, electrical machines, projectiles moving in electromagnetic field and electromagnetic launchers, we assume that a point in motion maintains its electromagnetic properties ε and μ (the substantial derivative is zero $\frac{ds\mu}{dt} = \frac{\partial \mu}{\partial t} + \overline{u} grad \mu$). Then, the partial derivatives of $\overline{D}$ and $\overline{B}$, vectors determining the electric and magnetic flux are replaced by their flux derivatives, with respect to time:

$$\frac{\partial D}{\partial t} \rightarrow \frac{d_f \overline{D}}{dt} \quad \text{and} \quad \frac{\partial \overline{B}}{\partial t} \rightarrow \frac{d_f \overline{B}}{dt} \qquad (3)$$

Then the fundamental balance of energy for electromagnetic field in a volume *v* surrounded by a closed surface s_v through which we calculate the Poynting's vector will transform, replacing the partial derivatives, with respect to time, with the flux derivatives into:

$$-\oint_{s_v}\oint (\overline{E} \times \overline{H}) \bullet \overline{ds} = \iiint_v \overline{E}\overline{J} dv + \iiint_v \left(\overline{E} \bullet \frac{\partial_f \overline{D}}{\partial t} + \overline{H} \bullet \frac{\partial_f \overline{B}}{\partial t} \right) dv \qquad (4)$$

which considers not only the electrodynamic phenomena, but also the electromechanical power conversion effects, due to the media motion.

Forces in electrical machines and electromagnetic accelerators

The fundamental balance of power [1, 5] will permit us to find the electromagnetic forces in three different formalisms:

1. As the global forces: generalized Lagrangian forces acting along generalized coordinates;
2. As a distribution of body forces in the projectile, accelerator, and power supply; and
3. As stresses using the artificial method of Maxwell's electromagnetic stress tensor, and as a generalization finding the origin and distribution of the recoil forces.

The theoretical tools which transform the expressions for forces, from a formalism to another are the constancy of the magnetic flux as the constraint for the Lagrangian transformation and the complex and subtle notion of flux derivative [1].

1. As a Lagrangian transformation, we choose one in which the sum of the Poynting's vector flux and the Joule losses is equal to zero during the transformation. Such constraint reduces the expression (4) to:

$$P_J + P_{S_o} = -\iiint_v \left(\bar{E}\frac{d_f\bar{D}}{dt} + \bar{H}\frac{d_f\bar{D}}{dt} \right) dv = 0 \tag{5}$$

$$\frac{d_f\bar{D}}{dt} = 0 \text{ and } \frac{d_f B}{dt} = 0 \text{ lead to the constraints } \psi_{el.s_o},\ \Phi_{m_{s_o}} = const. \tag{5a}$$

The only significant way to enforce the constraint is to keep fluxes (electric and magnetic) constant and, consequently, their derivative zero. Then the change in the energy must equal mechanical work and, as a result:

$$F_j = -\left(\frac{\partial W_{el}}{\partial x_j}\right)_{\psi_e = const.} - \left(\frac{\partial W_m}{\partial x_j}\right)_{\Phi_m = const.} \tag{6}$$

Formulas (5 and 6) show that the constancy of the flux (electric or magnetic) comes naturally from Maxwell's equations and the conservation of energy.

2. The body forces (forces per unit volume) are obtained as an extension of (6a, b), differentiating with respect to time to find the power:

$$-\left(\frac{dW}{dt}\right)_{\substack{\Psi_e = const.\\ \Phi_m = const.}} = \frac{dL}{dt} = \iiint_v \bar{f} \bullet \bar{u}\, dv \qquad (6a)$$

again by using the method of flux derivative, we obtain for the body force $\bar{f}$ an expression (7) containing six terms.

