ANGULAR COORDINATE REPETITIVE CONTROL OF MAGNETIC BEARINGS

Yohji Okada, Bunshu Nagai and Katsuhiko Karino
Department of Mechanical Engineering, Ibaraki University
4-12-1 Nakanarusawa, Hitachi 316 JAPAN
Phone +81-294-38-5025, Fax +81-294-38-1229
E-mail: okada@mech.ibaraki.ac.jp

SUMMARY

New repetitive control is applied to active magnetic bearings. It is intended for rotor magnetic bearing systems to reduce unbalance response automatically. Repetitive control is a kind of servo control which is expected to follow periodic command. The follow-up property is based on internal model principles, that is, the controller includes a repetitive-type transfer function. The system also has noise rejection properties from the repetitive disturbance produced by the rotor imbalance. A simple experiment is performed to confirm the proposed technique. The results show good disturbance rejection properties and robustness.

INTRODUCTION

Magnetic bearings have merits of supporting a rotor without physical contact [1] and producing a canceling force to the rotor disturbance. Unbalance response is one of the biggest problems for rotary machinery. Several methods are proposed to cancel this unbalance excitation: Mizuno and Higuchi proposed a disturbance observer-based controller [2]; Hisatani and Koizumi proposed an adaptive filter type controller [3]; Knospe and Tamer proposed a new technique called open loop control [4]. They are fundamentally based on a constant speed controller and requires an adaptive technique for the rotational speed. A rotary machine usually changes its speed. Standard disturbance cancellation control with constant time sampling does not work properly, because the internal model does not exactly match the variable speed imbalance.

This paper proposed a new type of repetitive control which uses angular coordinate sampling intervals [5], [6]. The frequency of this repetitive controller is automatically equalized to the rotor speed and is expected to reduce the rotor unbalance vibration. First, the plant is stabilized with the local PID controller which uses constant time sampling. Then the stabilized magnetic bearing is transformed into the angular coordinates of the rotor at the operating speed. Finally, the angular coordinate repetitive controller is designed using optimal regulator theory [6].

A test apparatus is made to confirm the proposed technique. The results are compared with and without repetitive control. The effect of rotor speed change and the robustness are also tested and discussed. The results show good unbalance rejection properties and robustness and robustness of the proposed repetitive controller.

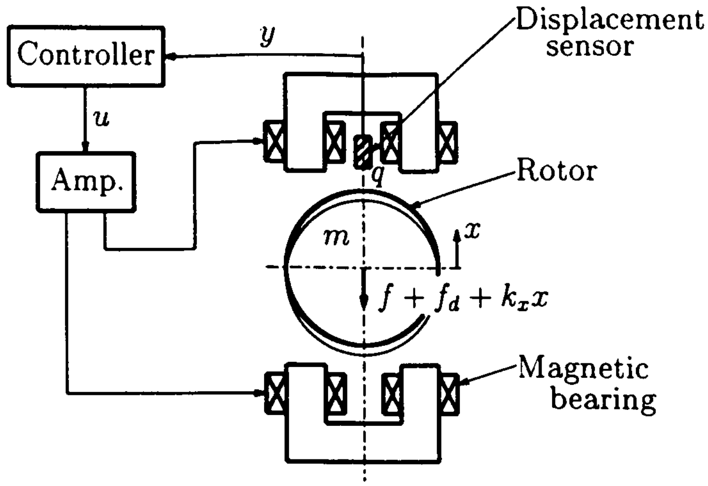

Figure 1: Model of magnetic bearing system

Table 1: The symbols of magnetic bearing system

m	:	Rotor mass	[kg]
k_x	:	Negative stiffness of magnetic bearing	[N/m]
K_f	:	Force factor of actuator	[N/V]
q	:	Sensor gain	[V/m]
x	:	Rotor displacement	[m]
u	:	Control input	[V]
y	:	Sensor output	[V]
f	:	Control force	[N]
f_d	:	Disturbance	[N]

MODELING AND LOCAL PID CONTROL

It is assumed that the coupling between two radial magnetic bearings are ignored and they can be controlled individually. This simplified radial magnetic bearing is shown schematically in Fig. 1. The symbols used are shown in Table 1. Only the vertical degree-of-freedom is indicated in the figure, and another set is necessary to control the horizontal position.

