ELIMINATION OF UNBALANCE VIBRATIONS IN VARIABLE SPEED MAGNETIC BEARINGS USING DISCRETE-TIME Q-PARAMETERIZATION CONTROL

JunHo Lee
Department of Electrical and Computer Engineering, Kanazawa University
2-40-20 Kodatsuno, Kanazawa 920, Japan

Abdelfatah M. Mohamed
Dept. of Electrical and Electronics Engineering, Assiut University

Fumio Matsumura
Department of Electrical and Computer Engineering, Kanazawa University
2-40-20 Kodatsuno, Kanazawa 920, Japan

ABSTRACT

In this paper we propose a controller design methodology using the discrete-time Q-parameterization control for variable speed magnetic bearings in order to achieve elimination of unbalance vibrations. Rotor unbalance usually generates sinusoidal disturbance forces, with frequency equal to the rotational speed. So in order to achieve asymptotic rejection of these disturbance forces, the Q-parameterization controller free parameter Q is chosen such that the controller has poles on the unit circle at $z = \exp^{jp_kT_s}$ for the different speeds of rotation p_k (T_s is the sampling period). First, we give a mathematical model for the magnetic bearing in state space form. Second, we explain the proposed discrete-time Q-parameterization controller design methodology. The controller free parameter Q is assumed to be a proper stable transfer function whose order equals twice the number of operating speeds. Third, we show that the controller free parameter which satisfies the design objectives can be obtained by simply solving a set of linear equations rather than solving a complicated optimization problem. We also show that the controller order equals: Number of degrees of freedom $\times$ (order of Q +3). Finally, several simulation and experimental results were obtained to evaluate the proposed controller. The results obtained showed the effectiveness of the proposed controller in eliminating the unbalance vibrations at the different speeds of rotation.

Keywords: Q-parameterization, Magnetic bearings, Vibrations, Rotor unbalance.

INTRODUCTION

Unbalance in the rotor of rotating machines causes vibrations due to the sinusoidal disturbance forces generated by the unbalance. This problem can be solved using active controlled magnetic bearing systems. There are several papers in the literature which deal with this problem [1]-[7] using either feedback control or Notch filters to eliminate these vibrations.

The Q-parameterization theory [8]-[9] provides a good tool for the controller design of magnetic bearing systems in order to achieve elimination of rotor vibrations [5]. This is because the controller free Q-parameter can be chosen such that asymptotic rejection of sinusoidal disturbances is achieved. The order of the Q-parameterization controller equals the order of the plant plus the order of the free parameter Q while in the other methods, the order of the controller equals the order of the plant plus the order of the weighting functions. Usually the Q-parameterization controller has lower order.

In this paper we extend the controller design methodology developed in [5] to achieve unbalance compensation for variable speed magnetic bearing systems, moreover we design the controller in the discrete-time domain. Since the frequency of the unbalance sinusoidal disturbance forces equals the rotational speed, we can achieve asymptotic rejection of these variable frequency disturbances by designing a controller which has poles on the unit circle at $z = \exp^{jp_kT_s}$ for the different speeds of rotation p_k, $k = 1, 2, ...r$ where r is the number of operating speeds and T_s is the sampling period. This can be done by a suitable choice of the controller free parameter Q. The controller free parameter Q is assumed to be a proper stable transfer function whose order equals twice the number of operating speeds. With this assumption the controller order is shown to equal: Number of degrees of freedom $\times$ (order of Q +3). We also show in this paper that the controller free parameter Q which satisfy our design objectives can be obtained by simply solving a set of linear equations rather than solving a complicated optimization problem as for example in the H_∞ synthesis control. A 36 state controller is obtained for the magnetic bearing operating at three different speeds. Several simulation and experimental results were obtained. The results showed that vibrations are eliminated at the different operating speeds using the proposed controller.

MATHEMATICAL MODEL FOR THE MAGNETIC BEARING

A- Equations of Motion

Consider the magnetic bearing system shown in Fig. 1.

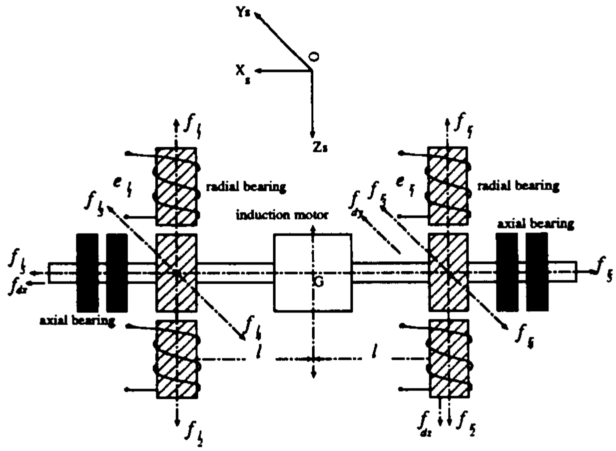

Fig. 1 Schematic diagram of the magnetic bearing system.

Table 1: Parameters of the Magnetic Bearing [9]

Parameter	Symbol	Value	Unit
Mass of the Rotor	m	1.39×10^1	kg
Moment of Inertia about X	J_x	1.348×10^{-2}	$kg \cdot m^2$
Moment of Inertia about Y	J_y	2.326×10^{-1}	$kg \cdot m^2$
Distance between Mass Center and Electromagnet	l	1.30×10^{-1}	m
Steady Attractive Force	$F_{l1,r1}$	9.09×10	N
	$F_{l2\sim4,r2\sim4}$	2.20×10	N
Disturbance Forces	$f_{dy}, f_{d\psi}, f_{dx}, f_{d\theta}$	?	N
Steady Current	$I_{l1,r1}$	6.3×10^{-1}	A
	$I_{l2\sim4,r2\sim4}$	3.1×10^{-1}	A
Steady Gap	G_{oj}	5.5×10^{-4}	m
Resistance	R	1.07×10	Ω
Inductance	L	2.85×10^{-1}	H
Rotational speed	p	$2\pi 30, 2\pi 20, 2\pi 10$	rad/sec