$$\bar{f} = \bar{f}_e + \bar{f}_m = \rho_v \bar{E} - \frac{E^2}{2} \bullet grad\varepsilon + grad\left(\frac{E^2}{2}\tau\frac{\partial\varepsilon}{\partial\tau}\right) + \bar{J} \times \bar{B} - \frac{H^2}{2} \bullet grad\mu + grad\left(\frac{H^2}{2}\tau\frac{\partial\mu}{\partial\tau}\right) \qquad (7)$$

If the magnetostriction term is considered to be less significant to the object of our research, the remaining two terms in the magnetic body force,

$$f_m = \bar{J} \times \bar{B} - \frac{H^2}{2} \bullet grad\mu \qquad (7a)$$

are very significant in showing the difference between iron core and air core machines. Fig. 1 shows an illustration of how the forces split in $\bar{J} \times \bar{B}$ forces and forces applied to the ferromagnetic materials of the slot making it obvious why in the ferromagnetic machines the majority of the stresses will apply on the strong ferromagnetic walls and a significantly smaller amount on the fragile windings. For the air core structures, we are able to use a much larger flux density - obtaining much larger power densities, at the price of a substantially higher excitation magnetomotive forces (obtained usually by self-excitation).

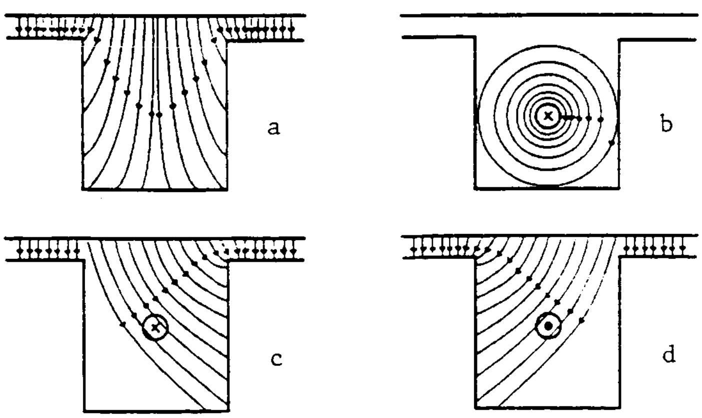

Figure 1. Why a large part of electromagnetic forces stress the ferromagnetic material.

3. The electromagnetic (Maxwell's) stress tensor gives as a third formalism [1] in which the global electromagnetic force $F(N)$ is obtained by integrating stresses $\overline{T}_u$ over an area s_v which, in turn, balances the body forces of electromagnetic origin , integrated over the volume V:

$$\overline{F} = \iiint_v \overline{f} dv = \oint\oint \overline{T}_n ds \qquad (8)$$

We obtain in the final the three components of the tensor on the three axes (x, y, z):

$$\overline{\Gamma}_x = \left[E_x\overline{D} + H_x\overline{B} - \overline{i}\left(\frac{\varepsilon E^2}{2} + \frac{\mu H^2}{2}\right)\right]$$
$$\overline{\Gamma}_y = \left[E_y\overline{D} + H_y\overline{B} - \overline{j}\left(\frac{\varepsilon E^2}{2} + \frac{\mu H^2}{2}\right)\right]$$
$$\overline{\Gamma}_z = \left[E_z\overline{D} + H_z\overline{B} - \overline{k}\left(\frac{\varepsilon E^2}{2} + \frac{\mu H^2}{2}\right)\right] \qquad (8c)$$

However, if the regime is not stationary, the equation (8) becomes:

$$\overline{F} + \frac{d}{dt}\iiint_v (\overline{D} \times \overline{B})dv = \oint\oint_{s_v} (\overline{\overline{T}})(-\overline{n})ds \qquad (8a)$$

leading to the fact that the resultant force in an isolated mechanical system interacting with an electromagnetic system is zero; expressed in terms of the mechanical and electromagnetic momentum where the electromagnetic momentum is $\overline{D} \times \overline{B} = \varepsilon\mu\overline{E} \times \overline{H} = \varepsilon\mu\overline{S}$).