Equations of Motion

Assuming that each control degree is independent, we have the following equation of motion in each direction.

$$m\frac{d^2x(t)}{dt^2} = f(t) + f_d(t) + k_x x(t) \quad f(t) = K_f u(t) \tag{1}$$

Equation (1) can be transformed into the following state equation.

$$\begin{cases} \dot{\boldsymbol{x}}(t) &= \boldsymbol{A}_c\boldsymbol{x}(t) + \boldsymbol{B}_c u(t) + \boldsymbol{E}_c f_d(t) \\ y(t) &= \boldsymbol{C}_c\boldsymbol{x}(t) \end{cases} \tag{2}$$

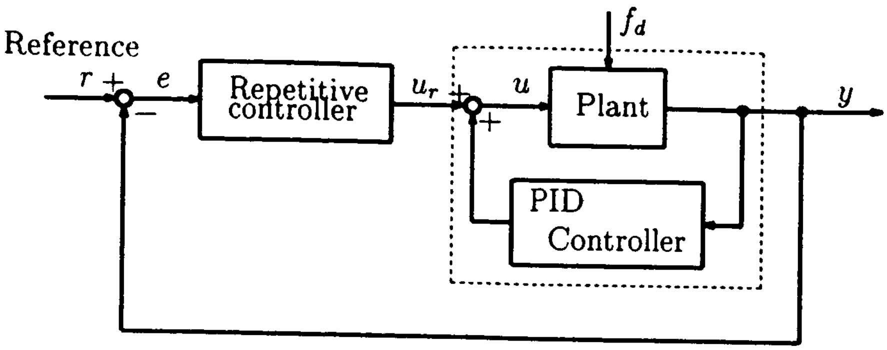

Figure 2: Block diagram of repetitive control system

$$A_c = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad B_c = \begin{bmatrix} 0 \\ \dfrac{K_f}{m} \end{bmatrix} \quad E_c = \begin{bmatrix} 0 \\ \dfrac{1}{m} \end{bmatrix}$$

$$C_c = \begin{bmatrix} q & 0 \end{bmatrix} \quad \boldsymbol{x}(t) = \begin{bmatrix} x(t) & \dot{x}(t) \end{bmatrix}^T$$

PID Control

The stabilizing controller used is the standard PID controller for each control degree of freedom.

$$G(s) = K_p + K_i \frac{1}{s} + K_d s \tag{3}$$

In this paper, Digital Signal Processor (DSP, TI TMS320C30) is used. The following digital PID controller

$$\begin{cases} x_1[k+1] &= e^{-T/T_d} x[k] + u[k] \\ x_2[k+1] &= x_2[k] + Tu[k] \\ y[k] &= \dfrac{K_d}{T}(e^{-T/T_d} - 1)x_1[k] + K_i x_2[k] + (K_p + \dfrac{K_d}{T})u[k] \end{cases} \tag{4}$$

$$\begin{aligned} u[k] &: \text{input signal} \\ x_1[k], x_2[k] &: \text{state variables} \\ y[k] &: \text{output signal} \end{aligned}$$

is installed, where T is the sampling interval and T_D is the derivative time constant [sec], respectively.

ANGULAR COORDINATE REPETITIVE CONTROL

PID controllers can stabilize rotor-magnetic bearing systems. However, unbalance excitation is one of the biggest problems of rotary machines. In the paper, angular coordinate repetitive control is proposed to reduce the unbalance response. First, the system is stabilized using the local PID controller, and then it is transformed into the angular coordinate of shaft position.

Table 2: The signals of repetitive control system

u	:	Control input	[V]
u_r	:	Repetitive control input	[V]
y	:	Control output	[V]
f_{dt}	:	Disturbance	[N]
r	:	Reference	[V]
e	:	Error	[V]

Finally, the repetitive controller is designed using optimal regulator theory. The designed controller has a strong merit that is independent from the rotational speed and the unbalance excitation is automatically reduced by this controller.

