RESISTIVE AND SUPERCONDUCTING MAGNET CONFIGURATIONS FOR LEVITATION OF WEAKLY DIAMAGNETIC MATERIALS

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SUMMARY

The magnetic fields that can be generated today by superconducting and resistive magnets are so strong that even weakly diamagnetic materials can be levitated. Materials with magnetic susceptibilities in the order of 10^{-5} require a product of field times field gradient of about 1000 T^2/m. Such values are easily obtained in standard 12-20 T, small bore (30-50 mm) resistive and superconducting solenoid magnets, available in many locations. To levitate larger objects, magnets with wider bores and different field configurations than solenoids are necessary, as for instance a split coil magnet with opposite field direction in the two halves. In this study, we characterize the levitation capability of some of the resistive magnets available at the NHMFL. We also examine different configurations, optimized for maximum levitation, such as strong gradient magnets and magnets in racetrack form, with the goal to determine the relationship between the limitation imposed by today's available conductor materials and the size of the body that can be levitated.

INTRODUCTION

High field superconducting and resistive magnets have been known to levitate weakly diamagnetic organic and non-organic materials with susceptibilities as low as 10^{-5}. Previous work at the NHMFL and the university of Nijmengen shows that matter with a specific gravity of about unity, and, therefore also living organisms such as frogs, peanuts and other substances can easily be levitated in fields of 10-16 T [1,2]. The levitation results from a balance between the magnetic force, F_m, acting on every molecule in the levitated specimen, and the gravitational force. Diamagnetism is a general attribute of matter, in a few materials it is covered by additional para- or ferromagnetism. The molecular diamagnetism is caused by electrons adjusting their orbits to reduce their energy in the presence of an external magnetic field. These magnetic levitation forces are many orders of magnitude weaker than forces on ferromagnetic materials.

An object will levitate when its magnetic force balances the gravitational force. The magnetic force per unit volume is related to the field and field gradient as,

$$\vec{F} = x\,(B.\nabla)\vec{B}\,,$$

(1)

where x is the magnetic susceptibility, and $\vec{B}$ is the magnetic field vector. In a solenoidal cylindrical coordinate system the total axial and radial forces, F_r and F_z, integrated over the levitation volume V are,

$$F_r = x\iiint [B_z\frac{dB_z}{dr}+B_r\frac{dB_r}{dr}]dv$$

$$F_z = x\iiint [B_z\frac{dB_z}{dz}+B_r\frac{dB_z}{dr}]dv$$

(2)

The radial force averages to zero and is self-stabilizing due to the homogeneous diamagnetic properties of the levitated material and the assumed symmetry of the suspended body. Though the net radial force is zero, there is a radial magnetic pressure acting on the levitated object. The radial pressure gets higher as the sample volume becomes comparable to the solenoid bore. The axial balancing force is dominated by the $B_z\ (dB_z/dz)$ term and is more uniform for a specimen with a radius small compared to the levitation solenoid bore.

Equating the axial force with the gravitational force, $F_g = \rho g V$, results in a relation between the magnetic susceptibility, the magnet field, bore, configuration, the specific gravity ρ,and the levitated object volume V. **In the case of a superconducting magnet, it is convenient** to correlate that relation to the maximum winding field, B_{max}, since it is the quantity that determines the superconducting current density. B_{max} in a single solenoid system occurs at the inner radius on the mid-plane. **In the case of a resistive magnet, it is natural** to correlate the levitation relation to the power consumption of the magnet. In this study we neglect (for both cases, superconducting or resistive magnets) the stress or power density constraints that may impose limits on the maximum field, and leave it for an engineering design to optimize the magnet configuration in view of the stress and power density. Later we show two examples, a small and a large bore resistive magnet system, that have been optimized for maximum central field and can be used to levitate small and medium size objects.

By equating the gravitational and levitation axial forces the lower bound on the magnetic susceptibility x that can be levitated as function of the maximum field or dissipated power W is,

$$x \geq \frac{\rho\,g}{\lambda_{B_{max}}}\,\frac{a_1}{B_{max}^2}, \quad or$$

$$x \geq \frac{\rho\,g}{\lambda_W}\,\frac{\rho_e\,a_1^2}{p_f\,W},$$

(3)

where ρ_e is the conductor electrical resistivity at the operating temperature (typically 2~2.5 x 10^{-8} ohm.m) and p_f is a filling factor accounting for insulation and cooling channels (typically 0.8-0.9).

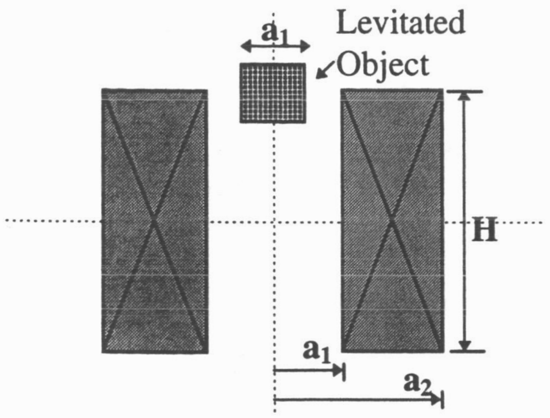


Fig. 1a Levitation in a single solenoid.

