

# **A NEW MODEL BASED METHOD FOR THE ACCURATE MEASUREMENT OF MAGNETIC BEARING FORCES**

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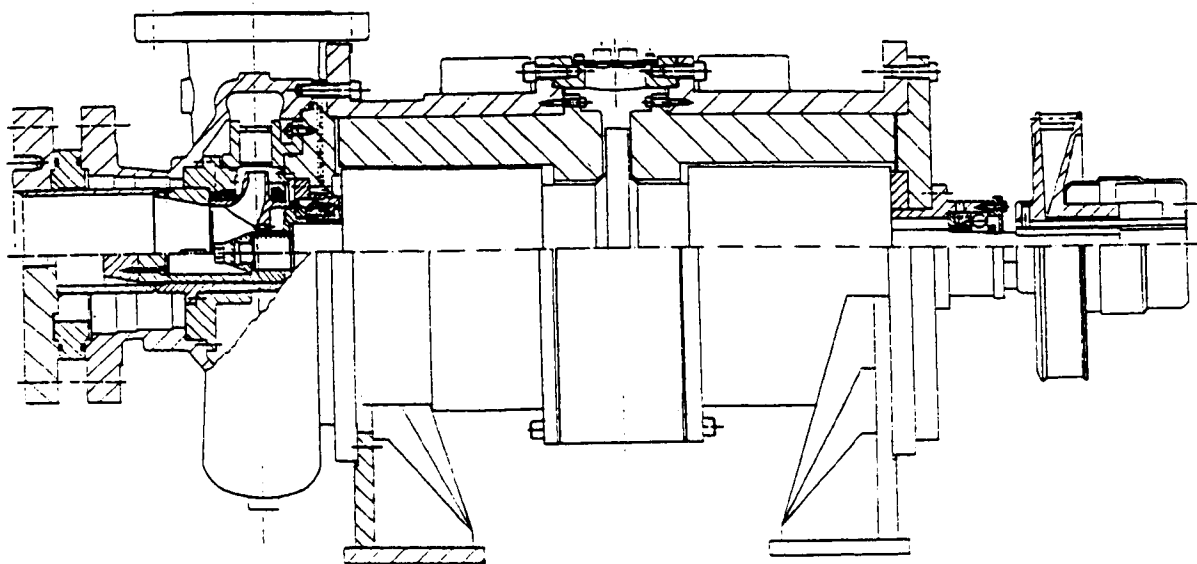
## **SUMMARY**

Active magnetic bearings (AMB) are typical mechatronic systems. Their employment in rotating machinery offers the possibility for active control of the dynamics of the shaft as well as use as a diagnosis tool. The use of an AMB can make turbomachinery safer and more efficient. In many cases an accurate knowledge of the bearing forces is demanded. A specific example where this topic is relevant is briefly described in this paper. It is a test-rig which was designed for the identification of physical parameters such as stiffness, damping and inertia coefficients of journal bearings, annular seals and others. The AMB are used as exciters in order to apply an artificial and controlled motion on a test shaft. For the identification of the unknown coefficients, it is necessary to measure time dependent displacements and forces, which are acting on the moving shaft, with sufficient accuracy. The measurement of the shaft movement, e.g. by eddy current sensors, with very low errors is state of the art whereas the determination of the bearing forces with high accuracy takes much more effort. A very simple and cheap possibility is based on the measurement of bearing coil currents and the air gap between rotor and magnet (i-s-method). The main advantage of this approach is that no additional sensor devices other than the ones which are required for the control system of the AMB, are necessary. But this method has the disadvantage of neglecting eddy current losses as well as hysteresis and saturation effects of the core material. Hence the measured force is polluted by an error of amplitude and phase with unknown quantities. Due to lamination of the magnets and the magnetized parts of the shaft in combination with comparatively low rotational speeds and excitation frequencies, eddy current losses are of less importance and the error is dominated by the hysteresis. The characteristic of the force error by using the i-s-method is shown by theory and with experimental help. In addition to that, the paper presents a new method that allows a prediction and, hence, correction of errors due to hysteresis and saturation effects. It is based on a model by which the hysteresis of the core material can be simulated. The results of the model are verified by measurements. In a specific test, it is shown

that the error of the force measurement can be reduced from 9% down to 1.5% in amplitude. The phase shift due to hysteresis losses is shown to be less than  $2^\circ$  for the core material used in the test-rig. Hence the capability of the method is proven, which allows a much more accurate identification of rotordynamic coefficients with the test rig. In addition to that, it offers the possibility of precise, simple and cheap force measurement in any machinery where AMB are used for shaft support.