Transformation from Time Coordinate to Angular Coordinate

The control system is schematically shown in Fig. 2. The meaning of signals used is summarized in Table 2. The local PID controller is included in the control object of the repetitive controller which is shown in the dotted block in Fig. 2. This control object is written by the following state equation, where $d(t)$ indicates the periodic disturbance.

$$\left\{ \begin{array}{lcl} \dfrac{d\boldsymbol{x}(t)}{dt} & = & \boldsymbol{A'}_c\boldsymbol{x}(t) + \boldsymbol{B}_c u_r(t) + \boldsymbol{E}_c d(t) \\ y(t) & = & \boldsymbol{C}_c\boldsymbol{x}(t) \end{array} \right. \tag{5}$$

$$\boldsymbol{A'}_c = \begin{bmatrix} 0 & 1 \\ \dfrac{K_f K_p}{m} & \dfrac{K_f K_d}{m} \end{bmatrix}$$

Next, let us transform the system into the angular coordinate domain. The transformation requires the following necessary condition.

$$\omega(t) = \frac{d\theta}{dt} > 0 \quad {}^{\forall}t > 0 \tag{6}$$

This requires the existence of the inverse function $t = t(\theta)$ from the rotating angular function $\theta = \theta(t)$. If the necessary condition of eqn. (6) is satisfied, then we can transform eqn. (5) into the following angular coordinate plant equation.

$$\left\{ \begin{array}{lcl} \dfrac{d\boldsymbol{x}(\theta)}{d\theta} & = & \tilde{\boldsymbol{A}}_c\boldsymbol{x}(\theta) + \tilde{\boldsymbol{B}}_c u_r(\theta) + \tilde{\boldsymbol{E}}_c d(\theta) \\ y(\theta) & = & \tilde{\boldsymbol{C}}_c\boldsymbol{x}(\theta) \end{array} \right. \tag{7}$$

$$\tilde{\boldsymbol{A}}_c = \frac{\boldsymbol{A'}_c}{\omega_r} \quad \tilde{\boldsymbol{B}}_c = \frac{\boldsymbol{B}_c}{\omega_r} \quad \tilde{\boldsymbol{C}}_c = \boldsymbol{C}_c \quad \tilde{\boldsymbol{E}}_c = \frac{\boldsymbol{E}_c}{\omega_r}$$

Equation (7) is the linearly approximate transformation from t to θ around the operating speed ω_r [rad/s]. Finally eqn. (7) is discretized using the following sampling angle

$$\Delta\theta = \frac{2\pi}{N} \text{ [rad]} \tag{8}$$

where N is the sampling number of one repetitive period in angular coordinate digital control.

Design of Repetitive Controller

Now the state equation is expanded to include the error of the signal. The expanded system includes the original control object as well as the repetitive error model. First we need to discretize the control object eqn. (7) using the sampling interval of eqn. (8). The control object and the error equations are written by

$$\boldsymbol{x}[k+1] = \boldsymbol{A}\boldsymbol{x}[k] + \boldsymbol{B}u_r[k] + \boldsymbol{E}d[k] \tag{9}$$

$$y[k] = \boldsymbol{C}\boldsymbol{x}[k] \tag{10}$$

$$e[k] = r[k] - y[k] \tag{11}$$

Next a new operator α is defined by

$$\alpha \equiv 1 - z^{-N} \tag{12}$$

where N is the sampling stage number of the repetitive period. That is, let L [sec] be the period of the repetitive disturbance and T_r be the sampling interval, we have the relation of $L = Nt_r$. Remember that z is the angular coordinate sampling operator and change its time interval. The operator α means the difference between the signal and the N sample advanced signal. Applying α to eqn. (11), we have

$$\alpha e[k+1] = \alpha r[k+1] - \alpha y[k+1] \tag{13}$$

Substitute eqns. (9) and (10) into eqn. (13), we have

$$\begin{aligned} e[k+1] = \; & e[k-N+1] + \alpha r[k+1] \\ & -\boldsymbol{C}\boldsymbol{A}\alpha\boldsymbol{x}[k] - \boldsymbol{C}\boldsymbol{B}\alpha u_r[k] - \boldsymbol{C}\boldsymbol{E}\alpha d[k] \end{aligned} \tag{14}$$

Applying α to eqn. (9), we have

$$\alpha\boldsymbol{x}[k+1] = \boldsymbol{A}\alpha\boldsymbol{x}[k] + \boldsymbol{B}\alpha u_r[k] + \boldsymbol{E}\alpha d[k] \tag{15}$$

Considering the periodic property of the reference and disturbance signals and combining eqns. (14) and (15), we can get the expanded state equation of eqn. (16). Where X is an expanded state variable.