Assuming that the rotor is a rigid floating body, the fundamental equations of motion of the rotor for the four radial degrees of freedom [5] are

$$
\begin{aligned}
\ddot{y}_s &= \frac{\alpha y_s}{m} + \frac{1}{m}(f_{l3} - f_{l4} + f_{r3} - f_{r4} + f_{dy}) \\
\ddot{z}_s &= \frac{\alpha z_s}{m} + \frac{1}{m}(f_{l2} - f_{l1} + f_{r2} - f_{r1} + mg + f_{dz}) \\
\ddot{\theta} &= -\frac{pJ_x}{J_y}\dot{\psi} + \frac{l}{J_y}(f_{l1} - f_{l2} + \frac{1}{2}f_{d\theta} + f_{r2} - f_{r1} + \frac{1}{2}f_{d\theta}) \\
\ddot{\psi} &= \frac{pJ_x}{J_y}\dot{\theta} + \frac{l}{J_y}(f_{l3} - f_{l4} + \frac{1}{2}f_{d\psi} + f_{r4} - f_{r3} + \frac{1}{2}f_{d\psi})
\end{aligned} \quad (1)
$$

The axial motion ($\mathbf{X}$ motion) is independent of the radial motion ($\mathbf{Y}, \mathbf{Z}, \Theta, \Psi$), can be controlled separately and it is not considered in this paper. All the parameters in Eq. (1) are defined in Table 1.

B- Electromagnetic Equations

1- Force Equation

The electromagnetic force f_j produced by the j th electromagnet can be expressed in terms of the coil current i_j and the gap length g_j as follows:

$$
f_j = k(\frac{i_j}{g_j})^2, \quad j = l1, l2, l3, l4, r1, r2, r3, r4 \quad (2)
$$

where k is a constant

2- Coil Voltage Equation

The voltage e_j across the j th electromagnet coil, can also be expressed in terms of i_j, electromagnet coil inductance L, and electromagnet coil resistance R as follows:

$$
e_j = L\frac{di_j}{dt} + Ri_j, \quad j = l1, l2, l3, l4, r1, r2, r3, r4 \quad (3)
$$

C- Linearization

Let F_{oj}, I_{oj}, G_{oj}, and E_{oj} be the nominal values of the force, coil current, gap length and electromagnet voltage of the jth electromagnet respectively, and let f'_j, i'_j, g'_j, e'_j be the deviation of these quantities from their nominal values. Then we can write $f_j = F_{oj} + f'_j$, $i_j = I_{oj} + i'_j$, $g_j = G_{oj} + g'_j$, $e_j = E_{oj} + e'_j$, where

$$
\begin{aligned}
f'_j &= c_j i'_j + d_j g'_j, \\
e'_j &= L\frac{di'_j}{dt} + Ri'_j
\end{aligned} \quad (4)
$$

and $c_j, , d_j$ are the linearization constants which are given by

$$
\begin{aligned}
c_j &= \frac{2kI_{oj}}{G_{oj}^2}, \\
d_j &= \frac{-2kI_{oj}^2}{G_{oj}^3}
\end{aligned} \quad (5)
$$

The gap deviations vector $\mathbf{g}$ can be expressed in terms of $y_s, z_s, \theta, \psi, l$ as follows:

$$\mathbf{g} = \begin{pmatrix} g'_{l1} \\ g'_{r1} \\ g'_{l3} \\ g'_{r3} \end{pmatrix} = - \begin{pmatrix} g'_{l2} \\ g'_{r2} \\ g'_{l4} \\ g'_{r4} \end{pmatrix} = \begin{pmatrix} (z_s - l\theta) \\ (z_s + l\theta) \\ (-y_s - l\psi) \\ (-y_s + l\psi) \end{pmatrix}. \tag{6}$$

Note that for the horizontal shaft magnetic bearing system

$$c_{l1} = c_{r1}, \ c_{l2} = c_{r2}, \ c_{l3} = c_{l4} = c_{r3} = c_{r4}$$

$$d_{l1} = d_{r1}, \ d_{l2} = d_{r2}, \ d_{l3} = d_{l4} = d_{r3} = d_{r4}$$

Assume that the coil voltages are controlled such that

$$e'_{l1} = -e'_{l2}, \ e'_{r1} = -e'_{r2}, \ e'_{l3} = -e'_{l4}, \ e'_{r3} = -e'_{r4} \tag{7}$$

then we have

$$i'_{l1} = -i'_{l2}, \ i'_{r1} = -i'_{r2}, \ i'_{l3} = -i'_{l4}, \ i'_{r3} = -i'_{r4} \tag{8}$$

$$\begin{aligned} f'_{l1} - f'_{l2} &= (c_{l1} + c_{l2})i'_{l1} + (d_{l1} + d_{l2})g'_{l1} \\ f'_{r1} - f'_{r2} &= (c_{r1} + c_{r2})i'_{r1} + (d_{r1} + d_{r2})g'_{r1} \\ f'_{l3} - f'_{l4} &= 2c_{l3}i'_{l3} + 2d_{l3}g'_{l3} \\ f'_{r3} - f'_{r4} &= 2c_{r3}i'_{r3} + 2d_{r3}g'_{r3} \end{aligned} \tag{9}$$

Also from Equation (6), we have

$$\begin{aligned} (g'_{l3} + g'_{r3}) &= -2y_s \\ (g'_{l1} + g'_{r1}) &= 2z_s \\ (g'_{l1} - g'_{r1}) &= -2l\theta \\ (g'_{l3} - g'_{r3}) &= -2l\psi \end{aligned} \tag{10}$$

