

MEASUREMENT AND CHARACTERIZATION OF FORCE DYNAMICS IN HIGH T_c
SUPERCONDUCTORS

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ABSTRACT

Magnetic bearing implementations using more exotic superconducting phenomena have been proliferating in recent years because they have important advantages over conventional implementations. For example, the stable suspension of an object in 6 degrees-of-freedom by superconducting means can be achieved without a control system and with the use of only a single superconductor. It follows that the construction becomes much simpler with decreased need for position sensors and stabilizers. However, it is recognized that the design of superconducting systems can be difficult because important characteristics relating to the 6 degree-of-freedom dynamics of an object suspended magnetically are not readily available and the underlying principles of superconducting phenomena are not yet completely understood. To eliminate some of the guesswork in the design process, this paper proposes a system which can resolve the mechanical properties of suspension by superconductivity and provide position and orientation dependent data about the system's damping, stiffness, and frequency response characteristics. This system employs an actively-controlled magnetically-suspended fine-motion device that can also be used as a 6 degree-of-freedom force sensor. By attaching the force sensor to a permanent magnet that is being levitated above a superconducting magnet, mechanical characteristics of the superconductor levitation can be extracted. Such information would prove useful for checking the validity of theoretical models and may even give insights into superconducting phenomena.

INTRODUCTION

Since the discovery of high-temperature superconductors, research in the field has been flourishing. In addition to numerous studies into the physics of the superconducting phenomena, a wide variety of applications research are energetically being pursued with the aim of using high T_c superconductors in magnetic bearing, levitation, and suspension systems.¹⁻⁵ It is widely known that when the non-uniform magnetic field of a permanent magnet interacts with a superconductor, stable levitation of the magnet over or under the superconductor can be achieved via the Meissner effect and strong flux pinning forces. Friction-free magnetic bearing or levitation devices using this phenomenon possess the important characteristic that neither a complicated control system nor power source is needed.

To design an actual levitation device using superconductors, the restoration and damping forces acting on the levitated magnetic element need to be precisely understood; and as a result, many fundamental aspects of these issues have been addressed by various research groups.^{3,5,6} To date, however, no complete physical model of forces in high T_c superconductors has been developed. Consequently, the design of applications such as magnetic bearings is still a trial-and-error process to some degree. Some of the important parameters needed for design are the magnitudes of vertical, lateral, and rotational forces along with their associated stiffness and damping characteristics. Static levitation pressure is an important force measure because it dictates the maximum load that the levitation system can bear and also levitation displacement under loaded conditions. Lateral and rotational forces govern the stability of the levitation. On the other hand, magnetic stiffnesses are important because they determine the system's natural resonant frequencies and are measures of the restorative forces acting on the levitated magnet when it is displaced by a transient disturbance. Damping, the dynamic characteristic which acts to suppress vibrational motion of the permanent magnet, is important because it contributes to levitational stability.

Several measurement techniques have been developed to obtain these quantitative characteristics experimentally. Some of the devices which have reportedly been used to measure static forces are a torsion balance,⁷ a single pan balance,^{3,8-11} and an elastic beam equipped with strain gauges.^{2,5,10,12} Dynamic response measurements have been made using gaussmeter probes with an oscillating table as a disturbance source³ and also by using the above strain gauge beam in combination with an optical tracking system.^{5,8}

However, all these methods suffer from the drawback that they limit measurement to only one degree-of-freedom at a single time. Also, even after allowing for reconfiguration between measurements, these methods can only be used to characterize forces in the three (x, y, z) translational degrees-of-freedom. Certainly, for many applications, knowledge of force characteristics in the remaining three (θ, ϕ, ψ) rotational degrees-of-freedom is important. Additionally, it may be valuable to be able to characterize any dynamic cross-coupling dependencies between motion in one degree-of-freedom and a resultant force in another. Such information should prove useful to not only the engineering design process but also for the validation of future theoretical superconductor models.