$$\boldsymbol{X}[k+1] = \boldsymbol{\Phi}\boldsymbol{X}[k] + \boldsymbol{G}\alpha u_r[k] \tag{16}$$

$$\boldsymbol{\Phi} = \begin{bmatrix} 0 & \cdots & 0 & 1 & -\boldsymbol{C}\boldsymbol{A} \\ 1 & \cdots & 0 & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & \cdots & 1 & 0 & 0 \\ 0 & \cdots & 0 & 0 & \boldsymbol{A} \end{bmatrix} \qquad \boldsymbol{G} = \begin{bmatrix} -\boldsymbol{C}\boldsymbol{B} \\ 0 \\ \vdots \\ 0 \\ \boldsymbol{B} \end{bmatrix}$$

$$\boldsymbol{X}[k] = \begin{bmatrix} e[k] & e[k-1] & \cdots & e[k-N+1] & \alpha\boldsymbol{x}[k] \end{bmatrix}^T$$

The controller is designed using optimum regulator theory. The performance index J is defined by

$$J = \sum_{k=0}^{\infty} \left\{ X^T[k] Q X[k] + \alpha u_r^T[k] h \alpha u_r[k] \right\} \tag{17}$$

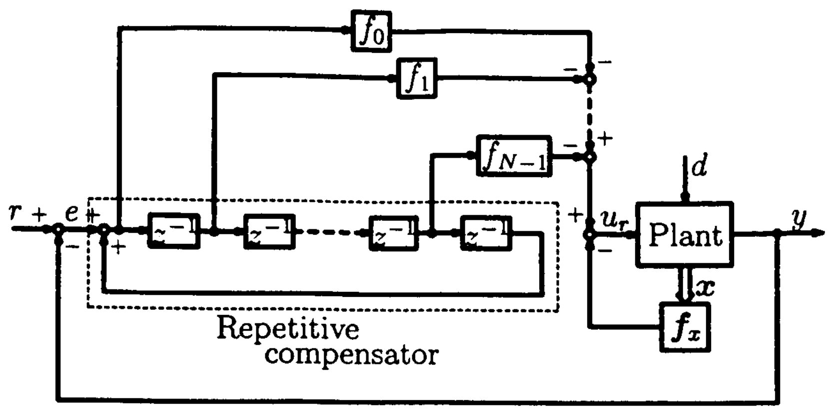

Figure 3: Block diagram of optimal repetitive control system

Where Q is the positive semi-definite weighting matrix for state variable and h is positive definite weight for actuating signal, respectively. Minimizing J, we have the following optimum actuating signal,

$$\begin{aligned} \alpha u_r[k] &= -\begin{bmatrix} f_0 & f_1 & \cdots & f_{N-1} & f_x \end{bmatrix} X[k] \\ &= -\begin{bmatrix} f_0 & f_1 & \cdots & f_{N-1} \end{bmatrix} \begin{bmatrix} e[k] \\ e[k-1] \\ \vdots \\ e[k-N+1] \end{bmatrix} - f_x \alpha x[k] \end{aligned} \tag{18}$$

The feedback gains are given by eqn. (19), where P is the positive definite symmetric solution of the Riccati equation (20).

$$\begin{bmatrix} f_0 & f_1 & \cdots & f_{N-1} & f_x \end{bmatrix} = \left[h + G^T P G \right]^{-1} G^T P \Phi \tag{19}$$

$$P = Q + \Phi^T P \Phi - \Phi^T P G \left[h + G^T P G \right]^{-1} G P \Phi \tag{20}$$

The optimum repetitive control system is schematically shown in Fig. 3. Remember that the local PID controller is included in the control object of the repetitive controller which is shown in the dotted block in Fig. 2.

EXPERIMENTAL RESULTS AND CONSIDERATIONS

To confirm the proposed technique, a test apparatus was made and tested. A schematic of the experimental setup is shown below.