Considering the mechanical momentum in addition to the electromagnetic one:

$$\frac{d}{dt}\left[G_{mech.} + \iiint_v (\overline{D} \times \overline{B})dv\right] = 0 \qquad (8d)$$

The conservation of momentum applies to the sum of both mechanical and electromagnetic momentum.

The expression above is important in evaluating the design questions related to recoil in electromagnetic guns. The volume integral of the electromagnetic force density, the Maxwell's stress tensor, and the electromagnetic momentum become tools in defining and calculating globally and locally the recoil in electromagnetic accelerators [1].

Intrinsic Characteristics of the Rotating Pulsed Power Supplies (RPPS) for Electromagnetic Launchers (EML)

The peculiar nature, "the strangeness", of the RPPS for EML is due not only to constraints related to their very high energy and very high power ratings under conditions of extreme transient characteristics (they operate under load for few milliseconds only), but also to strong differences with respect to other electromechanical power converters.

1) One, casually disregarded, but carrying important theoretical implication, is that such machines belong not to the customary constant voltage systems, but to a totally new world of constant current systems in which the supplied current is always the same and the voltage is variable, according to the power required by the load. Pestarini [12] has described in detail such a world mainly for d.c. machines, calling them Metadynes, and patenting numerous applications - the majority in complex drive systems.

Laithwaite alluded to it [6] and, as Pestarini did years before him, commented about how unprepared the common electrical machine designer is to adapt himself to a system of reference diametrically opposite to the usual practice and requirements.

2) The second intrinsic characteristic of the RPPS is the so-called "compensation." The compensation is actually a misnomer for a complex phenomenon used by the electrical machine designer to adapt the characteristics of the power supply to those needed by the electromagnetic launcher. The term is borrowed from the theory of operation of d. c. machines in which the armature current flows in series through a separate winding (compensation winding) embedded in the pole-pieces of the machine stator, canceling in large part the magnetic field of armature reaction. The success of the operation is assured by the constant (and opposite) relative position in space of the two magnetic fields - armature reaction and compensation - maintained by the brush and commutator system.

In the first approximation, the notion of armature reaction compensation can be generalized to a synchronous generator by using a conductive uniform shield - placed on the excitation poles and opposing the armature. During the machine discharge into the low impedance load (such as the electromagnetic launcher), image currents in the shield produce a magnetic field opposite to the armature reaction field, canceling a large part of it and achieving an important reduction of the internal impedance of the power supply, thus increasing the efficiency of the power transfer. A refinement of the method involves an anisotropic shield, permitting compensation while reducing losses by providing magnetic decoupling during the self-excitation process.

Armature reaction compensation applied to pulsed synchronous generators by means of a conductive shield (continuous or partially distributed) is always imperfect due to the phase shift of the induced currents and their resultant magnetic field and the transient field penetration. Such apparent imperfections are actually used by electrical machine designers in order to achieve the goal of variability of the internal electromagnetic field structure of the RPPS in order to adapt itself to the continuous changing power requirements of the EML and launch package in every instant of acceleration [2, 3, 4]. For example, to the condition of a constant acceleration impressed on the projectile requiring, generally, a constant current power supply under a variable, continuously increasing voltage, which drops sharply at the end of launching.

Phasorial formalism in the Blondel's two reaction theory of synchronous machines, as well as in Doherty and Nickle's, and in Park's transformation [9, 10, 11] shows that the general theory of electrical machinery was conceived for steady-state analysis. Even Kron's [11] generalized theory and its "Application of Tensors to the Analysis of Rotating Machines" is still, in large part, devoted to the same type of treatment. However, the generalized electrical machine theory and the notion of primitive machines offers the best analytical tools for the treatment and evaluation of quality factors (goodness) of the RPPS for EMLs.