## INTRODUCTION

Figure 1 shows a test rig which is designed for the identification of rotordynamic parameters, such as stiffness, damping and masses. The knowledge of these coefficients allows a comprehensive description of the dynamic characteristics of components which are used in fluid handling machinery. The test results of a long annular seal, an impeller and a hydrostatic bearing have been published up to now. Those results can be used for deepening the knowledge about the behavior of the components themselves and their influence on the rotordynamics of the whole turbomachine. The test rig uses active magnetic bearings (AMB) for rotor support. In parallel, the AMB-system is used to apply a well defined relative motion between rotor and stator, by which the fluid forces in the test-component are generated. In order to identify the rotordynamic coefficients, it is necessary to measure displacements and forces simultaneously. The measurement of displacements is done by eddy current probes with an accuracy of 1% or better.



*Fig. 1: Sectional drawing of a AMB Component Tester used for identification of physical parameters*

The forces are measured indirectly by using values from the magnets. The force, which is acting on the shaft due to a single magnet as shown in Fig. 2, can be calculated from the change of the magnetic field energy in the air gaps. Assuming a homogenous magnetic field in the air gaps and neglecting stray fluxes yields the single force  $f$ .

$$f_{\text{real}} = \frac{\Phi_L^2}{A \cdot \mu_0} \cdot \cos \alpha = \frac{B_L^2 \cdot A}{\mu_0} \cdot \cos \alpha \quad (1)$$

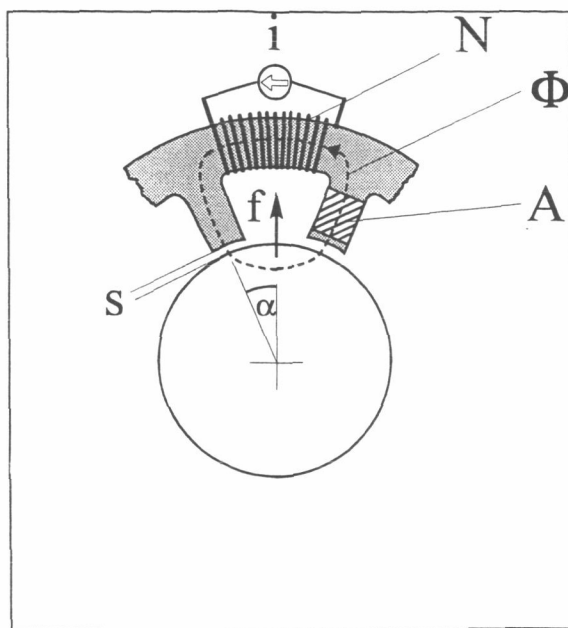


Figure 2: Single magnet of a radial magnetic bearing

The flux density  $B_L$  can be substituted by the magnetic field strength  $H_L$ .

$$B_L = \mu_0 \cdot H_L \quad (2)$$

Introducing the formulation for  $H_L$  as follows

$$H_L = \frac{1}{2 \cdot s} \cdot (N \cdot i - l_{FE} \cdot H_{FE}) \quad (3)$$

allows a formulation for the single force  $f$  according to equation (4).

$$f_{\text{real}} = A \cdot \mu_0 \cdot \cos \alpha \cdot \frac{1}{4 \cdot s^2} \cdot (N \cdot i - l_{FE} \cdot H_{FE})^2 \quad (4)$$

This formulation contains, besides known geometrical and material dependent values, the coil current  $I$  and the air gap  $s$ , which are measurable physical values. The magnetic field strength in the laminated core  $H_{FE}$  is unknown and not possible to measure. Hence it is neglected, which finally gives a basic equation where the force of a single magnet can be calculated from  $i$  and  $s$ . Therefore, we call it the  $i$ - $s$ -method. The resulting force is not the real force but an indirectly measured force, including error.