It follows that a new high T_c superconductor force measurement system using an actively-controlled magnetically-levitated fine-motion mechanism is proposed in this paper. The mechanism would have the combined function of multiple degree-of-freedom force and position sensing

of a permanent magnet levitating above a superconductor. When used in actively-controlled force measurement mode, the sensing device could be used to perform automated static force measurements of the superconductor's spatially dependent, intrinsically hysteretic forces and torques. Additionally, when used in position control mode, the sensing device could be used to perform automated characterization of force dynamics in the superconductor.

Essentially, it is proposed that the actively-compensated magnetic servo levitation device can be used as a tool to characterize the passive compensation of the superconductor–magnet interaction. When the superconductor is not in its superconducting state (i.e. no force interaction with the magnet), the transient response of the magnetic suspension system can be determined. Then, by measuring the change in the entire system's transient response when the superconductor is switched to its superconducting state, it should be possible to quantify the superconductor's contribution to the system's force dynamics.

MEASUREMENT PRINCIPLE

Six Degree-of-Freedom Measurement Device Using Magnetic Servo Levitation

The type of device being proposed for use in a superconductor force measurement system is a 6 degree-of-freedom (DOF) actively-controlled magnetic servo levitation mechanism that can function as a precision force/position controllable actuator. The mechanism's levitated component, or flotor, is a free-moving rigid body that can be position-sensed and controlled in as many as 6 DOF. Because the motion of the flotor is free from troublesome friction and backlash, control can be achieved using PD digital control without difficulty. Some examples of relevant devices which fall into this category are the "magic" wrist developed at IBM¹³ and the magnetically supported intelligent hands developed at Tokyo University.^{14,15} Typical force and position sensing resolutions on the order of 1 mN and 1 μ m can be expected from these devices.

Despite possessing flexibility in physical design, as represented by the unique constructions of each of these examples, these devices have some common features. In general, actuation is provided by either the attractive forces of strategically placed electromagnets¹⁴⁻¹⁶ or the Lorentz forces of voice coil motors.¹³⁻¹⁶ Flotor position-sensing is generally done by non-contact means such as gap sensors or position sensitive detectors.¹⁶ For the purpose of using this type of device to measure superconductor forces, a construction which has sensing near the flotor's end-point (i.e. the permanent magnet levitating over the superconductor) would be best. Reference 16 gives an extensive discussion of related design issues so they will not be discussed further here.

Important to this discussion, however, is the understanding of the mechanism's dynamics and control. To do this, we will refer to a conceptual schematic view of a magnetic servo levitation device shown in Figure 1. The actual structure of the device shown here is not important since any of the previously mentioned example devices could be employed. Yet, it is significant to note the relationship between the permanent magnet attached to the magnetic servo levitation device's end-point and the bulk high T_c superconductor arranged below. Due to the limited motion range of magnetic servo levitation, the device would be mounted on a stage that would provide gross movement capability as needed. Figure 2 is

a photograph of the setup which we have begun to use in our experiments. Inside the dark plastic case and above the superconductor vessel is the "intelligent hand", developed at Tokyo University, being used as a force sensor.¹⁴