Substituting Equations (9)–(10) in Equation (1), we then have

$$\begin{aligned} \ddot{y}_s &= \frac{1}{m}(\alpha - 4d_{l3})y_s + \frac{2}{m}c_{l3}(i'_{l3} + i'_{r3}) + \frac{1}{m}f_{dy} \\ \ddot{z}_s &= \frac{1}{m}(\alpha - 2(d_{l1} + d_{l2}))z_s - \frac{1}{m}(c_{l1} + c_{l2})(i'_{l1} + i'_{r2}) + \frac{1}{m}f_{dz} \\ \ddot{\theta} &= -\frac{pJ_x}{J_y}\dot{\psi} + \frac{l}{J_y}\Big((d_{l1} + d_{l2})(-2l\theta) + (c_{l1} + c_{l2})(i'_{l1} - i'_{r1})\Big) + \frac{l}{J_y}f_{d\theta} \\ \ddot{\psi} &= \frac{pJ_x}{J_y}\dot{\theta} + \frac{l}{J_y}\Big(2d_{l3}(-2l\psi) + 2c_{l3}(i'_{l3} - i'_{r3})\Big) + \frac{l}{J_y}f_{d\psi} \end{aligned} \tag{11}$$

Let $e_y = e'_{l3} + e'_{r3}$, $e_z = e'_{l1} + e'_{r1}$, $e_\theta = e'_{l1} - e'_{r1}$, $e_\psi = e'_{l3} - e'_{r3}$,
then from Equation (4) we have

$$\begin{aligned} e_y &= L\frac{d(i'_{l3} + i'_{r3})}{dt} + R(i'_{l3} + i'_{r3}) \\ e_z &= L\frac{d(i'_{l1} + i'_{r1})}{dt} + R(i'_{l1} + i'_{r1}) \\ e_\theta &= L\frac{d(i'_{l1} - i'_{r1})}{dt} + R(i'_{l1} - i'_{r1}) \\ e_\psi &= L\frac{d(i'_{l3} - i'_{r3})}{dt} + R(i'_{l3} - i'_{r3}) \end{aligned} \tag{12}$$

D- State Space Representation

Let

$$\mathbf{x_y} = \begin{pmatrix} y_s \\ \dot{y}_s \\ i_y \end{pmatrix}, \ \mathbf{x_z} = \begin{pmatrix} z_s \\ \dot{z}_s \\ i_z \end{pmatrix}, \ \mathbf{x}_\theta = \begin{pmatrix} l\theta \\ l\dot{\theta} \\ i_\theta \end{pmatrix}, \ \mathbf{x}_\psi = \begin{pmatrix} l\psi \\ l\dot{\psi} \\ i_\psi \end{pmatrix},$$

$$\mathbf{x_g} = \begin{pmatrix} \mathbf{x_y} \\ \mathbf{x}_\psi \\ \mathbf{x_z} \\ \mathbf{x}_\theta \end{pmatrix}, \ \mathbf{u} = \begin{pmatrix} u_y \\ u_\psi \\ u_z \\ u_\theta \end{pmatrix} = \begin{pmatrix} e_y \\ e_\psi \\ e_z \\ e_\theta \end{pmatrix}, \ \mathbf{y} = \begin{pmatrix} y_s \\ l\psi \\ z_s \\ l\theta \end{pmatrix}, \ \mathbf{f_d} = \begin{pmatrix} f_{dy} \\ f_{d\psi} \\ f_{dz} \\ f_{d\theta} \end{pmatrix}, \ \mathbf{z_r} = \begin{pmatrix} y_s \\ l\psi \\ z_s \\ l\theta \end{pmatrix}$$

Then from Equations (11) and (12) we have

$$\begin{aligned} \dot{\mathbf{x}}_\mathbf{g} &= \mathbf{A_g}(p)\mathbf{x_g} + \mathbf{B_g}(\mathbf{u}+\mathbf{v}) + \mathbf{E_g}\mathbf{f_d} \\ \mathbf{y} &= \mathbf{C_g}\mathbf{x_g} + \mathbf{n} \\ \mathbf{z_r} &= \mathbf{C_g}\mathbf{x_g}, \end{aligned} \tag{13}$$

where $\mathbf{z_r}$ is the variable that needs to be regulated, $\mathbf{v}$ represents actuator noise, $\mathbf{n}$ represents sensor noise and

$$\mathbf{A_g}(p) = \begin{pmatrix} A_y & 0 & 0 & 0 \\ 0 & A_\psi & 0 & A_{\psi\theta} \\ 0 & 0 & A_z & 0 \\ 0 & A_{\theta\psi} & 0 & A_\theta \end{pmatrix}, \ \mathbf{B_g} = \begin{pmatrix} b_y & 0 & 0 & 0 \\ 0 & b_\psi & 0 & 0 \\ 0 & 0 & b_z & 0 \\ 0 & 0 & 0 & b_\theta \end{pmatrix},$$

$$\mathbf{C_g} = \begin{pmatrix} c_y & 0 & 0 & 0 \\ 0 & c_\psi & 0 & 0 \\ 0 & 0 & c_z & 0 \\ 0 & 0 & 0 & c_\theta \end{pmatrix}, \ \mathbf{E} = \begin{pmatrix} e_{dy} & 0 & 0 & 0 \\ 0 & e_{d\psi} & 0 & 0 \\ 0 & 0 & e_{dz} & 0 \\ 0 & 0 & 0 & e_{d\theta} \end{pmatrix},$$

$$A_y = \begin{pmatrix} 0 & 1 & 0 \\ \frac{(\alpha-4d_{l3})}{m} & 0 & \frac{2c_{l3}}{m} \\ 0 & 0 & \frac{-R}{L} \end{pmatrix}, \ A_z = \begin{pmatrix} 0 & 1 & 0 \\ \frac{(\alpha-2(d_{l1}+d_{l2}))}{m} & 0 & \frac{-(c_{l1}+c_{l2})}{m} \\ 0 & 0 & \frac{-R}{L} \end{pmatrix},$$

$$A_\theta = \begin{pmatrix} 0 & 1 & 0 \\ \frac{-2(d_{l1}+d_{l2})}{m_1} & 0 & \frac{(c_{l1}+c_{l2})}{m_1} \\ 0 & 0 & \frac{-R}{L} \end{pmatrix}, \ A_\psi = \begin{pmatrix} 0 & 1 & 0 \\ \frac{-4d_{l3}}{m_1} & 0 & \frac{2c_{l3}}{m_1} \\ 0 & 0 & \frac{-R}{L} \end{pmatrix},$$

$$A_{\theta\psi} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{-pJx}{J_y} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \ A_{\psi\theta} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{pJ_x}{J_y} & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$b_y = b_z = b_\theta = b_\psi = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{L} \end{pmatrix}, \ c_y = c_z = c_\theta = c_\psi = (1 \quad 0 \quad 0),$$

$$e_{dy} = e_{dz} = \begin{pmatrix} 0 \\ \frac{1}{m} \\ 0 \end{pmatrix}, \ e_{d\theta} = e_{d\psi} = \begin{pmatrix} 0 \\ \frac{1}{m_1} \\ 0 \end{pmatrix}, \ m_1 = J_y/l^2$$