Fig. 2 shows the diagram of a synchronous machine as a primitive machine with fictitious axis coils, using the two-axis (direct, *D*, and quadrature *Q*) theory [10]. When compared to the normal, primitive machine, takes into account the compensation winding as a nonuniformly-distributed shield. It can characterize in this way a wide spectrum of "compensation" windings, from continuous shields to very selective and asymmetrical windings, obtained through modification of the parameters of the *m* winding in the direct axis and the *n* windings in the quadrature axis, correspondingly.

Since the pulsed power supply has a highly transient behavior, the equations are written using operational calculus formalism, replacing the phasor formalism used for steady state. The equations for the system of coils are written in function of

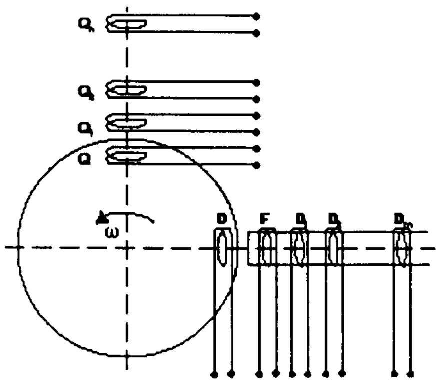

Figure 2. Generalized "primitive" synchronous machine with several (nonuniform) compensation windings.

their self and mutual inductances and, for armature coils including the rotation voltage term, leading in the case of fig. 2 to a set of (m + 2) equations containing (m + 2) currents, as well as the excitation (field) voltage v_f and the magnetic flux linkage in the direct axis ψ_d. The relation between the Laplace transforms of flux, current, and voltage is:

$$\bar{\psi}_d = \frac{X_d(p)}{\omega_O}\bar{i}_d + \frac{G(p)}{\omega_O}\bar{v}_f \qquad (10)$$

where ω_O allows for the proper dimensionality in (1) above.

$X_d(p) = \dfrac{a_{(m+1)}p^{m+1} + a_m p^m + \ldots + a_1 p + a_0}{b_{(m+1)}p^{m+1} + b_m p^m + \ldots + b_1 p + b_0}$ is the direct-axis operational impedance . $G(p)$ has the

same denominator as $X_d(p)$, but a numerator of m order only. For the quadrature axis, the elimination of currents from a set of (n + 1) equations gives, similarly:

$$\overline{\psi}_q = \frac{X_q(p)}{\omega_0}\overline{i}_q \tag{10}$$

where $X_q(p)$ is the quadrature axis operational impedance being the quotient of two polynomials of order n while $\overline{\psi_q}$ is the Laplace transform of the magnetic flux in the quadrature axis.

Equations (9) and (10) apply to the transient regimes. During the steady state operating condition when all the variables in the two-axis reference frame are constant, $p = 0$ and as an example, the direct axis operational impedance $X_d(p)$ degenerates into the direct axis synchronous reactance X_d.

Despite idealizations and linearizations, the operational impedances, $X_d(p)$ and $X_{(q)}(p)$, characterize and define very efficiently a large spectrum of different transient behaviors of RPPS for EMLs. In a preliminary design characterization, the direct axis operational impedance $X_d(p)$ is often used in the form of an operational admittance $Y_d(p)$ expanded into partial fractions. As an example, in the well-known short-circuit characterization of synchronous generators:

$$Y_d(p) = \frac{1}{X_d(p)} = \frac{(1+T_{d'_0}p)(1+T_{d''_0}p)}{(1+T_{d'}p)(1+T_{d''}p)}Y_d \tag{11}$$

where $T_{d'_0}$, $T_{d''_0}$, $T_{d'}$, and $T_{d''}$ are the principal time constants of the synchronous machine. After expanding into partial fractions and using the canonical notations, we obtain the classical:

$$Y_d(p) = \frac{1}{X_{d'}} + \left(\frac{1}{X_{d'}} - \frac{1}{X_d}\right)\frac{T_{d'}p}{1+T_{d'}p} + \left(\frac{1}{X_{d''}} - \frac{1}{X_{d'}}\right)\frac{T_{d''}p}{1+T_{d''}p}. \tag{11a}$$

The last partial fraction represents the compensation winding as one of the three branches in parallel in the equivalent, modified, direct axis equivalent circuit. A similar modified quadrature axis equivalent admittance is used for design evaluation. It is interesting to see the operational admittance transformed for sinusoidal changes of frequency ω in which replacing p by $j\omega$, we obtain the operational admittance frequency locus in fig. 3 (direct axis).