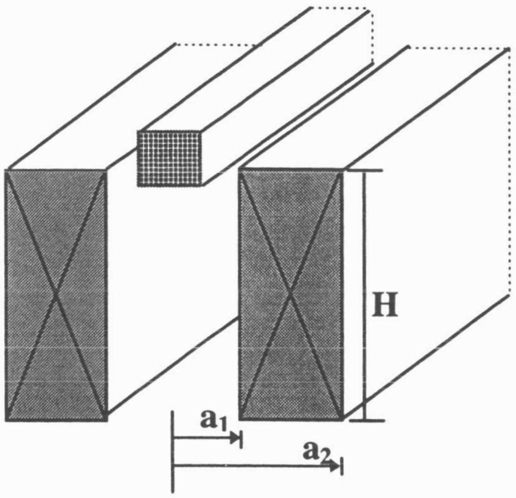
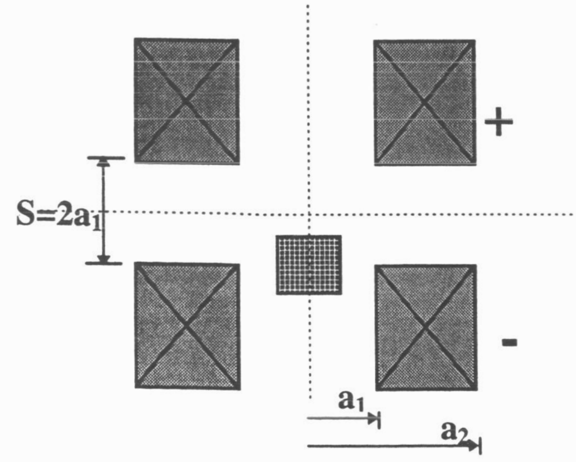


Fig.1b Levitation in a single racetrack magnet



. Fig. 2a Levitation in a split Helmholtz solenoid.

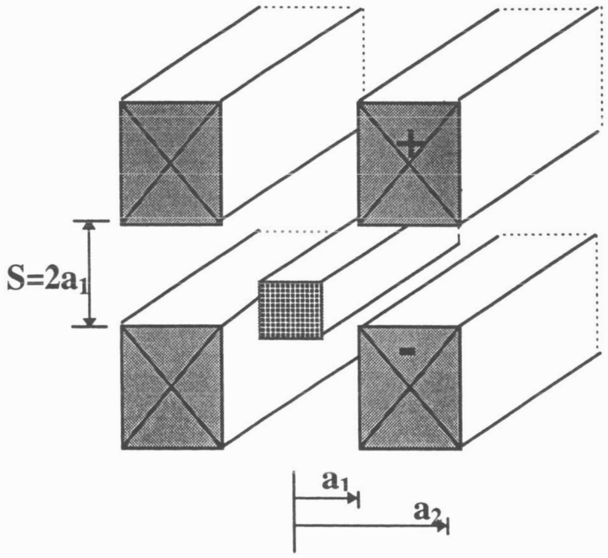


Fig. 2b Levitation in a split Helmholtz race-track magnet

The levitation factor λ is a dimensionless constant that relates to bore diameter, magnet geometry, to field or power. It is a geometry factor, like the Fabry factor, that defines the optimum magnet geometry and allows for a comparison between different magnet designs. We have computed the levitation factor for several solenoidal and racetrack geometries. Solenoids can have uniform current density, or 1/r current distribution (Bitter solenoids $J = J_0 a_1/r$. J_0 is the current density at the inner radius a_1). Fig. 1a shows a sketch of the winding cross-section, and the levitated volume that is centered at the winding opening where we encounter the highest product of B x grad B. The volume occupies a cross-sectional area with length and thickness a_1, i.e., half the bore diameter. The bottom of the levitated volume is at $z = H/2 - a_1/2$ and the top is at $z = H/2 + a_1/2$. This volume has the highest product of field and field gradient. The levitation winding has two aspect ratios; $\alpha = a_2/a_1$ and $\beta = H/2a_1$ where a_2 is the outer radius and H is the height. We also investigated racetrack coils, since they allow to suspend longer objects. We assumed an aspect ratio where each turn circumference is 20 times its radial distance a_1.

In addition to the simple geometry, shown in Fig. 1a and 1b, we have considered a split Helmholtz pair geometry made of two halves with opposite current direction that can be either solenoids with uniform current density, 1/r Bitter type current density, or race-track winding as shown in Fig. 2a and 2b. The split gap between the two halves is kept equal to the inner diameter, or the inner radial spacing in case of a racetrack. The change in the levitation force relative to the convenient choice of a split gap that is slightly different from the bore size, is small and is, therefore, neglected in this article. The levitated volume is centered in the highest product of field times field gradient region that is axially bound by $Z_{min} = -1.4a_1$ and $Z_{max} = -0.4a_1$. The radial width is again a_1.