$$f_m = \frac{1}{4} \cdot A \cdot \mu_0 \cdot \cos \alpha \cdot N^2 \cdot \frac{i^2}{s^2} \quad (5)$$

### ERRORS DUE TO THE I-S METHOD

Now the consequences of neglecting the field strength in the core material are demonstrated again for a single magnet. The combination of eq. (4) and (5) yields an expression which represents the difference between the measured force by the  $i$ - $s$ -method and the real force. Hence it is formulation for the force measurement error.

$$(\sqrt{f_m} - \sqrt{f_{\text{real}}}) = \frac{1}{2 \cdot s} \cdot \sqrt{A \cdot \mu_0 \cdot \cos \alpha} \cdot l_{FE} \cdot H_{FE} \quad (6)$$

It appears that the measured force is always higher than the real force. It is simple to understand, because the measured coil is not completely transformed in a magnetic force. A part of it has to be used for magnetization of the iron, which is neglected by the  $i$ - $s$ -method. It is also obvious now that the force measurement error must be dependent on the properties of the core material and on the real magnetic force because  $H_{FE}$  is related to the flux density in the core  $B_{FE}$ , which again is related to the real force as it is shown in eq. (7). Hence the characteristics of  $H_{FE}$  over  $B_{FE}$  is given by a hysteresis loop, which is determined by the material used.

$$\sqrt{f_{\text{real}}} = \sqrt{\frac{A \cdot \cos \alpha}{\mu_0}} \cdot B_{FE} \quad (7)$$

Figure 3 shows the outer hysteresis loop of the material used for the bearings, without the virgin curve.

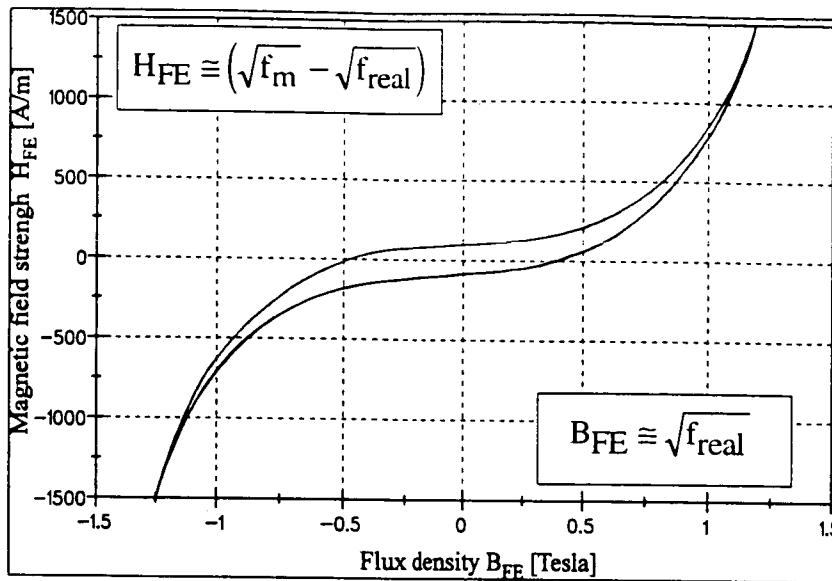


Figure 3: *Characteristic hysteresis loop of laminated core material*

Following the expressions of eq. (6) and (7) the difference between i-s-measured force and the real force, hence the force measurement error, must have the same characteristic dependence on the real force as is shown in figure 3. This has been proven by an experiment with the test rig. The net real force is measured by calibrated load cells and the i-s-method yields the net measured force. The result for one pair of magnets acting in opposition to each other is presented in figure 4. It shows the typical error characteristic depending on the bearing load. Equation (6) shows that the force measurement error also depends on the air gap  $s$ . By calibration this dependence can be eliminated with an electronic circuit. It also has to be noted that the major loop in figure 4 results from two minor loops from both single magnets.