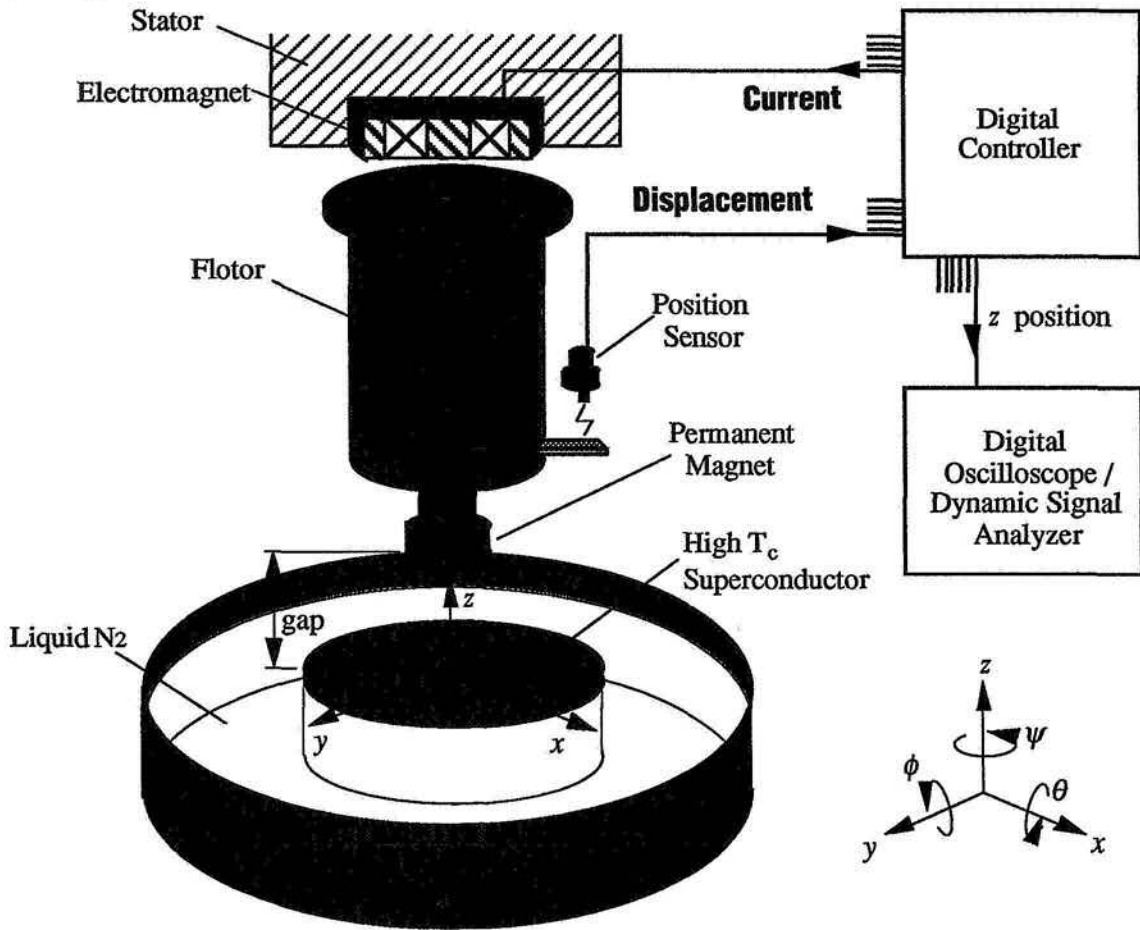


Figure 1. Schematic of magnetic servo levitation measuring system.

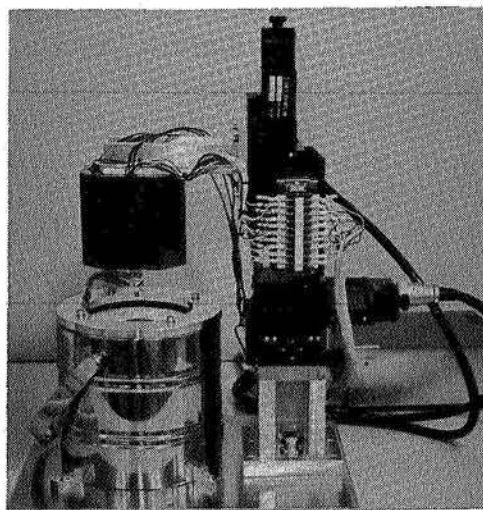


Figure 2. Photograph of measurement system using "magnetically supported intelligent hand".

Static Force Measurement

Although the flotor has forces acting on it in 6 DOF, only those acting in the translational z direction will be considered for simplicity. As shown, the forces acting against the flotor's levitation are gravity, Mg , and an external force, $f_{sc}(t)$, due to the superconductor–magnet interaction. Oppositely, the levitation forces provided by the actuators are a constant gravitational offset force, f_o , and a digitally controlled force, $f(t)$, which acts to isolate the flotor from any external disturbances and maintain the desired reference position, $z_d(t)$. It follows that the flotor's equation of motion can be written as

$$M \frac{\partial^2 z(t)}{\partial t^2} = f_o + f(t) - Mg - f_{sc}(t) \quad (1)$$

Because $f_o = Mg$, equation (1) can be reduced to

$$M \frac{\partial^2 z(t)}{\partial t^2} = f(t) - f_{sc}(t) \quad (2).$$

This can be viewed as a general equation of motion that can be applied to all remaining DOF since gravitational force considerations are no longer necessary. Because the controlled force, $f(t)$, is known and $f(t) - f_{sc}(t) = 0$ in the static case, the magnetic levitation device can be used to measure the external force of the superconductor, $f_{sc}(t)$.