The levitation factor has been computed for the four geometries shown in figures 1 and 2 for maximum field, i.e., superconducting magnets, or power, i.e., resistive magnets. The results are shown in figure 3a-d and table 1.

Discussion of the Results

The levitation factor for superconducting solenoids with two different current density distributions, and also for two magnet geometries are displayed in Fig. 3a and 3b. It follows that there is no significant difference for the first five cases, and the levitation factor λ is about 2×10^5. The split coil racetrack shows a geometric suspension efficiency of only half of the others. As first approximation it follows from equation (3) that $a_1/B^2 = 2.10^{-4}$ for a susceptibility of 10^{-5}. With today's technology, superconducting magnets can be built that generate 20 T in bores up to 150 mm diameter [3]. This coincides with the above relationship, i.e., we can suspend objects up to 80 - 100 mm diameter. A racetrack coil would have the advantage that longer objects can be levitated. Assuming a length of 5 times the bore diameter, objects of 100 mm x 800 mm would float in a magnetic field. The construction of the superconducting magnet would be a challenging but feasible task, involving over 10 t of conductor and reinforcement structure.

The levitation factors of the resistive magnets (Fig. 3c and 3d) give very similar results for solenoids, i.e., the current density distribution has no impact on the levitation factor. All other geometries, like split and racetrack, have lower efficiencies. Resistive magnets have the advantage that they are not limited in magnetic field, however, at the expense of excessive power requirements. For example, to levitate a human body using a racetrack resistive magnet with a 500 mm bore and 2.5 m length, the magnet would have to generate a maximum field of 38 T, and the power demand would be of the order of 1000 MW. Stress and power density constraints may increase the power and winding field requirements even more.

As a demonstration of the levitation capabilities at the NHMFL, we show in Fig. 4a and 4b the lower bound on the values of magnetic susceptibilities of materials that can be levitated in our 50 mm and 200 mm bore magnets. In our 50 mm magnet we can levitate diamagnetic materials with $x = 0.5 \times 10^{-5}$ and higher, while in our 200 mm bore x has to be higher than 1.0×10^{-5}. The profile of the levitation force is also shown in Fig. 4. The total volume that can be levitated is limited by the region where $F_z > 1$ and by the bore diameter. For both magnets the useful F_z region is about 30 mm thick, i.e., objects of up to 750 cm^3 can be suspended.

Conclusion

Magnets with small bore diameters have strong gradients. The condition of levitation of diamagnetic matter is, therefore, easy to achieve for small objects. Large bore magnets have lower gradients. To achieve the same B x grad B product the field must be increased. Today's conductor materials set limits on the achievable fields that restrict the object size to about 100 mm diameter for superconducting magnets and to 180 mm diameter for a 20 MW resistive magnet. Large objects would require higher power consumption or unavailable higher field materials in case of superconducting magnets. For example to levitate a human body only resistive magnet in a racetrack configuration could satisfy the field and the volume requirements. A field of almost 40 T is needed, and the continuous power demand would be about 1 GW.

Acknowledgment.

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References

1. M. V. Berry, A. K. Geim, submitted to Eur. J. Phys.

2. J. Brooks, J. Perenboom, private communications.

3. W. D. Markiewicz et al, in "High Magnetic Fields: Applications, Generation, Materials", ed. H. J. Schneider-Muntau, World Scientific, 1997, p. 287-297

Table 1.a Levitation factor in a single solenoid, Bitter solenoid or race track magnet system.