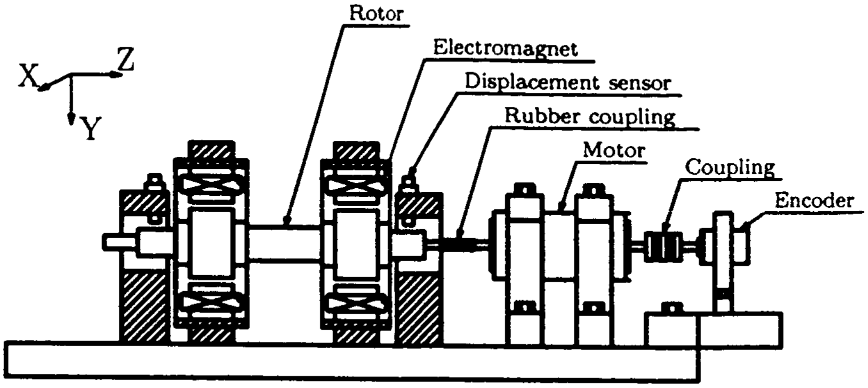

Figure 4: Scheme of experimental setup

Control System

As mentioned before, a digital control system is used, employing a digital signal processor. The control system is shown schematically in Fig. 5. The system parameters used are

$$
\begin{array}{lcll}
m & = & 0.403 & [\mathrm{kg}] \\
K_f & = & 2.104 & [\mathrm{N/V}] \\
q & = & 1000 & [\mathrm{V/m}] \\
K_p & = & 4000 & [\mathrm{V/m}] \\
K_d & = & 25 & [\mathrm{V{\cdot}s/m}] \\
K_i & = & 0 \text{ (for PD) or } 0.35 \text{ (for PID)} & [\mathrm{V/s{\cdot}m}] \\
T & = & 0.0001 & [\mathrm{sec}]
\end{array}
$$

The repetitive controller is designed using the nominal rotating speed of $\omega = 2\pi \times 20$ [rad/s] = 20 [Hz]. The repetitive sampling stage number is $N = 15$. The Riccati equation is solved using the following weightings;

$$Q(1,1) = 10.0 \quad , \quad h = 1.0$$

Results and Discussions

Figure 6 shows the time response of the rotor which runs at 20 [Hz]. In this case, integral control is not used to clarify the effect. The static disturbance is considered as a kind of periodic function and is expected to be canceled with the repetitive control. First the rotor runs without repetitive control. Then the repetitive control is turned on at the sampling number of 130. The periodic disturbances as well as the static error is quickly decreased without increasing the actuating signal. These results indicate the superiority of the proposed optimum repetitive control.