Dynamic Force Measurement

To consider the dynamics of the system, equation (2) can be expressed by its Laplace transform as

$$s^2 MZ(s) = F(s) - F_{sc}(s) = F_T(s) \quad (3)$$

Momentarily ignoring the presence of any external superconductor force, a block diagram for a possible digital controller for the magnetic levitation system is shown in Figure 3. $D(z)$ is a discrete time compensator of the digital controller, $G_p(s)$ is the transfer function of the plant, and $H(s)$ is the transfer function of the feedback channel. Included are the A/D and D/A converters which interface the digital controller to the physical system. If we assume that a simple unity-feedback digital PD control is implemented, then $H(s) = 1$ and the compensator's transfer function can be written as the z-transform

$$D(z) = d_z + \frac{k_z(z-1)}{Tz} \quad (4)$$

where T is the A/D converter sampling period and d_z and k_z are the desired damping and spring constants chosen by the operator and entered into the computer which supervises the digital control system.

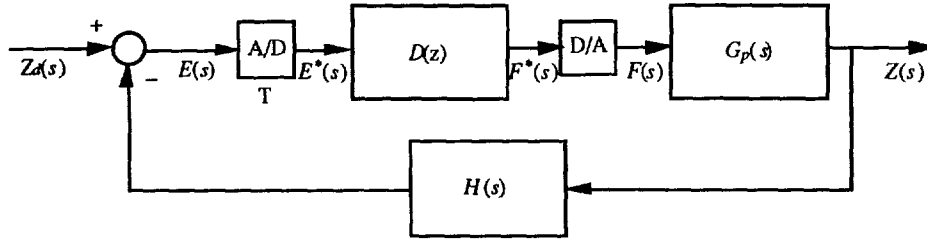


Figure 3. Block diagram of digital controller for magnetic servo levitation.

In practice, because magnetic levitation devices normally employ only position sensors, the velocity signal which is needed for PD control must be either calculated by numerical differentiation of the position signal or estimated by an adaptive control scheme.^{15,16} To ensure accurate velocity calculation it is important to keep the sampling period, T , at a minimal value. Regardless of the actual implementation details of the digitally controlled magnetic levitation system, it is our goal to relate its actual measurable response with a purely analog model. One reason for doing this is that a simple transfer function for the sampling process in Figure 3 does not exist and the analysis is complicated.¹⁷ Therefore, we will assume that the experimental determined transient response of the system can be modelled within a limited frequency range by the analog system whose block diagram is shown in Figure 4.

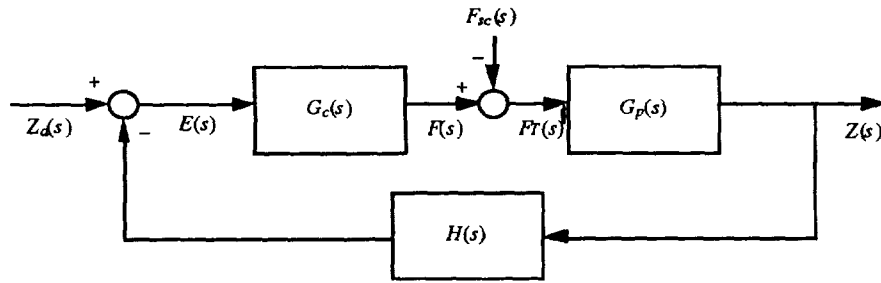


Figure 4. Block diagram of analog model of magnetic servo levitation system.