An alternative approach is the use of Kron's re-establishment of transient dynamical equations from the equivalent circuits and the introduction, by using Kron's method of reference frames, of an interconnection clause [11]. In the first approach (fig. 4), if the transient equations of the machine (pulsed generator) or of a group of machines (pulsed generator and electromagnetic launcher) are required, they can be obtained from the equivalent circuit containing the variable frequency f_{ds}, feature. The transient system is found by replacing f by the p/j operator, where j is the imaginary unit. The transformation for the reference frames is described in [11]. The interconnection clause is used not only to study the power supply (compensated synchronous generator) together with the electromagnetic launcher, but also to include the process of self-excitation as the interconnection of separate equivalent circuits.

Quality Indicators and Electrodynamics Similitude Criteria for Classical Electrical Machines

The theory of electrical machine design [7, 8, 9] called in [13] "an esoteric preoccupation of the few" uses a complex methodology to arrive to an optimal design. Such methodology can be formally checked and verified at each logical step, the procedures, being almost canonical, matured for more than a century and reaching asymptotically a level which has remained almost flat for the last forty years. Several quality indicators, similitude criteria, and scaling laws will be reminded to the reader.

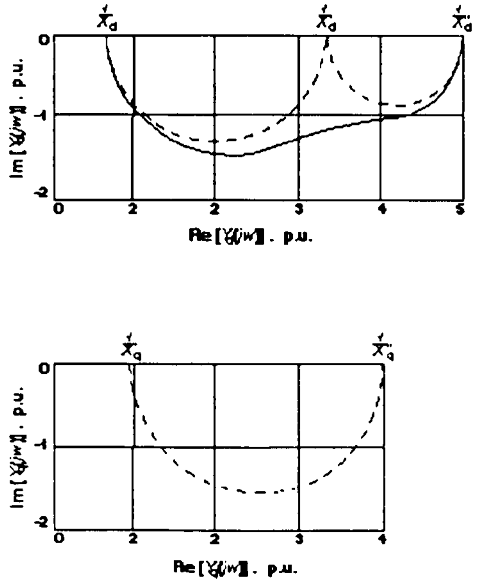

Figure 3. Operational admittance direct axis (d).

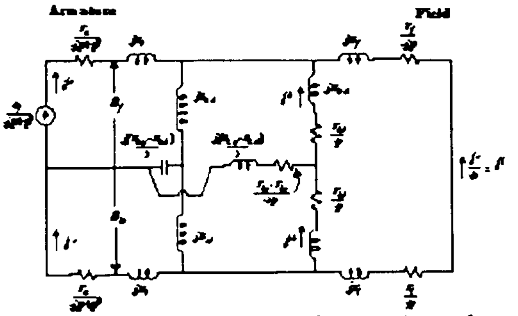

Figure 4. Generalized transient frequency locus on equivalent circuit (Kron).

Electrodynamics Similitude and Scaling Relations

The apparent interior power: $S_i = mEI$ (VA) is the product of number of phases m, phase current $I(A)$ and phase electromotive force $E(v)$. $E = \pi\sqrt{2}\Phi f w k_w (V)$ where: Φ is the fundamental of main magnetic flux (Wb), $f = pn =$ frequency (rps), and k_w = winding factor. The flux Φ can be expressed as:

$$\Phi = B_\delta \frac{2}{\pi} \tau_p l_i \tag{12}$$

where: B_δ = flux density in the air gap (T), τ_p - polar pitch (m), and l_i i= ideal length (m). If the phase current I is expressed in terms of the sheet of current density $A(A/m)$:

$$A = \frac{2mIw}{\pi D} \tag{13}$$

Substituting (13), (14), and (15) in (13'), the fundamental formula relating the apparent power, geometrical dimension and the rotational velocity n (rps) is obtained:

$$S_i = \frac{\pi k_w}{2\sqrt{2}}(2p\Phi)(\pi DA)\ n = \frac{\pi^2 k_w}{\sqrt{2}} D^2 l_i\ n\ AB_\delta = (2p)\sqrt{2}f\ k_w(\tau_p)^2 l_i B_\delta A. \tag{14, 15, 16}$$