The subscripts y, z, θ and ψ denote the $\mathbf{Y}$, $\mathbf{Z}$, $\mathbf{\Theta}$ and $\mathbf{\Psi}$ motions respectively. Note that if the rotational speed $p = 0$ the system can be divided into four separate SISO systems. However if $p \neq 0$ the system can be divided into three separate subsystems, two SISO systems and one, two-input two-output system. In both cases the electromagnets coil voltage are given as follows

$$\begin{aligned} e'_{l1} &= -e'_{l2} = (u_z + u_\theta)/2, \\ e'_{r1} &= -e'_{r2} = (u_z - u_\theta)/2, \\ e'_{l3} &= -e'_{l4} = (u_y + u_\psi)/2, \\ e'_{r3} &= -e'_{r4} = (u_y - u_\psi)/2 \end{aligned} \tag{14}$$

In this paper we design the controller for $p = 0$.

DISCRETE-TIME Q-PARAMETERIZATION CONTROL

The discretization of each subsystem defined by Eq. (14) using a zero order hold at a sampling time of T_s *sec* [15] yields the following SISO discrete-time control system:

$$\begin{aligned} x(k+1) &= A_d x(k) + b_d u(k) + b_d v(k) + e_d f_d(k) \\ y(k) &= c_d(k) + n(k) \\ z_r(k) &= c_d x(k) \end{aligned} \qquad (15)$$

A- The Q-parameterization theory

The Q-parameterization theory [11]-[12] states that the set of all stabilizing controllers of a given plant $G(z)$ can characterized by one free parameter (one-parameter-control feedback) or two free parameters (two-parameter-control feedback) namely Q_1 and Q_2 .

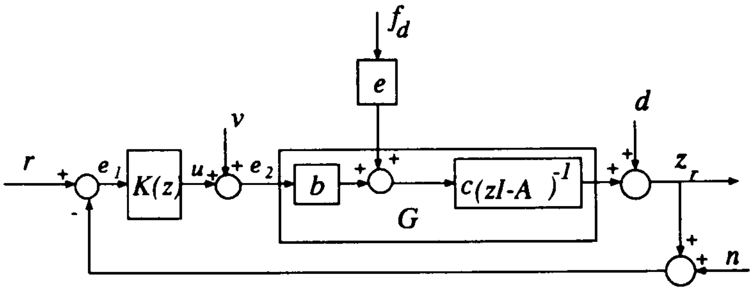

Fig. 2 One-Parameter-Control Feedback System.

Consider the one-parameter-control feedback system shown in Fig. 2, for controlling any of the SISO subsystems described by Eq. (13), where $z_{ref} \in \mathbb{R}$ is the reference (command) input signal, $v \in \mathbb{R}$ is the actuator noise, $n \in \mathbb{R}$ is the sensor noise, $f_d \in \mathbb{R}$ is the disturbance force, $d \in \mathbb{R}$ is the output disturbance, $u \in \mathbb{R}$ is the controller output, $z \in \mathbb{R}$ is the plant output to be regulated, and $K \in R$ is the stabilizing controller for $G(s)$. Note that v, n, and f_d may also represent model uncertainties. In order to characterize the set of all stabilizing controllers K for $G(z)$, first we need to construct a doubly coprime factorization (see [11] for details) $N, D, \tilde{N}, \tilde{D}, X, Y, \tilde{X}, \tilde{Y} \in RH_\infty$ for $G(z)$. First we choose real matrices f_1 and f_2 such that the matrices $A_o := A_d - b_d f_1$ and $\tilde{A}_o := A_d - f_2 c_d$ are stable (all the eigenvalues of A_o and $\tilde{A}_o$ lie inside the unit circle), then the doubly coprime factorization $N(z), D(z), \tilde{N}(z), \tilde{D}(z), X(z), Y(z), \tilde{X}(z), \tilde{Y}(z) \in RH_\infty$ for $G(z)$ is given as follows

$$\begin{aligned} N(z) &= c_d(zI - A_o)^{-1} b_k \\ D(z) &= I - f_1(zI - A_o)^{-1} b_d \\ \tilde{N}(z) &= c_d(zI - \tilde{A}_o)^{-1} b_d \\ \tilde{D}(z) &= I - c_d(sI - \tilde{A}_o)^{-1} f_2 \\ X(z) &= f_1(zI - \tilde{A}_o)^{-1} f_2 \\ Y(z) &= I + f_1(zI - \tilde{A}_o)^{-1} b_d \\ \tilde{X}(z) &= f_1(zI - A_o)^{-1} f_2 \\ \tilde{Y}(z) &= I + c_d(zI - A_o)^{-1} f_2 \end{aligned} \qquad (16)$$

Then the set of all stabilizing controllers for $G(z)$ is given by

$$K(z) = \{(Y(z) - Q(z)\tilde{N}(z))^{-1}(X(z) + Q(z)\tilde{D}(z)),\ Q(z) \in RH_\infty,\ \left|Y(z) - Q(z)\tilde{N}(z)\right| \neq 0\}. \qquad (17)$$

B- Controller Objectives

The following controller objectives are imposed

1. We need to achieve robust stability against speed and other parameter variation; and achieve fast and well damped transient response
2. We need to achieve rejection of low frequency **disturbances**
3. We need to achieve asymptotic rejection of the class of sinusoidal disturbance with frequency equal to the rotational speed p, in order to compensate for the unbalance.