Solenoid	λ_w	λ_w	λ_w	λ_w	λ_w	λ_{Bmax}	λ_{Bmax}	λ_{Bmax}	λ_{Bmax}	λ_{Bmax}
	$\beta=0.6$	$\beta=0.8$	$\beta=1.0$	$\beta=2.0$	$\beta=3.0$	$\beta=0.6$	$\beta=0.8$	$\beta=1.0$	$\beta=2.0$	$\beta=3.0$
1.01	2.04E-10	2.04E-10	1.88E-10	1.12E-10	7.71E-11	1.78E+05	2.11E+05	2.23E+05	2.14E+05	1.96E+05
1.1	1.79E-09	1.82E-09	1.69E-09	1.03E-09	7.09E-10	1.75E+05	2.07E+05	2.18E+05	2.08E+05	1.95E+05
1.3	4.08E-09	4.29E-09	4.08E-09	2.58E-09	1.79E-09	1.71E+05	2.00E+05	2.10E+05	1.97E+05	1.84E+05
1.5	5.28E-09	5.71E-09	5.55E-09	3.64E-09	2.54E-09	1.70E+05	1.96E+05	2.05E+05	1.89E+05	1.74E+05
2.0	6.05E-09	6.91E-09	7.00E-09	5.03E-09	3.59E-09	1.68E+05	1.89E+05	1.95E+05	1.74E+05	1.57E+05
3.0	5.04E-09	6.11E-09	6.55E-09	5.50E-09	4.12E-09	1.60E+05	1.79E+05	1.83E+05	1.59E+05	1.38E+05
5.0	3.00E-09	3.82E-09	4.31E-09	4.40E-09	3.63E-09	1.44E+05	1.59E+05	1.64E+05	1.44E+05	1.22E+05
Bitter	$\beta=0.6$	$\beta=0.8$	$\beta=1.0$	$\beta=2.0$	$\beta=3.0$	$\beta=0.6$	$\beta=0.8$	$\beta=1.0$	$\beta=2.0$	$\beta=3.0$
1.01	2.04E-10	2.04E-10	1.88E-10	1.12E-10	7.71E-11	1.78E+05	2.11E+05	2.23E+05	2.14E+05	1.96E+05
1.1	1.79E-09	1.82E-09	1.70E-09	1.03E-09	7.10E-10	1.75E+05	2.07E+05	2.18E+05	2.08E+05	1.95E+05
1.3	4.16E-09	4.36E-09	4.14E-09	2.61E-09	1.81E-09	1.71E+05	2.01E+05	2.11E+05	1.98E+05	1.85E+05
1.5	5.52E-09	5.94E-09	5.75E-09	3.74E-09	2.61E-09	1.70E+05	1.97E+05	2.06E+05	1.90E+05	1.76E+05
2.0	6.94E-09	7.80E-09	7.81E-09	5.48E-09	3.89E-09	1.69E+05	1.92E+05	1.98E+05	1.78E+05	1.61E+05
3.0	7.21E-09	8.45E-09	8.79E-09	6.89E-09	5.06E-09	1.65E+05	1.85E+05	1.90E+05	1.67E+05	1.47E+05
5.0	6.50E-09	7.84E-09	8.40E-09	7.37E-09	5.72E-09	1.58E+05	1.76E+05	1.81E+05	1.58E+05	1.36E+05
RT	$\beta=0.6$	$\beta=0.8$	$\beta=1.0$	$\beta=2.0$	$\beta=3.0$	$\beta=0.6$	$\beta=0.8$	$\beta=1.0$	$\beta=2.0$	$\beta=3.0$
1.01	4.97E-11	5.43E-11	5.37E-11	3.87E-11	2.84E-11	1.02E+05	1.37E+05	1.59E+05	1.75E+05	1.57E+05
1.1	4.49E-10	4.98E-10	4.98E-10	3.67E-10	2.70E-10	1.03E+05	1.36E+05	1.57E+05	1.70E+05	1.58E+05
1.3	1.09E-09	1.25E-09	1.28E-09	9.82E-10	7.33E-10	1.06E+05	1.36E+05	1.54E+05	1.62E+05	1.50E+05
1.5	1.50E-09	1.76E-09	1.83E-09	1.47E-09	1.12E-09	1.06E+05	1.34E+05	1.50E+05	1.56E+05	1.43E+05
2.0	1.99E-09	2.43E-09	2.64E-09	2.34E-09	1.83E-09	1.07E+05	1.31E+05	1.43E+05	1.45E+05	1.31E+05
3.0	2.12E-09	2.73E-09	3.10E-09	3.18E-09	2.64E-09	1.05E+05	1.25E+05	1.35E+05	1.33E+05	1.17E+05
5.0	1.83E-09	2.45E-09	2.90E-09	3.51E-09	3.21E-09	9.62E+04	1.13E+05	1.21E+05	1.19E+05	1.05E+05

Table 1.b Levitation factor in a split Helmholtz solenoid, Bitter solenoid or race track magnet system.