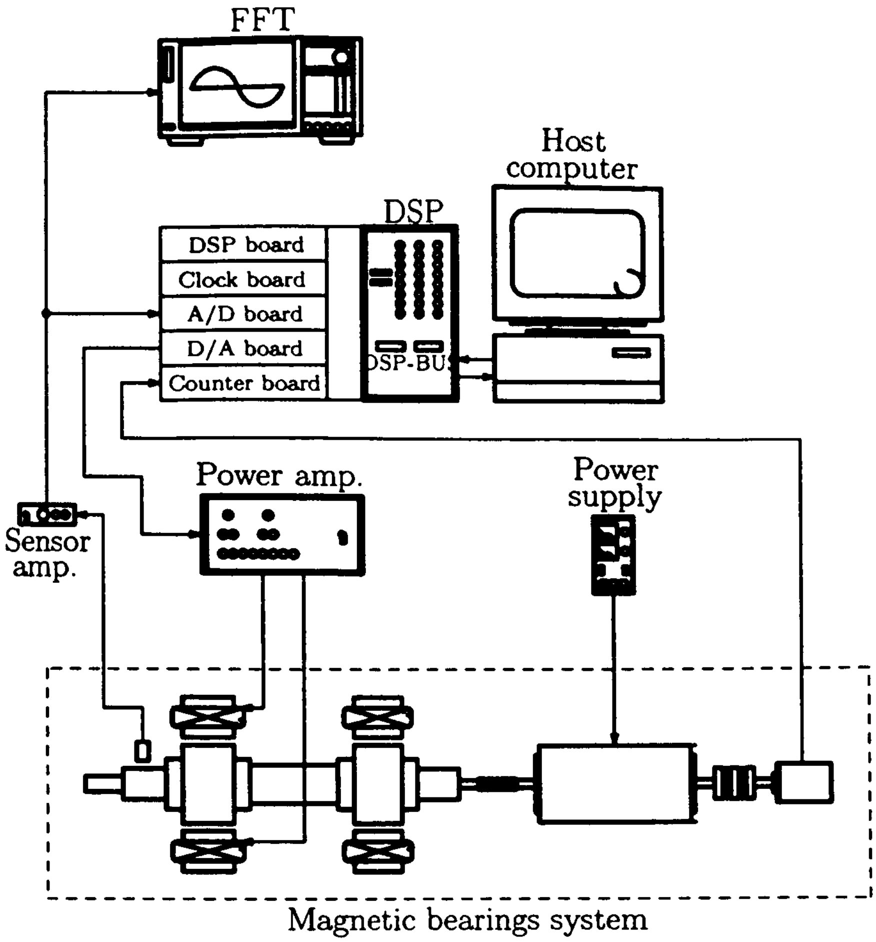

Figure 5: Control system of magnetic bearings

Next, the robustness of the rotational speed change is tested and the results are shown in Fig. 7. The controller is designed using the nominal speed of 20 [Hz], while the responses are tested by changing the rotational speed of 15, 20 and 25 [Hz]. The response with the repetitive controller is indicated by the solid line and is compared with the response without repetitive control. Unbalance response is always reduced even though the rotational speed changes about 25%. These responses indicate the robustness of the proposed optimum repetitive controller.

Finally, repetitive control is applied to the PID controlled magnetic bearings. The responses are shown in Fig. 8. The top graph indicates the horizontal displacement. First, the rotor runs at 20 [Hz] without repetitive control. The repetitive control is turned on at the sampling of 80 and the periodic response decays quickly. The middle graph indicates the expanded response for 500 or 560 sampling which is compared with the response without the good disturbance rejection property of the proposed repetitive control.

In this paper, the repetitive number for one revolution is chosen as 15 and it is relatively small. However, the optimum repetitive controlled indicates relatively good unbalance rejection properties. Increasing the sampling number will decrease higher order repetitive unbalance. Hence a repetition number more than 30 is recommended for the practical digital control system.

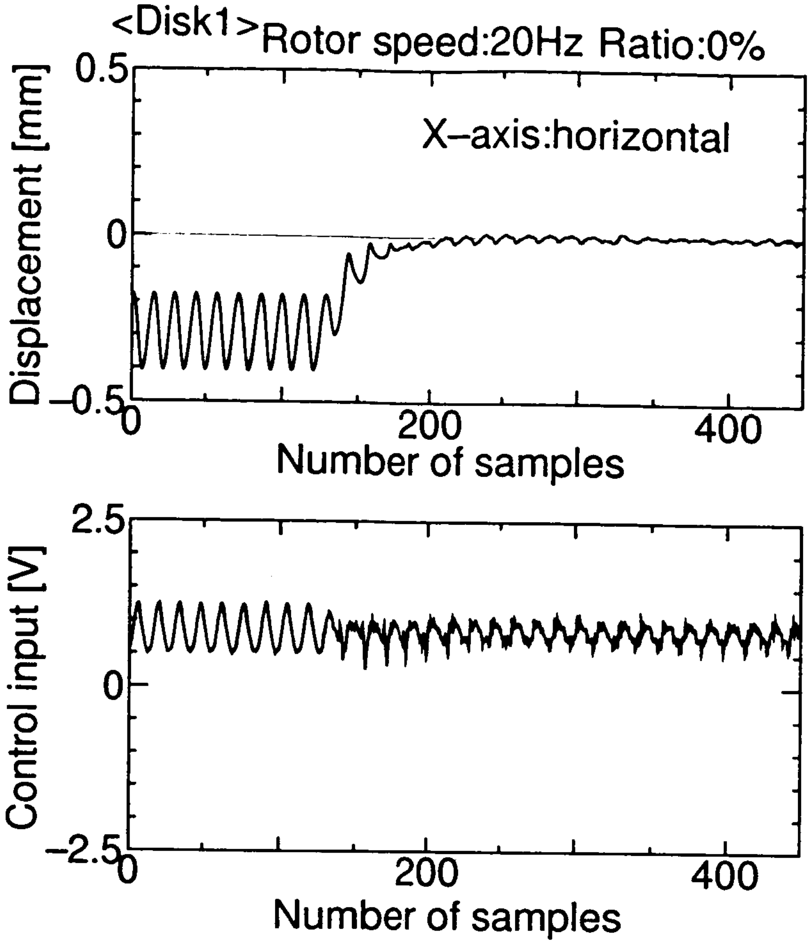

Figure 6: Transient response of optimal repetitive control system

CONCLUSIONS

A new repetitive control is proposed using the angular coordinate sampling domain. The controller is designed using the optimum regulator theory. The designed controller is applied to a solid rotor and tested. The following results are obtained.