Again, assuming initially that $f_{sc}(t) = 0$ and unity feedback is employed, the transfer function for the system in Figure 4 can be written as

$$T(s) = \frac{Z(s)}{Z_d(s)} = \frac{G_c(s)G_p(s)}{1 + G_c(s)G_p(s)} \quad (5)$$

where the plant transfer function, $G_p(s)$, can be found from equation (3) and expressed as

$$G_p(s) = \frac{Z(s)}{F_T(s)} = \frac{1}{Ms^2} \quad (6)$$

For simple characterization of the magnetic levitation device's dynamics, we model the compensator, $G_c(s)$, with linear mechanical elements of damping and elastance to give a second-order system. The transfer function for this compensator is then given by

$$G_c(s) = d_0s + k_0 \quad (7)$$

where d_o is the damping coefficient and k_o is the spring constant. The system's transfer function then reduces to the expression

$$T(s) = \frac{d_o s + k_o}{M s^2 + d_o s + k_o} \quad (8).$$

To experimentally quantify the effective damping coefficient and spring constant of the magnetic levitation device, a dynamic signal analyzer can be used to measure the frequency response and then automatically calculate the poles and zeros of equation (8). Alternatively, this information can also be revealed by observing the transient response of the position signal with an oscilloscope. Although, both impulse and step responses of the system can reveal the same information, in this discussion only the impulse response will be dealt with. Practically, an input which approximates an impulse can be provided by rapidly increasing and decreasing the flotor's desired servo position, z_d , via the computer supervising the digital control system.

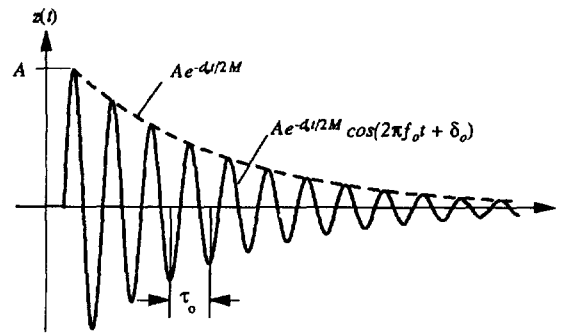


Figure 5. Impulse response for second-order system model of magnetic servo levitation.

Figure 5 shows a typical second-order response to an impulse input. The position signal in this figure exhibits damped oscillation, resembling a cosine curve with decaying amplitude, as the flotor settles toward the desired reference position. It follows that from the envelope of the curve, which decays exponentially with time, the damping coefficient, d_o , can be calculated. By measuring the quasiperiod of the signal, τ_o , which is defined as the time between successive maxima or minima, the quasicyclic frequency of the oscillation, f_o , can be calculated using the relation ¹⁸

$$f_o = \frac{1}{\tau_o} \quad (9).$$

It is then possible to calculate the spring constant of the system by ¹⁸

$$k_o = 4\pi^2 M f_o^2 + \frac{d_o^2}{4M} \quad (10).$$

The second term in equation (10) cannot be neglected, as is often done, unless damping is very weak.

Next, we consider the case for which the superconductor's interaction with the permanent magnet contributes an external force on the flotor and permanent magnet combination. We choose to model the effect of the superconductor levitational force, $F_{sc}(s)$, with a second-order mechanical system that includes damping and elastance. The transfer function of this position compensation by the superconductor is defined by

$$G_{sc}(s) = \frac{Z(s)}{F_{sc}(s)} = d_{sc}s + k_{sc} \quad (11).$$

Figure 6, a now modified block diagram of the magnetic servo levitation system, shows the stabilizing negative-feedback force loop that acts the flotor by the superconductor in response to a change in position.

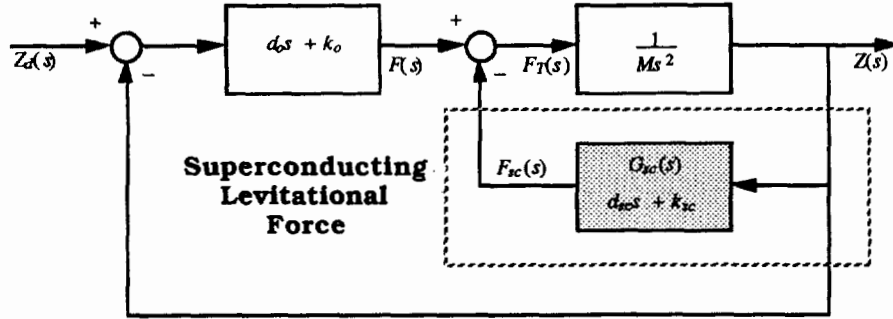


Figure 6. Block diagram of magnetic servo levitation system model including superconductor forces.