Solenoid	λ_w	λ_w	λ_w	λ_w	λ_w	λ_{Bmax}	λ_{Bmax}	λ_{Bmax}	λ_{Bmax}	λ_{Bmax}
	$\beta=0.6$	$\beta=0.8$	$\beta=1.0$	$\beta=2.0$	$\beta=3.0$	$\beta=0.6$	$\beta=0.8$	$\beta=1.0$	$\beta=2.0$	$\beta=3.0$
1.01	9.76E-11	9.40E-11	8.51E-11	5.00E-11	3.42E-11	1.01E+05	1.23E+05	1.35E+05	1.52E+05	1.56E+05
1.1	8.61E-10	8.42E-10	7.70E-10	4.59E-10	3.14E-10	1.75E+05	2.00E+05	2.06E+05	1.90E+05	1.77E+05
1.3	1.99E-09	2.00E-09	1.87E-09	1.15E-09	7.92E-10	1.75E+05	1.96E+05	2.01E+05	1.81E+05	1.67E+05
1.5	2.59E-09	2.68E-09	2.55E-09	1.62E-09	1.13E-09	1.78E+05	1.95E+05	1.98E+05	1.74E+05	1.59E+05
2.0	3.02E-09	3.27E-09	3.23E-09	2.24E-09	1.58E-09	1.86E+05	1.97E+05	1.96E+05	1.63E+05	1.44E+05
3.0	2.51E-09	2.87E-09	2.99E-09	2.39E-09	1.77E-09	1.97E+05	2.02E+05	1.97E+05	1.52E+05	1.26E+05
5.0	1.39E-09	1.67E-09	1.82E-09	1.76E-09	1.42E-09	2.05E+05	2.05E+05	1.97E+05	1.43E+05	1.11E+05
Bitter	$\beta=0.6$	$\beta=0.8$	$\beta=1.0$	$\beta=2.0$	$\beta=3.0$	$\beta=0.6$	$\beta=0.8$	$\beta=1.0$	$\beta=2.0$	$\beta=3.0$
1.01	9.76E-11	9.40E-11	8.51E-11	5.00E-11	3.42E-11	1.01E+05	1.23E+05	1.35E+05	1.52E+05	1.56E+05
1.1	8.63E-10	8.43E-10	7.71E-10	4.59E-10	3.15E-10	1.75E+05	2.00E+05	2.06E+05	1.90E+05	1.77E+05
1.3	2.02E-09	2.03E-09	1.89E-09	1.16E-09	8.01E-10	1.75E+05	1.96E+05	2.02E+05	1.82E+05	1.68E+05
1.5	2.71E-09	2.78E-09	2.63E-09	1.67E-09	1.16E-09	1.78E+05	1.96E+05	1.99E+05	1.76E+05	1.60E+05
2.0	3.44E-09	3.68E-09	3.59E-09	2.44E-09	1.71E-09	1.84E+05	1.97E+05	1.97E+05	1.66E+05	1.47E+05
3.0	3.55E-09	3.95E-09	4.00E-09	3.00E-09	2.18E-09	1.91E+05	2.00E+05	1.98E+05	1.57E+05	1.34E+05
5.0	3.05E-09	3.49E-09	3.64E-09	3.04E-09	2.32E-09	1.96E+05	2.02E+05	1.98E+05	1.51E+05	1.23E+05
RT	$\beta=0.6$	$\beta=0.8$	$\beta=1.0$	$\beta=2.0$	$\beta=3.0$	$\beta=0.6$	$\beta=0.8$	$\beta=1.0$	$\beta=2.0$	$\beta=3.0$
1.01	2.29E-11	2.40E-11	2.32E-11	1.60E-11	1.15E-11	3.71E+04	4.80E+04	5.52E+04	7.00E+04	7.47E+04
1.1	2.07E-10	2.20E-10	2.15E-10	1.51E-10	1.10E-10	7.84E+04	9.86E+04	1.12E+05	1.34E+05	1.39E+05
1.3	5.05E-10	5.51E-10	5.50E-10	4.04E-10	2.96E-10	1.07E+05	1.33E+05	1.49E+05	1.49E+05	1.33E+05
1.5	6.97E-10	7.78E-10	7.89E-10	6.03E-10	4.46E-10	1.14E+05	1.38E+05	1.50E+05	1.45E+05	1.28E+05
2.0	9.21E-10	1.07E-09	1.13E-09	9.39E-10	7.16E-10	1.23E+05	1.42E+05	1.50E+05	1.37E+05	1.17E+05
3.0	9.53E-10	1.16E-09	1.27E-09	1.22E-09	9.82E-10	1.34E+05	1.48E+05	1.52E+05	1.29E+05	1.06E+05
5.0	7.42E-10	9.36E-10	1.07E-09	1.20E-09	1.06E-09	1.43E+05	1.53E+05	1.54E+05	1.23E+05	9.51E+04