For classical machines, the expressions (16a, b, c) are usually translated in the simple form of a "machine constant" as the power per unit speed obtained from the armature unit volume. A more elaborate analysis shows that the power increases faster than the cube of the linear dimensions of the electric machine, let the current density J be expressed as:

$$J = \frac{At_1}{H_{cu}b_{cu}} = \frac{A}{h_c\beta\ k_{cu}}\left(\frac{A}{m^2}\right),\ \beta = \frac{b_c}{t_1}, \text{ and } k_{cu} = h_{cu}b_{cu}/h_c b_c \tag{17a, b, c}$$

where h_c is the height of the slot and , the ratio between the slot width to the slot pitch, and the copper filling coefficient of the slot. The average flux density along the tooth is obtained from:

$$B\zeta k_{Fe} = B_\delta \tag{18}$$

where $\zeta = \frac{b}{t_1}$ is the ratio between the average width of the tooth to the slot pitch and k_{Fe} is the ratio between the net length of the active ferromagnetic iron to the total length of the armature. Then, (14, 15, 16) can be transformed as:

$$S_i = \frac{\pi^2 k_w}{\sqrt{2}} \beta k_{cu}\zeta k_{Fe} D^2 l_i n h_c JB = (2p)f\sqrt{2}\ k_w \frac{\tau^4}{\lambda} k_{cu}\ \ \beta_1 JB \text{ (where } \delta = \tau/l_i), \tag{19a, b}$$

leading to the conclusion that in electrical machines, with similar geometry the apparent power increases proportionally to the linear dimensions to the fourth:

$S_i = kL^4$ or $L = \frac{1}{k'}\sqrt[4]{S_i}$ or $L = \frac{1}{k'}(S_i)^{\frac{1}{4}}$. The weight of machine G is: $G = k_2 L^3 = k_2'(S_i)^{\frac{3}{4}}$.

Active Conductor Electromagnetic Power Conversion Densities

In spite of the appearance of describing the local power density, the density obtained previously is an average one since its definition involves the rated apparent power of the entire machine.

The local power density in the condition of lossless electromechanical power conversion per unit volume of active conductor of the generator, is:

$$-(\bar{J} \times \bar{B}) \bullet \bar{u} = (\bar{u} \times \bar{B}) \bullet \bar{J} \tag{20}$$

which, multiplied by the elementary volume, *dv* is the truly local definition of instantaneous conversion. The "active conductor" is the seat of the power conversion and by the principle of equality of action and reaction, is equally felt in both the stator and rotor of the electromechanical converter.

The flux density $\bar{B}$ in expression (12) is the vectorial sum of the impressed excitation field $\bar{B}_{imp}$ and the induced field by the armature reaction, $\bar{B}_{a,ind}$ (invoking the Lenz rule) such that:

$$\left[-\bar{J} \times \left(\bar{B}_{imp} - \bar{B}_{a,ind}\right)\right] \bullet \bar{u} = \left[\bar{u} \times \left(\bar{B}_{imp} - \bar{B}_{a,ind}\right)\right] \bullet \bar{J}. \tag{21}$$

A "compulsator" exploits the manner in which the two $\bar{B}$ vectors add (or rather subtract) in different moments of discharge. Controlling the compensation, by eliminating partially the direct axis shield and allowing the armature reaction in the initial moment of discharge to almost completely demagnetize the armature field $B_{imp} - B_{a,ind} \approx 0$; and gradually, after that to reach a high degree of compensation in the latest moments of launching, the machine will apply a voltage linearly increasing from zero to maximum, the ideal one for EMLs.