The transfer function for the entire system now takes the form

$$T(s) = \frac{Z(s)}{Z_d(s)} = \frac{G_c(s)G_p(s)}{1 + [G_c(s) + G_{sc}(s)]G_p(s)} \quad (12)$$

which can be reduced further to

$$T(s) = \frac{d_o s + k_o}{Ms^2 + (d_o + d_{sc})s + (k_o + k_{sc})} = \frac{d_o s + k_o}{Ms^2 + ds + k} \quad (13).$$

By comparing equations (8) and (13) we can see the effect of adding the superconductor to the system is just a simple increase in total system damping and elastance. The entire system's damping is the simple sum of the independent damping coefficients respectively associated with the magnetic levitation portion of the system and the superconductor portion of the system. With a similar relationship for the system's elastance we have

$$d = d_o + d_{sc} \quad \text{and} \quad k = k_o + k_{sc} \quad (14) \quad (15).$$

Thus, we can calculate the damping coefficient and spring constant of the superconductor-magnet interaction if the transient response of the entire system is measured. A typical impulse response of this system would take the form shown in Figure 7. Because of increased damping and elastance due to the superconductor, this oscillatory signal's decay rate and the quasifrequency are both greater than those for the magnetic levitation device alone as shown in Figure 5. By employing the method described before to extract the damping coefficient, d , and spring constant, k , the relationships in equations (14) and (15) can be used to calculate d_{sc} and k_{sc} .

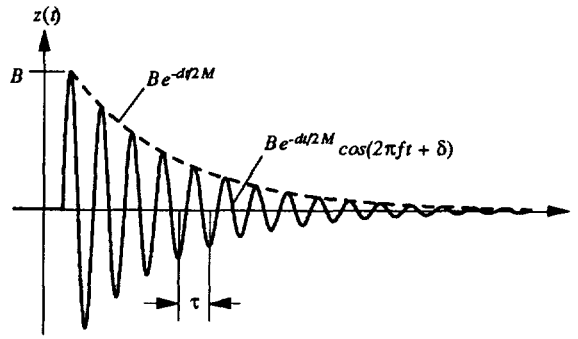


Figure 7. Impulse response for second-order system model of magnetic servo levitation including superconductor forces.

COMPLETE 6 DOF CHARACTERIZATION OF SUPERCONDUCTOR FORCES

Static Force Measurements

The static superconductor forces acting on a permanent magnet directly determine some very important performance quantifiers for any suspension application they may be used in. For example, in a magnetic bearing application, the levitation pressure governs the maximum vertical load that can be applied to the bearing. Similarly, lateral forces govern the maximum horizontal load that the bearing can stably support. Rotational forces, in turn, dictate how the bearing may perform if torsional loading conditions cause the permanent magnet to twist away from an equilibrium orientation. In all cases, these static forces also determine the loaded bearing's displacement from an unloaded equilibrium position.

The hysteretic nature of the static forces in superconductors is well known.^{3,5,6,12,19} For example, if the height of the magnet over the superconductor is cycled at *very low* frequency the levitation force exhibits hysteresis which depends on whether the magnet is approaching or retreating from the superconductor. Thus, not only are the force magnitudes functions of position and orientation of the permanent magnet over the superconductor, but also the directions the magnet has been moved to arrive at that position and orientation. Whether the force is actually repulsive or attractive not only depends on position and orientation but also on superconductor composition⁵ and whether cooling of the superconductor was done in zero magnetic-field cooled (ZFC) or field cooled (FC) conditions.^{3,4}