- The angular coordinate repetitive controller can reduce the unbalance response automatically even though the rotating speed changes.
- The repetitive control system is perfectly stabilized by optimum regulator theory.
- It is argued that the proposed repetitive control is one of the most suitable unbalance cancellation techniques for rotating machinery.

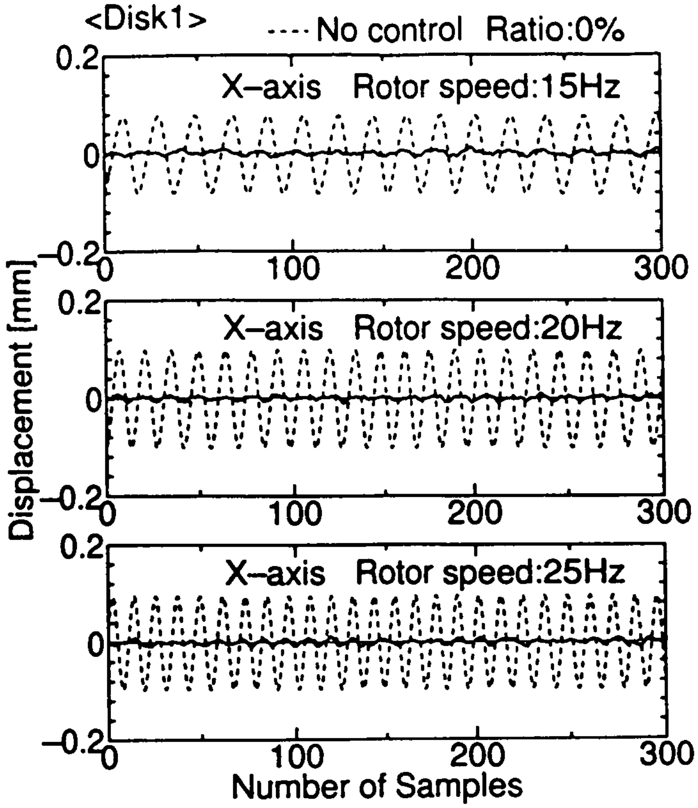

Figure 7: Time response of optimal repetitive control system

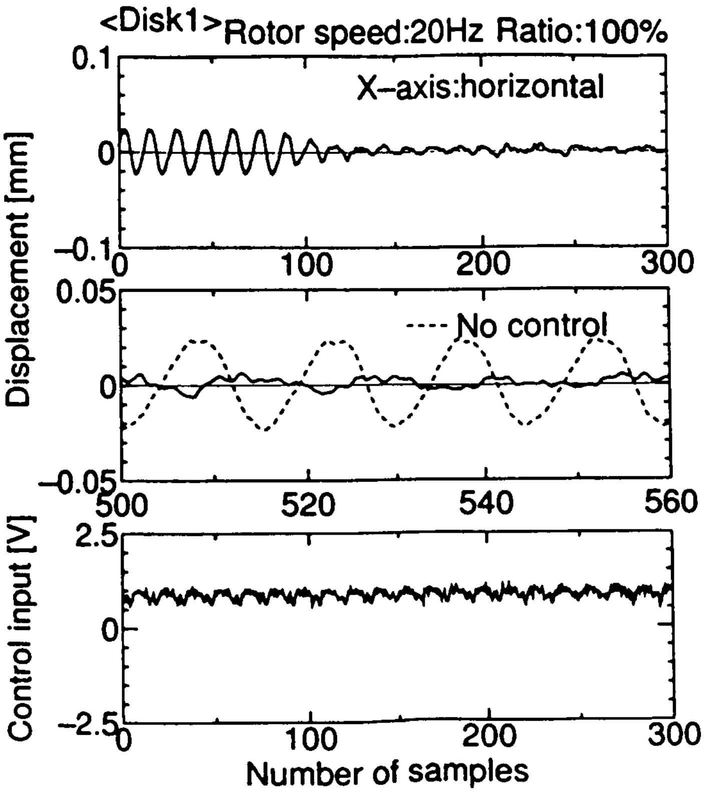

Figure 8: Response of disturbance cancellation and optimal repetitive control system

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