C- Controller Synthesis

1. In order to satisfy requirement No. 1, the closed loop poles must be located at a prescribed region in the open left half plane. This can be achieved by choosing $N, D, \tilde{N}, \tilde{D}, X, Y, \tilde{X}, \tilde{Y}, Q \in D_s$ where D_s is a subset of RH_∞ defined as shown in Fig. 3.

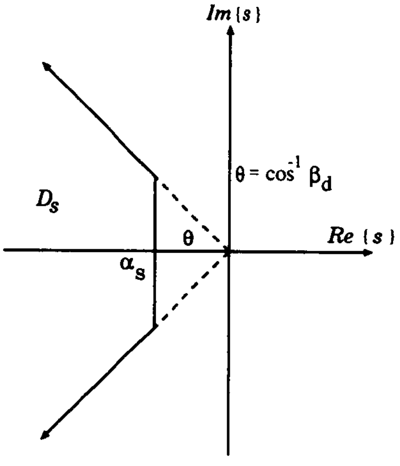

Fig. 3 Generalized region of stability.

2. In order to satisfy requirement No. 2, the controller must have a pole at $z = 1$. This can be achieved by choosing $Q(z)$ such that the following identity holds

$$K(z=1) = \infty \tag{18}$$

From Eq. (17), $Q(z)$ must satisfy the following Equation

$$Y(z=1) - Q(z=1)\tilde{N}(z=1) = 0 \tag{19}$$

3. In order to satisfy requirement No. 3, the controller free parameter $Q(z)$ must be chosen such that the controller has poles at $z = \exp^{\pm jpT_s}$. This can be achieved by choosing $Q(z)$ such that the following identity holds

$$K(z = \exp^{jpT_s}) = \infty \tag{20}$$

Let us assume that the operating speeds of the magnetic bearing are $p_1, p_2, \ldots p_r$. Then at each operating speed the frequency of the generated unbalance sinusoidal disturbance forces equals p_k, $k = 1, 2, \ldots r$. In order to achieve asymptotic rejection for the class of sinusoidal disturbance with variable frequencies, the following condition is imposed on $Q(z)$

$$K(z = \exp^{jp_k T_s}) = \infty, \qquad k = 1, 2, ...r \tag{21}$$

From Equation (17) we have

$$Y(z = \exp^{jp_k T_s}) - Q(z = \exp^{jp_k T_s})\tilde{N}(z = \exp^{jp_k T_s}) = 0, \qquad k = 1, 2, ...r \tag{22}$$

Equations (22) are in fact $2r$ equations, r equations for the real part and r equations for the imaginary part. Equations (19) and (22) indicate that we have $2r+1$ equations in $Q(z)$, this suggests that $Q(z)$ can take the form

$$Q(z) = a_0 + \frac{a_1}{(z - z_1)} + \frac{a_2}{(z - z_2)} + ... + \frac{a_n}{(z - z_n)} \tag{23}$$

where $a_0, a_1, ...a_n \in \mathbb{R}$ are free design parameters and $z_1, z_2, ...z_n > \alpha_s \in \mathbb{R}$ are fixed. Note that Q is a proper stable transfer function whose order equals twice the number of operating speeds.

Theoretically we should be able to design a controller which achieves asymptotic rejection for the class of sinusoidal disturbance of variable frequencies. However in this case as the order of Q gets higher, so does the controller. In this case the practical implementation of the controller becomes difficult. Model reduction techniques [14] must be used to reduce the order of the controller. In this paper we design a controller for magnetic bearings rotating at three different speeds p_1, p_2, and p_3, so $Q(z)$ is chosen as follows:

$$Q(z) = a_0 + \frac{a_1}{(z - z_1)} + \frac{a_2}{(z - z_2)} + \frac{a_3}{(z - z_3)} + \frac{a_4}{(z - z_4)} + \frac{a_5}{(z - z_5)} + \frac{a_6}{(z - z_6)} \tag{24}$$

Then we have

$$Q(z = 1) = a_0 + \frac{a_1}{(1 - z_1)} + \frac{a_2}{(1 - z_2)} + \frac{a_3}{(1 - z_3)} + \frac{a_4}{(1 - z_4)} + \frac{a_5}{(1 - z_5)} + \frac{a_6}{(1 - z_6)} \tag{25}$$

$$Q(zp_k) = a_0 + \frac{a_1}{(zp_k - z_1)} + \frac{a_2}{(zp_k - z_2)} + \frac{a_3}{(zp_k - z_3)} + \frac{a_4}{(zp_k - z_4)} + \frac{a_5}{(zp_k - z_5)} + \frac{a_6}{(zp_k - z_6)}, \qquad k = 1, 2, 3 \tag{26}$$

where $zp_k = \exp^{jp_k T_s}$ Eqs. (25) and (26) are in fact seven linear equations in the seven unknown free design parameters a_0, a_1, a_2, a_3, a_4, a_5, a_6. In order to solve Eqs. (25) and (26) for a_0, a_1, a_2, a_3, a_4, a_5, a_6 we need first to solve Eqs. (19) and (22) for $Q(z = 1)$ and $Q(z = \exp^{jp_k T_s})$, $k = 1, 2, 3$. Eqs. (19) and (22) are also linear equations in $Q(z = 1)$ and $Q(z = \exp^{jp_k T_s})$, $k = 1, 2, 3$. From Eqs. (19) and (22) we have

$$\begin{aligned} Q(z = 1) &= Y(z = 1)\tilde{N}^{-1}(z = 1), \\ Q(z = \exp^{jp_k T_s}) &= Y(z = \exp^{jp_k T_s})\tilde{N}^{-1}(z = \exp^{jp_k T_s}), \qquad k = 1, 2, 3 \end{aligned} \tag{27}$$