In any case, we would like to express the 6 DOF static forces and torques acting on the magnet in vector form so that they are in a convenient form to be used in modelling or simulations of a suspension application using superconductors. We can write

$$f_{sc}(t) = f_{sc}(\mathbf{r}(t), \text{sign}(\dot{\mathbf{r}})) = \begin{bmatrix} f_x \\ f_y \\ f_z \\ \tau_\theta \\ \tau_\phi \\ \tau_\psi \end{bmatrix} \quad (16)$$

where the variable $\mathbf{r}(t)$ is a 6 DOF position and orientation vector and the variable $\mathbf{sign}(\dot{\mathbf{r}})$ is a vector indicating on which branches of the hysteresis curves for each DOF the static force and torque measurements have been made. These vectors are defined as

$$\mathbf{r}(t) = \begin{bmatrix} x \\ y \\ z \\ \theta \\ \phi \\ \psi \end{bmatrix} \quad \text{and} \quad \mathbf{sign}(\dot{\mathbf{r}}) = \begin{bmatrix} \mathit{sign}(\dot{x}) \\ \mathit{sign}(\dot{y}) \\ \mathit{sign}(\dot{z}) \\ \mathit{sign}(\dot{\theta}) \\ \mathit{sign}(\dot{\phi}) \\ \mathit{sign}(\dot{\psi}) \end{bmatrix} \quad (17) \quad (18).$$

where the elements, $\mathit{sign}(\dot{n})$, can take one of two values: + or -, indicating whether the position/orientation value n is steadily being changed in positive or negative steps along its respective hysteresis curve. It should be noted that to consider the force measurements to be truly static, the magnet should have no velocity during measurement. However, it has been reported that an extremely low constant velocity on the order of 100 $\mu\text{m/s}$ or less can be tolerated.⁵

Considering that each element in the vector f_{sc} is effectively a function of 12 variables due to hysteresis in all 6 DOF, the number of measurements that are required for its identification over reasonable magnet motion ranges can be large. However, this measurement process could be automated by a computer system host to the force measuring magnetic levitation device that would systematically measure forces on all hysteresis branches for many combinations of positions and orientations. This process, not requiring human intervention, would be exceedingly hard to duplicate, if not impossible, using current force measurement technology.

One frequently used method to get an approximation of magnetic stiffness in superconductors is to use static force measurements. As one traces a major hysteresis loop, a small deviation from the loop can be made by momentarily reversing the direction of motion and a minor hysteresis loop can be traced out before returning to the major loop. By averaging the slope of these minor loops, a magnetic stiffness measurement can be made.^{3,5,8} However, it has been reported that the values obtained this way may not actually reflect the true magnetic stiffness because they are dependent on the displacement size used.⁵ Thus, it would be prudent to also determine stiffness by a dynamic method which can also yield important damping information at the same time. This will be covered in the next section.

Dynamic Force Measurements

The goal of measuring the dynamic characteristics of superconductors is to quantify the two important parameters of damping and elastance. With this data, it is then possible to design a magnetic suspension application using the superconductor-magnet interaction phenomenon with complete understanding of the transient response which can be expected when the system is assembled and operating. Therefore, it is important to identify damping coefficients and spring constants that describe the dynamic behaviour of the superconductor-magnet combination. These characteristics can then be summarized for 6 DOF in the following respective matrix forms:

$$\mathbf{d}_{sc} = \mathbf{d}_{sc}(\mathbf{r}, \text{sign}(\dot{\mathbf{r}})) = \begin{bmatrix} d_{xx} & d_{xy} & d_{xz} & d_{x\theta} & d_{x\phi} & d_{x\psi} \\ d_{yx} & d_{yy} & . & . & . & . \\ d_{zx} & . & d_{zz} & . & . & . \\ d_{\theta x} & . & . & d_{\theta\theta} & . & . \\ d_{\phi x} & . & . & . & d_{\phi\phi} & . \\ d_{\psi x} & . & . & . & . & d_{\psi\psi} \end{bmatrix} \quad (19)$$

and

$$\mathbf{k}_{sc} = \mathbf{k}_{sc}(\mathbf{r}, \text{sign}(\dot{\mathbf{r}})) = \begin{bmatrix} k_{xx} & k_{xy} & k_{xz} & k_{x\theta} & k_{x\phi} & k_{x\psi} \\ k_{yx} & k_{yy} & . & . & . & . \\ k_{zx} & . & k_{zz} & . & . & . \\ k_{\theta x} & . & . & k_{\theta\theta} & . & . \\ k_{\phi x} & . & . & . & k_{\phi\phi} & . \\ k_{\psi x} & . & . & . & . & k_{\psi\psi} \end{bmatrix} \quad (20)$$