The function  $f(B)$  is a formulation of the hysteresis curve and  $g(B)$  determines the thickness of the loops. The formulation used is restricted to the points where the upper and the lower curve meet each other, i.e. the case of complete saturation is not treated.

$$f(B) = A_1 \cdot \tan(A_2 \cdot B) \quad \text{for } |B| \leq B_s \quad (9)$$

$$g(B) = \frac{df}{dB} \cdot \left[ 1 - A_3 \cdot \exp\left(\frac{A_4 \cdot |B|}{B_s - |B|}\right) \right] \quad \text{for } |B| \leq B_s \quad (10)$$

The coefficients  $A_1 \dots A_4$  are calculated iteratively. The characteristic values for the remanent flux density the coercive field strength and the gradients  $O_i$  are known for the core material in the bearings.

$$2 \cdot A_2 \cdot \mu_s \cdot H_s = \sin(2 \cdot A_2 \cdot B_s) \quad (11)$$

$$A_1 = H_s \cdot \cot(A_2 \cdot B_s) \quad (12)$$

$$B_s = \frac{1}{A_2} \cdot a \cos \sqrt{A_1 \cdot A_2 \cdot \mu_s} \quad (13)$$

$$A_3 = 1 - \frac{1}{A_1 \cdot A_2} \cdot \left( \frac{1}{\mu_c} + \gamma \cdot H_c \right) \quad (14)$$

$$A_4 = \frac{B_r - B_s}{B_r} \cdot \ln \left[ \frac{1}{A_3} - \frac{\cos^2(A_2 \cdot B_r)}{A_1 \cdot A_2 \cdot A_3} \cdot \left( \frac{1}{\mu_r} + \gamma \cdot A_1 \cdot \tan(A_2 \cdot B_r) \right) \right] \quad (15)$$

Integration of eq. (8) yields two integral equations, which allows a description of each point of the outer B-H-curve between upper and lower boundaries. The upper and the lower curves of the loop are formulated separately.

$$H_u(B) = f(B) + e^{-\gamma \cdot B} \cdot \int_{-B_{\max}}^B \left[ g(\xi) - \frac{df}{dB}(\xi) \right] \cdot e^{\gamma \cdot \xi} d\xi \quad (16)$$

$$H_d(B) = f(B) - e^{\gamma \cdot B} \cdot \int_B^{B_{\max}} \left[ g(\xi) - \frac{df}{dB}(\xi) \right] \cdot e^{-\gamma \cdot \xi} d\xi \quad (17)$$

The inner loops of the hysteresis are given by eq. (18) and (19):

$$H_{iu} = f(B) + \cosh[\gamma \cdot (B_{\min} - B)] \cdot \int_{B_{\min}}^B \left( g(\xi) - \frac{df}{dB}(\xi) \right) \cdot \sinh[\gamma \cdot (B_{\min} - \xi)] d\xi \quad (18)$$

$$H_{id} = f(B) - \cosh[\gamma \cdot (B - B_{\max})] \cdot \int_{B_{\max}}^B \left( g(\xi) - \frac{df}{dB}(\xi) \right) \cdot \sinh[\gamma \cdot (B_{\max} - \xi)] d\xi \quad (19)$$

With the help of equations (1), (4), (5) and the hysteresis equations (16) to (19) the related real magnetic force, which is applied to the shaft by a pair of magnets, can be correlated to the measured force by the i-s-method. Hence, the force error due to neglecting the magnetization of the iron can be calculated.

## RESULTS

The measured force error loop from fig. 4 is used to tune the free parameters of the model. It represents the outer hysteresis loop, which is given by eq. (16) and (17). The comparison between simulated and measured results for one pair of radial magnets is shown in figure 5 on the left-hand side.