For large RPPS, this solution leads to a more compact system than the one proposed in [14] by Driga in which, in a polyphase compensated generator, the output wave shape, and magnitude are solid-state controlled, thus decoupling the velocity of the generator and output wave control. The same relation applies to the launcher in which the back *emf* is locally described as the motional field $(\bar{u}_a \times \bar{B}_a)$, increasing linearly with the projectile velocity $\bar{u}_a$ and, for a constant current (and constant acceleration), increasing ideally from zero to the exit velocity.

The "Goodness Factor" Criterion: Is It Too Simplistic?

The criterion of "Goodness" of an electrical machine was introduced by Laithwaite in [6] as a means of comparing the relative performance (from the design point of view) of different electrical machines. It had to be an objective measure of the ability of the electrical machine to convert power electromechanically - a property which is more general than, for example, the efficiency which can be increased at the expense of other indicators of vital importance to the machine operation.

Since the electromechanical power conversion is determined by the coupling of the electric and magnetic circuits, Laithwaite defined it as the proportionality:

$$G \approx \frac{1}{\text{Resistance}} \times \frac{1}{\text{Reluctance}} \times \text{ frequency or } G \approx \left(\frac{A_e}{\rho l_e}\right) \times \left(\frac{\mu_0 A_m}{l_m}\right) \times f. \tag{22}$$

This is a simplistic approach to quality of a design and is based on the ability of an electric circuit to produce current *(I)* for a given electromotive force *(E)*, namely (I/E) combined with the ability of a magnetic circuit to produce flux for a given magnetomotive force I', namely (Φ/I). The product (I/E) X (Φ/I) is made dimensionless by multiplying it with the frequency. It is an imperfect approach to "goodness," but leads to interesting inferences. For instance: In the example considered by Laithwaite [6], the length of the magnetic circuit is equal to the thickness of the gap. What happens to the "goodness factor" in the case of air core machines considered, almost exclusively as RPPS for EMLs and made entirely of "gap?"

According to the previous paragraph, the generalization of the "goodness factor" for pulsed, transient machines requires the replacement of the simple reluctance and resistance by corresponding very complex operational impedances in a dimensionless form, containing (as suggested by Kron) a connection clause. Such clause permits adding to the same expression the contribution of the launcher and the self-excitation stages, considered as intermediate interconnected machines. In such generalized "goodness factor," the frequency is replaced by the square root of the ratio between the machine discharge time t_d and the thermal time constant of the power supply τ_t namely $\sqrt{\frac{t_d}{\tau_t}}$. This factor takes into account the scaling of the machine from steady-state to short pulse discharge. Then the "generalized goodness" is

$$GG \approx \frac{\overline{Y}_1(p) \bullet \overline{Y}_2(p) \ldots \overline{Y}_k(p)}{\omega_{01} \bullet \omega_{02} \ldots \omega_{0k}} \sqrt{\frac{t_d}{\tau_t}} \tag{23}$$

where the factors $\omega_{01} \ldots \omega_{0k}$ at the denominator make dimensionless the product of the partial operational admittances at the numerator.

CONCLUSION

By using Maxwell's equations for moving media, three different formulations for electromagnetic forces in electrical machines are introduced, obtaining from them a definite quality assessment for the electromagnetic recoil and force distribution in air and iron core machines.

The introduction of a generalized "goodness factor" for rotating pulsed power supplies and for electromagnetic launchers is possible through the use of interconnected operational admittances, not only for the power supply, but for accelerator and self-excitation system, as well. Such a "goodness factor" is actually a complex transient function describing the comparative interplay of several magnetic and electric circuits in the condition of rapid conversion. Additionally, several other quality indicators and similitude criteria from the classical electrical machine theory must be taken into account.

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