Then the design parameters a_0, a_1, a_2, a_3, a_4, a_5, a_6 can easily be found by solving the following set of linear equations: Let $zpp_{kj} = 1/(zp_k - z_j)$, $k = 1, 2, 3$, $j = 1, 2, ...6$. Then we have

$$\begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{pmatrix} = \begin{pmatrix} 1 & 1/(1 - z_1) & 1/(1 - z_2) & 1/(1 - z_3) & 1/(1 - z_4) & 1/(1 - z_5) & 1/(1 - z_6) \\ 1 & \Re(zpp_{11}) & \Re(zpp_{12}) & \Re(zpp_{13}) & \Re(zpp_{14}) & \Re(zpp_{15}) & \Re(zpp_{16}) \\ 0 & \Im(zpp_{11}) & \Im(zpp_{12}) & \Im(zpp_{13}) & \Im(zpp_{14}) & \Im(zpp_{15}) & \Im(zpp_{16}) \\ 1 & \Re(zpp_{21}) & \Re(zpp_{22}) & \Re(zpp_{23}) & \Re(zpp_{24}) & \Re(zpp_{25}) & \Re(zpp_{26}) \\ 0 & \Im(zpp_{21}) & \Im(zpp_{22}) & \Im(zpp_{23}) & \Im(zpp_{24}) & \Im(zpp_{25}) & \Im(zpp_{26}) \\ 1 & \Re(zpp_{31}) & \Re(zpp_{32}) & \Re(zpp_{33}) & \Re(zpp_{34}) & \Re(zpp_{35}) & \Re(zpp_{36}) \\ 0 & \Im(zpp_{31}) & \Im(zpp_{32}) & \Im(zpp_{33}) & \Im(zpp_{34}) & \Im(zpp_{35}) & \Im(zpp_{36}) \end{pmatrix}^{-1} \begin{pmatrix} Q(z = 1) \\ \Re(Q(z = zp_1)) \\ \Im(Q(z = zp_1)) \\ \Re(Q(z = zp_2)) \\ \Im(Q(z = zp_2)) \\ \Re(Q(z = zp_3)) \\ \Im(Q(z = zp_3)) \end{pmatrix} \tag{28}$$

where $\Re(\bullet)$ and $\Im(\bullet)$ denotes the real and imaginary parts of ($\bullet$).

RESULTS AND SIMULATION

The method of controller design discussed in section IV is applied to a magnetic bearing system whose parameters are given in Table 1. The μ synthesis Toolbox [16] with Simulink were used for the design and simulation. The system is discretized using a zero order hold at a sampling time $T_s = 158\mu sec$. The controller $\mathbf{K}(s)$ is designed at a speed $p = 0$ rad/sec and must be able to keep the system stable for a speed range $(0 - 2\pi 250)$ rad/sec. The operating speeds are assumed to be $p_1 = 2\pi 30$ rad/sec, $p_2 = 2\pi 20$ rad/sec, and $p_3 = 2\pi 10$ rad/sec. The generalized region of stability D_s is defined by the following parameters: $\alpha_s = 0.995$, $\beta_d = 0.707$ to insure a certain degree of stability against parameter variation and to get fast and well damped transient response. $z_1 = 0.99$, $z_2 = 0.981$, $z_3 = 0.984$, $z_4 = 0.981$, $z_5 = 0.978$, $z_6 = 0.975$. The following results are obtained: The controller free parameters $a_0, a_1, a_2, a_3, a_4, a_5, a_6$ which satisfies Eqs. (19) and (22), for each subsystem of Eq. (13) were found to be

$$\begin{aligned}
Y-\text{motion}: a_0 &= 7.7\times 10^7,\ a_1 = 1.44\times 10^7,\ a_2 = -1.589\times 10^8 a_3 = 6.649\times 10^8,\ a_4 = -1.325\times 10^9,\\
&\quad a_5 = 1.262\times 10^9,\ a_6 = -4.618\times 10^8\\
\Psi-\text{motion}: a_0 &= 7.63\times 10^7,\ a_1 = 1.427\times 10^7,\ a_2 = -1.574\times 10^8 a_3 = 6.583\times 10^8,\ a_4 = -1.312\times 10^9,\\
&\quad a_5 = 1.250\times 10^9,\ a_6 = -4.573\times 10^8\\
Z-\text{motion}: a_0 &= -5.258\times 10^7,\ a_1 = -9.55\times 10^6,\ a_2 = 1.053\times 10^8 a_3 = -4.406\times 10^8,\ a_4 = 8.782\times 10^8,\\
&\quad a_5 = -8.369\times 10^8,\ a_6 = 3.061\times 10^8\\
\Theta-\text{motion}: a_0 &= 5.209\times 10^7,\ a_1 = 9.461\times 10^6,\ a_2 = -1.043\times 10^8 a_3 = 4.363\times 10^8,\ a_4 = -8.696\times 10^8,\\
&\quad a_5 = 8.288\times 10^8,\ a_6 = -3.031\times 10^8
\end{aligned} \tag{29}$$

Substituting the Q 's in Eq. (17) we get (after model reduction) a 9 state controller for each of the four subsystems. The overall controller of the whole system has 36 states and is formulated as follows:

$$\mathbf{K}(z) = \begin{pmatrix} K_Y(z) & 0 & 0 & 0 \\ 0 & K_\Psi(z) & 0 & 0 \\ 0 & 0 & K_Z(z) & 0 \\ 0 & 0 & 0 & K_\Theta(z) \end{pmatrix} \tag{30}$$

where $K_Y(z)$, $K_\Psi(z)$, $K_Z(z)$, $K_\Theta(z)$ are the controllers of the Y, Ψ, Z, Θ subsystems respectively.

Table 2: Controller order Vs. Number of operating speeds

Number of operating speeds	Degree of controller
$r = 1$	20
$r = 2$	28
$r = 3$	36
$r = 4$	44
$r = 5$	52
$r = 6$	60
...	...
$r = N$	No.of degrees of freedom $\times(2N+3)$

An interesting observation from the controller design procedure explained in the previous section is shown in Table 2. Table 2 shows a relationship between the number of operating speeds and the degree of the controller. We can conclude from this table that the degree of the controller equals: number of degrees of freedom $\times(2N+3)$ where N is the number of operating speeds or equals: number of degrees of freedom $\times$ (order of Q +3).