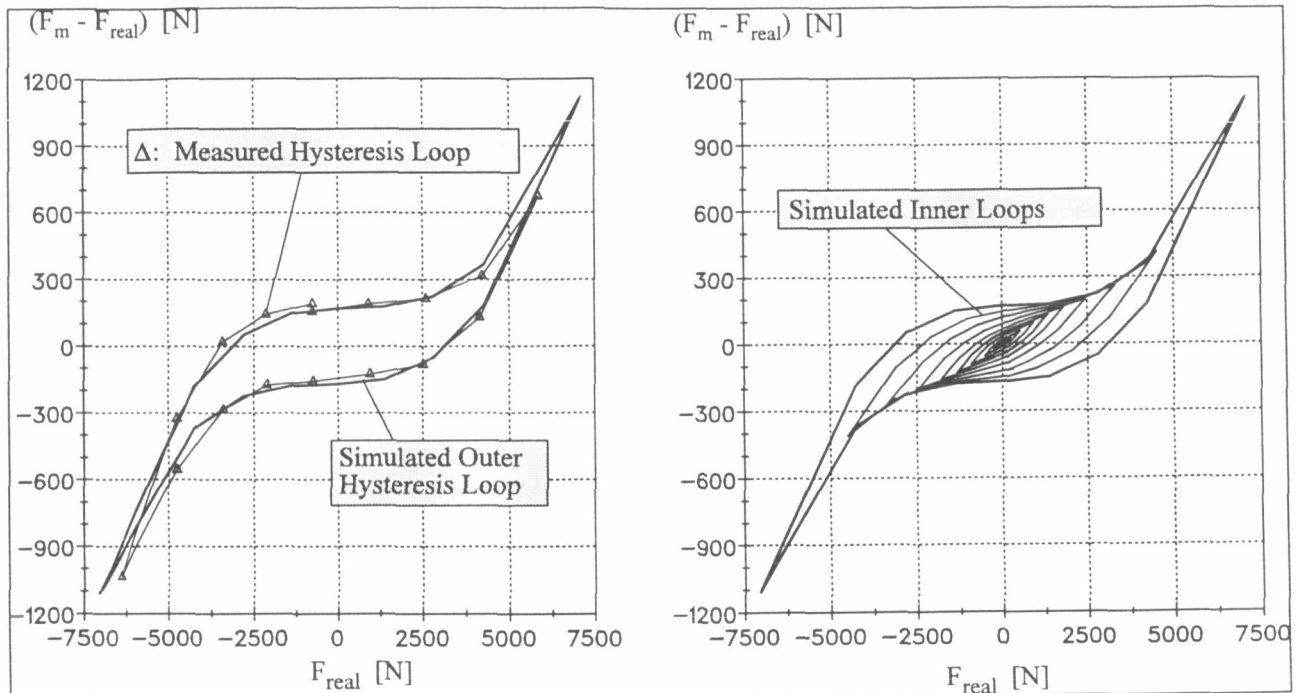


Fig. 5: Correlation of measured and simulated force error hysteresis loops (left) and simulated inner loops (right) of one pair of radial magnets

Specifically in the working range of the magnets there is a very good correlation between theory and experiment. The difference at higher force levels may be the result of incipient saturation of the amplifiers of the magnets.

On the right side of figure 5, some simulated inner loops are presented. Those loops are the basis for calculating the force errors for any desired force amplitude. The end points of the inner loops are drifting away from the outer one when there are no effects from saturation anymore. This results in a minimum of force error at 3200 N force level (see fig. 6). The increase of error for lower real forces is due to the stronger contraction of the loop on the  $F_{real}$ -axis.

It should be realized that the i-s force measurement method leads to an error in force amplitude of at least 5.8% for each pair of magnets. Tremendous deviations are expected when the full saturation of the material is reached. In addition to that, it has to be recognized that low force levels result in errors of about 10% for one pair of magnets.