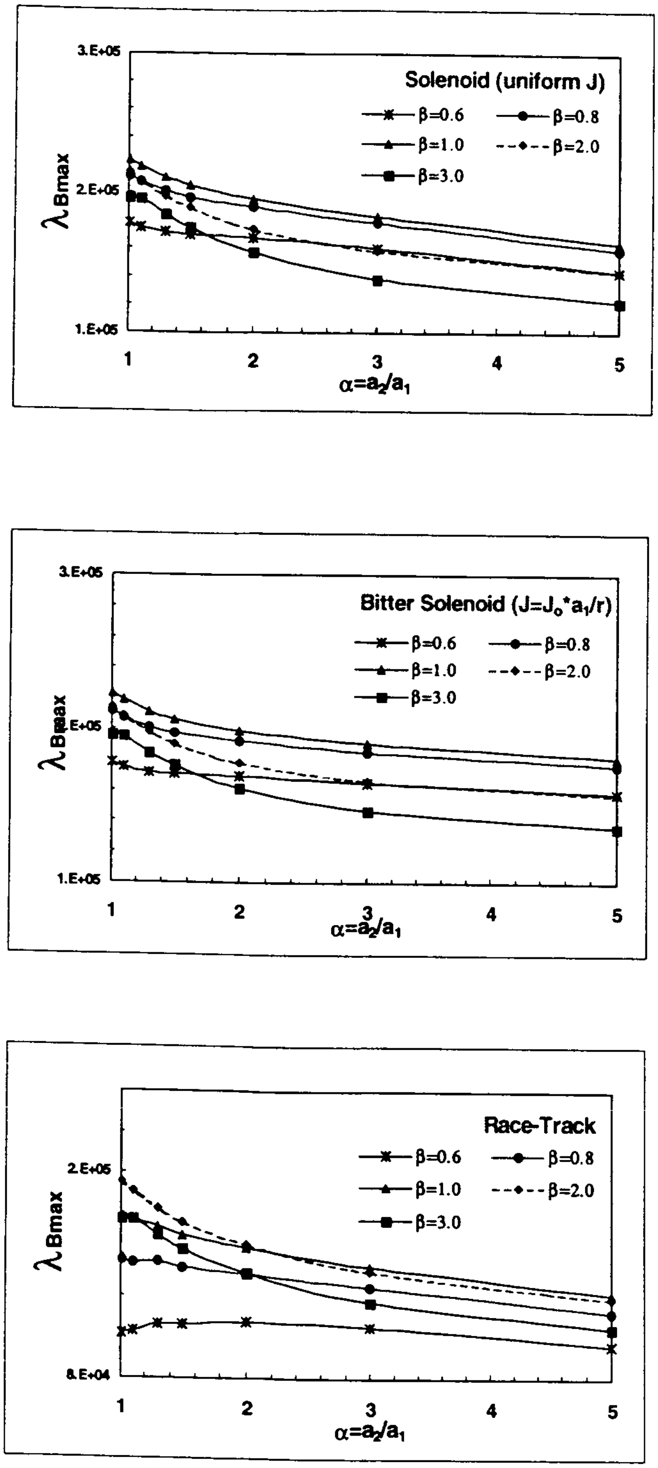

Fig. 3a Levitation factor for Helmholtz split pair relative to maximum winding field, B_{max}.

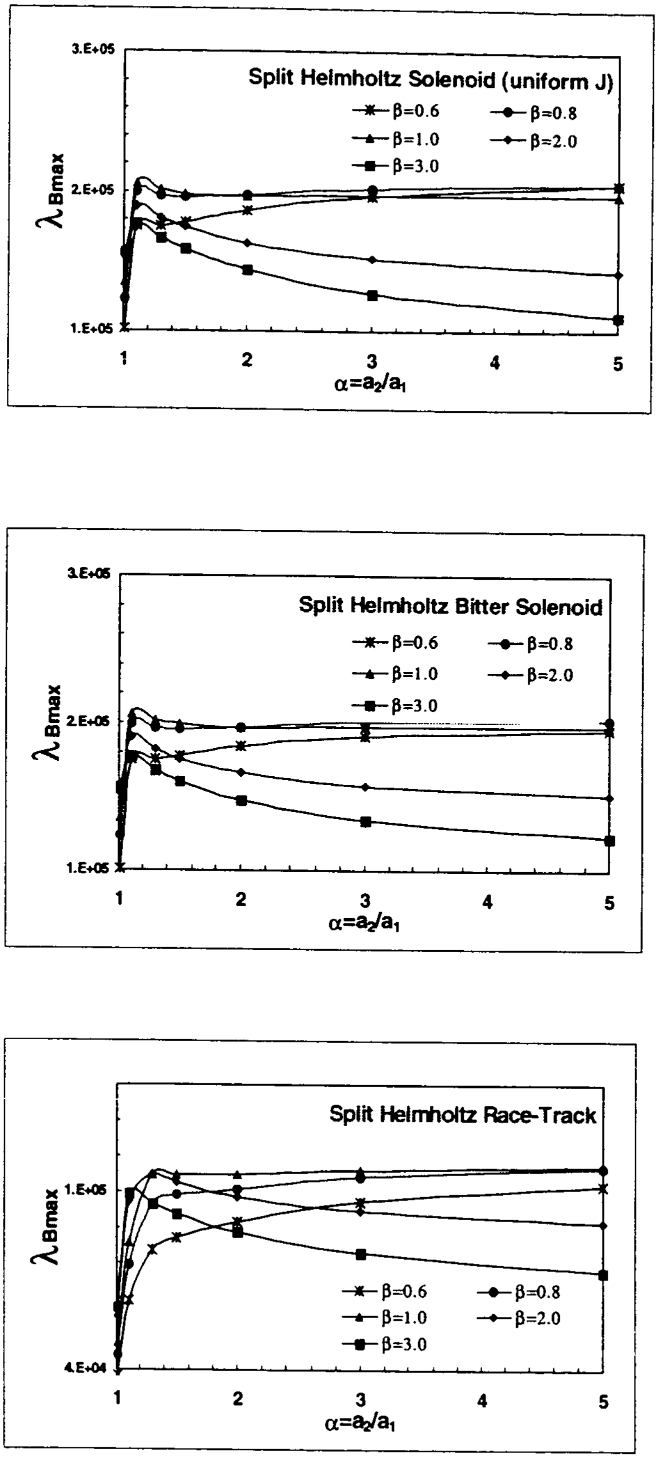

Fig. 3b Levitation factor for single coils relative to maximum winding field, B_{max}.