A plot of the singular values of the loop gain GK is shown in Fig. 4. High loop gain at low frequency and low gain at high frequency are achieved. A plot of the singular values of sensitivity is shown in Fig. 5. From Fig. 5 we can see that the sensitivity is small at low frequencies which means good disturbance rejection for the class of step disturbances and approaches zero at the frequencies $\omega_1 = 2\pi 30$ rad/sec, $\omega_2 = 2\pi 20$ rad/sec and $\omega_3 = 2\pi 10$ rad/sec which means asymptotic rejection of the unbalance sinusoidal disturbance forces at these speeds.

In this design, we ignored the interference terms, which express the gyroscopic effect, as $p = 0$. We therefore verify the robust stability of the system against the changes in the rotor speed. Let the perturbed plant $p \neq 0$ be denoted by G_p and the additive perturbation Δ_p from G is as follows:

$$\Delta_p = G_p - G \tag{31}$$

The robust stability is guaranteed if the following inequality holds:

$$\overline{\sigma}(\Delta_p) < \frac{1}{\overline{\sigma}(K(I+GK)^{-1})} \tag{32}$$

Fig. 6 shows the singular values plot of $1/\overline{\sigma}(K(I+GK)^{-1}$ and $\overline{\sigma}(\Delta_p)$ at $p = 2\pi 250$ rad/sec. From Fig. 6 we can see that the system is stable up to a speed $p = 2\pi 250$ rad/sec. Fig. 7, Fig. 8, Fig 9 show the gap deviations due to unbalance and electromagnet forces acting on the rotor. In Fig. 7, 8, and 9 we can see the good suppression of the imbalance forces at the variable three design speeds.

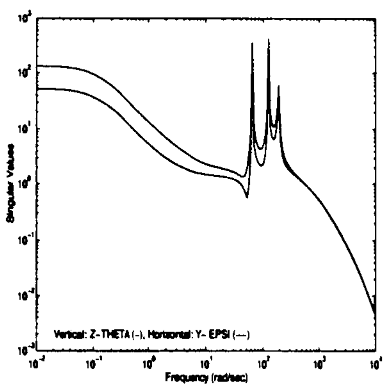

Fig. 4 Singular values of the loop gain GK.

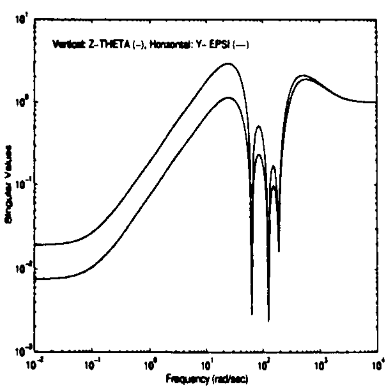

Fig. 5 Singular values of the sensitivity.

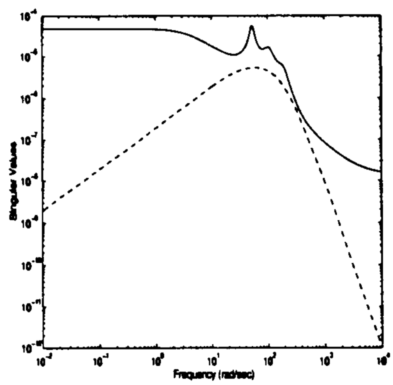

Fig. 6 $1/\overline{\sigma}(K(I+GK)^{-1})$ (-) and $\overline{\sigma}(\Delta_p)$ (- -) $p = 2\pi 250$.

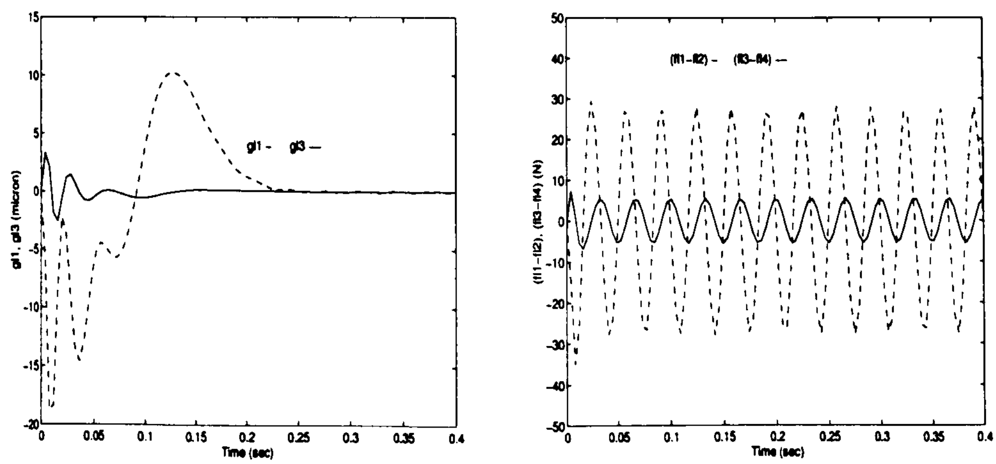

Fig. 7 Airgap variations due to unbalance and magnetic forces acting on the bearing for $p = 2\pi 30$.

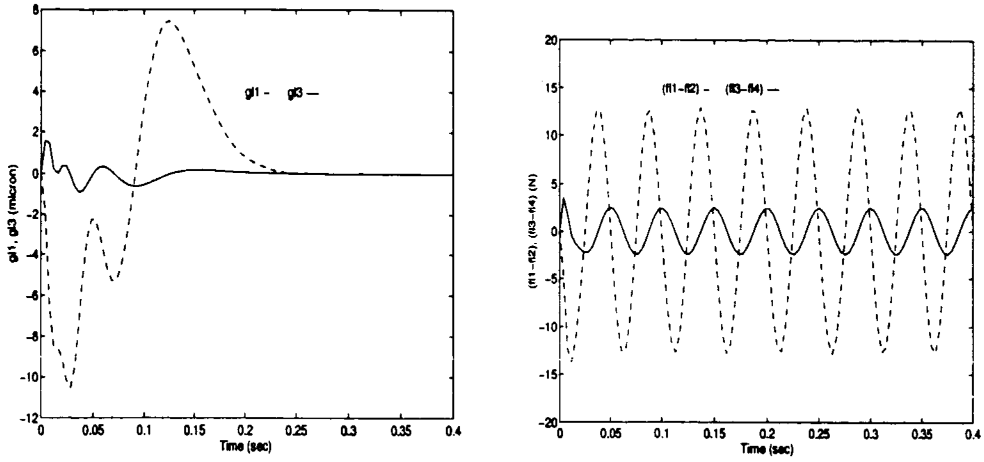

Fig. 8 Airgap variations due to unbalance and magnetic forces acting on the bearing for $p = 2\pi 20$.