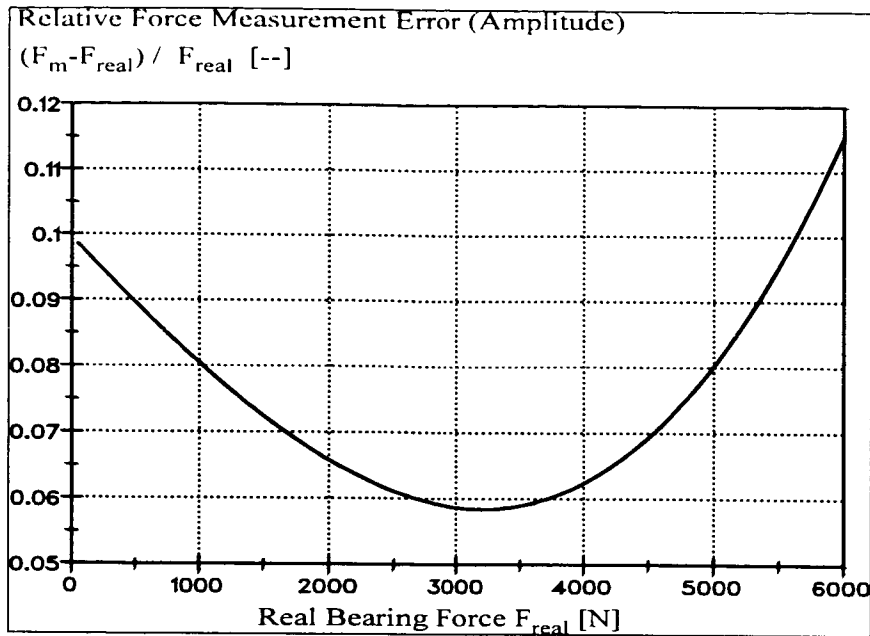


Fig. 6: Model based calculated relative force measurement error due to hysteresis

## APPLICATION AND VERIFICATION

In order to test and verify the method, a specific test setup was created, where the AMB test-rig was used for identification of the well known bending stiffness of a clamped beam in the way as described, for example in [1]. In a simplified sketch, figure 6 shows the test configuration.

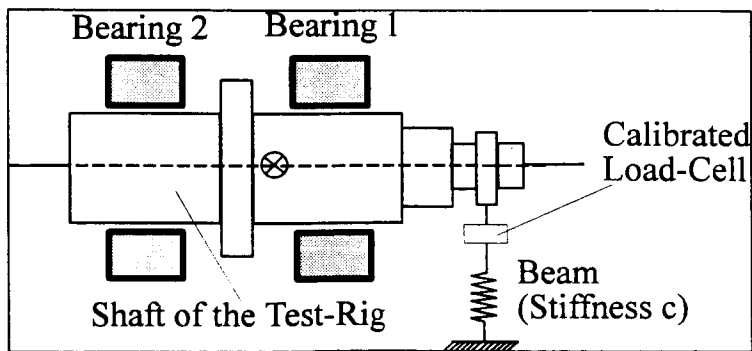


Fig. 6: Sketch of the test configuration for identification of a stiffness

The measured forces at bearing 1 and 2 together with the artificial pure lateral shaft motion are in direct correlation with the beam stiffness  $c$ . An additional load cell is used for reference.

The stiffness identified by the load cell is identical to the analytically calculated value of 4.58 MN/m. Using the bearing forces delivered by the i-s-method for identification results in a stiffness of 4.95 MN/m. Hence it is 8% higher than the real value. The application of the model

based error calculation first needs the real forces at the magnetic bearings, which are accurately determined from geometrical values, the mass of the shaft and the real stiffness  $c$  of the beam. At bearing 1 there is a force magnitude of  $F_1=237$  N and at bearing 2 only  $F_2=53$  N is reached. The hysteresis model delivers theoretical relative errors in amplitude of  $F_1=9.5\%$  and  $F_2=9.8\%$ . Hence this theory forecasts a relative error of the stiffness of  $+9.4\%$ . It has to be noted that for this application the magnitude of the bearing forces are quite low. In spite of that, the result of the error calculation is very close to reality.

## SUMMARY AND CONCLUSIONS

It has been demonstrated that the characteristic error of applying the i-s-method for measuring AMB-forces is based on the hysteresis of the magnetized core material. With help of a hysteresis model it was possible to simulate the force error characteristic of a radial bearing used in a test-rig. The method was verified by a specific test.

It has to be noted that the model still does not fully match reality. First of all the rotation of the shaft, which also causes losses in the rotor material, is not included. Furthermore, the effect of frequency dependence on the hysteresis curves was only estimated in this case. All these effects are taken into account by measuring the magnetic flux. The application of Hall Sensors allows force calculations with an accuracy of up to  $1.5\%$  for static and  $5.5\%$  for dynamic forces.

However, the i-s-method could be applied without additional hardware on existing magnetic bearings and the hysteresis calculation allows a much more accurate identification of physical parameters with help of the AMB's.

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