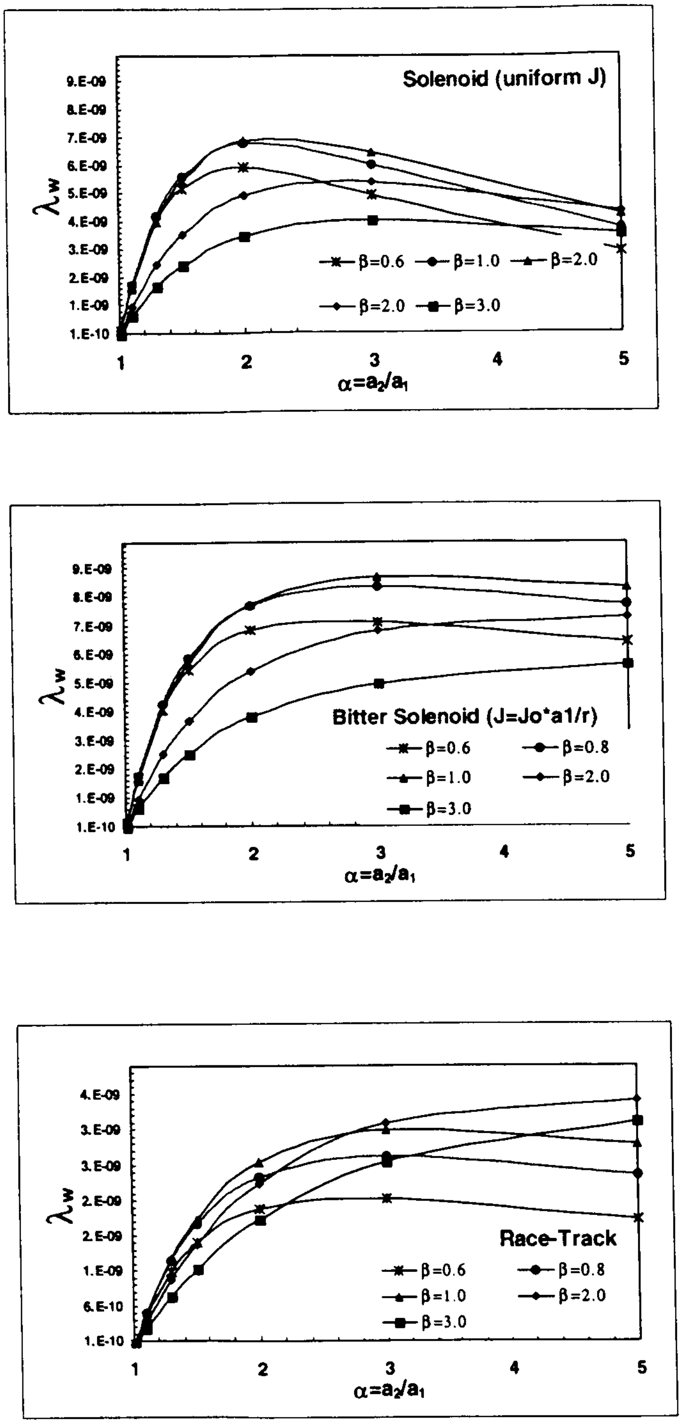

Fig. 3c Levitation factor for resistive Helmholtz split pair relative to power consumption, W.

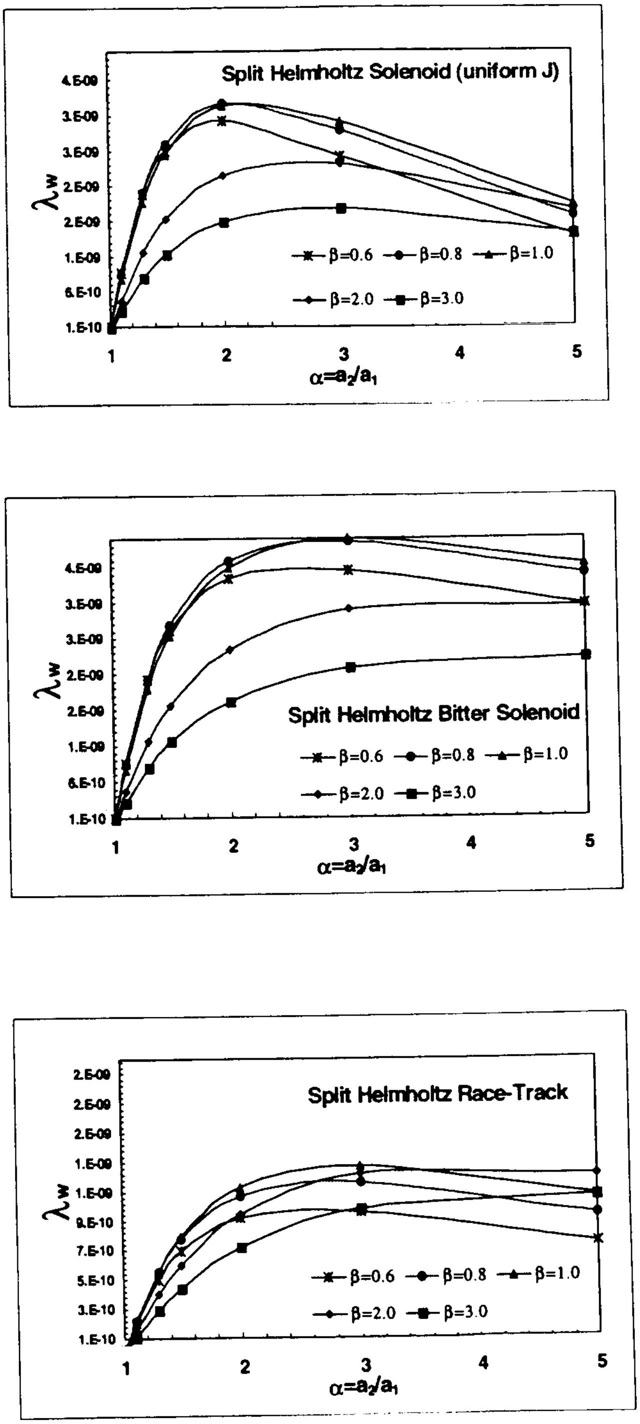

Fig. 3d Levitation factor for resistive single coil relative to power consumption, W.