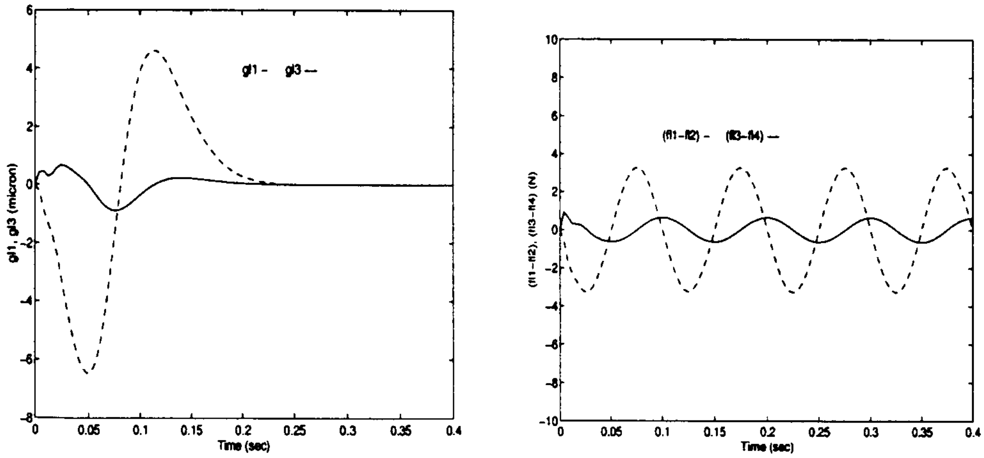

Fig. 9 Airgap variations due to unbalance and magnetic forces acting on the bearing for $p = 2\pi 10$.

CONCLUSIONS

In this paper we employed the discrete-time Q-parameterization control to design a controller which achieves elimination of unbalance vibrations in variable speed magnetic bearings. The free controller parameter is chosen such that the controller has poles on the unit circle at $z = \exp^{j p_k T_s}$ for the different speeds of rotation p_k , and satisfies other control objectives. This insures asymptotic rejection of the unbalance disturbance forces generated by the unbalance. The controller free parameter Q is assumed to be a proper stable transfer function whose order equals twice the number of operating speeds. We showed that the free controller parameter is obtained by solving a set of linear equations rather than solving a complicated optimization problem. We also showed that the controller order equals: Number of degrees of freedom $\times$ (order of Q +3). The controller is designed at speed $p = 0$ and the good simulation results that were obtained at speeds $p = 2\pi 30$ rad/sec, $p = 2\pi 20$ rad/sec and $p = 2\pi 10$ rad/sec showed the robustness of the proposed controller.

Elimination of unbalance vibrations in a variable speed magnetic bearing can also be achieved by making the rotor rotate around its axis of inertia at the different operating speeds (automatic balancing). In this case the rotor will be free from vibrations. This can be done using the same procedures explained in section III and [5].

References

1. Mizuno, T., and Higuchi, T., "Design of Magnetic Bearing Controllers Based on Disturbance Estimation," Proceedings of the 2nd International Symposium on Magnetic Bearings, pp. 281-88, July 12-14, 1990, Tokyo Japan.
2. Larsonneur R., and Herzog, R., "Feedforward Compensation of Unbalance: New Results and Application Examples," Proceedings ITUAM Symposium on Active Control of Vibration, pp. 45-52, Septmber 1994, Bath, U.K.
3. Knospe, C. R., "Robustness of Unbalance Response Controllers," Proceedings of the 3rd International Symposium on Magnetic Bearings, pp. 580-589, Virginia, USA, July 1992.
4. Herzog, R., Bühler , P., Gähler , C., and Larsonneur, R., "Unbalance Compensation Using Generalized Notch Filters in the Multivariable Feedback of Magnetic Bearings," *IEEE Transactions on Control System Technology,* Vol. 4, No. 5, pp. 580-586, September. 1996.
5. Mohamed, A., and Busch-Vishniac, I.,"Imbalance Compensation and Automatic Balancing in Magnetic Bearing Systems Using The Q-Parameterization Theory," *IEEE Transactions on Control System Technology,* Vol. 3, No. 2, pp. 202-211, June 1995.
6. Matsumura, F., Namerikawa, T., Hagiwara, K., and Fujita, M., "Application of Gain Scheduled H_∞ Robust Controllers to a Magnetic Bearing," *IEEE Transactions on Control System Technology,* Vol. 4, No. 5, pp. 484-493, September 1996.
7. Sivrioglu, S., and Nonami, K., "LMI Based Gain Scheduled H_∞ Controller Design for AMB Systems under Gyroscopic and Unbalance Disturbance," Proceedings of the 5th International Symposium on Magnetic Bearings, pp.191-196, August 28-30, 1996, Kanazawa, Japan.
8. Vidyasagar, M., Control System Synthesis: a Factorization Approach, Cambridge, Mass.: MIT Press, 1985.
9. Boyd, S., Barratt, C., *Linear Controller Design: Limits Of Performance,* Prentice Hall Information and Science Series New Jersey 1991.
10. McFarlane, D. and Glover, K.,"Robust Controller Design Using Normalized Coprime Factor Plant Descriptions," *Lecture Notes in Control and Information Sciences,* Vol. 138, Springer-Verlag, 1990.
11. Ogata, K., *Discrete Time Control Systems,* Prentice-Hall Inc., Englewood Cliffs, New Jersey 1987.
12. Balas, G., Doyle, J. C., Glover, K., Packard, A., and Smith, R., *μ -Analysis and Synthesis Toolbox,* The Math Works Inc., 1993.