R	B_r	B_z	Bt	F_z/x	F_z
m	T	T	T	N/m3	kg/kg
0.000	0.000	16.719	16.719	-2.063E+09	1.031
0.005	0.384	16.736	16.740	-2.071E+09	1.036
0.010	0.772	16.789	16.807	-2.096E+09	1.048
0.015	1.168	16.886	16.926	-2.130E+09	1.065
0.020	1.569	17.037	17.109	-2.154E+09	1.077
0.025	1.967	17.259	17.371	-2.119E+09	1.060
				Average=	1.02590

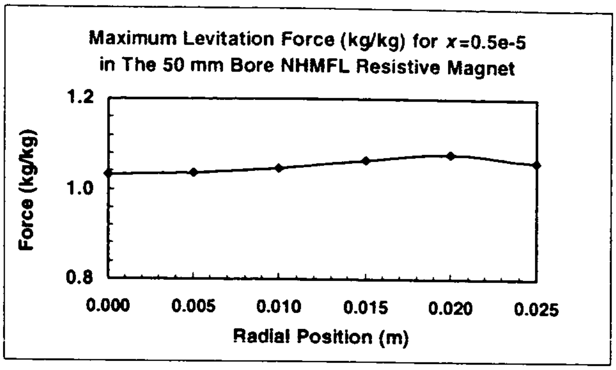

Fig. 4a Levitation force distribution in the NHMFL 50 mm bore, 22 T resistive magnet at z= 0.09 m from mid-plane

R	B_r	B_z	Bt	F_z/x	F_z
m	T	T	T	N/m3	kg/kg
0.000	0.000	0.000	0.000	-7.80E+08	0.9360
0.000	0.000	15.728	15.728	-7.88E+08	0.9454
0.020	0.613	15.742	15.754	-8.12E+08	0.9741
0.040	1.249	15.783	15.832	-8.53E+08	1.0234
0.060	1.935	15.844	15.962	-9.08E+08	1.0898
0.080	2.715	15.905	16.135	-9.51E+08	1.1416
				Average=	1.0655

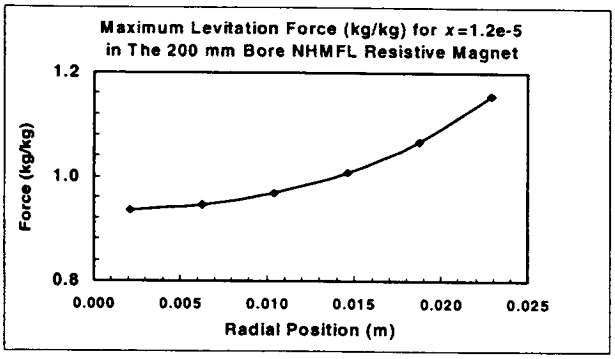

Fig. 4b Levitation force distribution in the NHMFL 200 mm, 20 T bore resistive magnet at z = 0.17 m from mid-plane

R	B_r	B_z	Bt	F_z/x	F_z
m	T	T	T	N/m3	kg/kg
0.000	0.000	0.000	0.000	-1.03E+09	1.0345
0.000	0.000	-19.278	19.278	-1.04E+09	1.0422
0.040	-2.708	-19.312	19.501	-1.07E+09	1.0671
0.080	-5.467	-19.413	20.168	-1.12E+09	1.1170
0.120	-8.336	-19.574	21.275	-1.21E+09	1.2076
0.160	-11.398	-19.781	22.830	-1.38E+09	1.3759
				Average=	1.19902

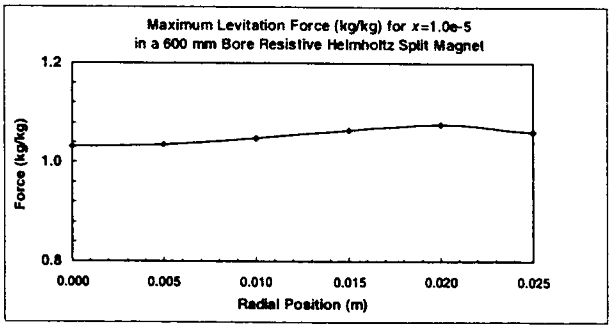

Fig. 4c Levitation force distribution in a 600 mm bore, 38 T maximum field resistive split Helmholtz magnet at z = -0.29 mm from mid-plane.