The individual elements of the damping coefficient matrix, \mathbf{d}_{sc} , can be defined by

$$d_{ab} = d_{ab}(\mathbf{r}, \text{sign}(\dot{\mathbf{r}})) = -\frac{\Delta f_a}{\dot{b}} \quad \text{for } a, b = x, y, z \quad (21)$$

and

$$d_{ab} = d_{ab}(\mathbf{r}, \text{sign}(\dot{\mathbf{r}})) = -\frac{\Delta \tau_a}{\dot{b}} \quad \text{for } a, b = \theta, \phi, \psi \quad (22)$$

which show that damping is the oscillation reducing force that occurs in the direction a when the magnet has velocity in direction b . The individual elements of the spring constant matrix, \mathbf{k}_{sc} , can be defined by

$$k_{ab} = k_{ab}(\mathbf{r}, \text{sign}(\dot{\mathbf{r}})) \approx -\frac{\Delta f_a}{\Delta b} \quad \text{for } a, b = x, y, z \quad (23)$$

and

$$k_{ab} = k_{ab}(\mathbf{r}, \text{sign}(\dot{\mathbf{r}})) \approx -\frac{\Delta \tau_a}{\Delta b} \quad \text{for } a, b = \theta, \phi, \psi \quad (24)$$

which show that elastance is the restorative force exerted in direction a on the magnet when it is displaced by a *small* amount in direction b from some equilibrium position. Like static forces and torques, the dynamic characteristics of damping and stiffness for the superconductor are functions of magnet position and orientation and also functions of which branches of the familiar hysteresis loops the superconductor–magnet interaction is operating on. Thus, it follows that each element of the damping coefficient and spring constant matrices is a function of 12 variables.

The impulse (or step) response analysis techniques discussed previously could also be automated by a computer system hosting the actively controlled magnetic levitation system. The damping coefficient and spring constant matrices could be systematically identified by making measurements on all branches of the force–position hysteresis loops for many combinations of positions and orientations.

To our knowledge, none of the existing methods that have been reported in literature are capable of

identifying the full damping and elastance matrices. In fact, these one DOF implementations even require reconfiguration to measure only the diagonal elements of d_{xx} , d_{yy} , d_{zz} , k_{xx} , k_{yy} , and k_{zz} .^{3,5,8}

The advantage of expressing damping and elastance properties in matrix form is that an equation of motion can then be written for the magnet–superconductor interaction (without a magnetically levitated measuring device attached) in the form

$$\mathbf{M} \frac{\partial^2 \mathbf{r}(t)}{\partial t^2} - \mathbf{d}_{sc} \frac{\partial \mathbf{r}(t)}{\partial t} - \mathbf{k}_{sc}(\mathbf{r}(t) - \mathbf{r}(0)) = 0 \quad (25)$$

where $\mathbf{r}(0)$ is the initial equilibrium position and orientation and \mathbf{M} is the inertial matrix of the magnet which can be expressed as

$$\mathbf{M} = \begin{bmatrix} M & & & & & \\ & M & & & & \\ & & M & & & \\ & & & J_\theta & & \\ & & & & J_\phi & \\ & & & & & J_\psi \end{bmatrix} \quad (26).$$

Equation (25) fully describes the dynamic behaviour of the superconductor–magnet levitation sub-system. It follows that the sub-system has specific mechanical properties that can be simulated by computer and can be utilized as a single module in a larger application. It is important to remember however, that \mathbf{d}_{sc} and \mathbf{k}_{sc} are not constant, but instead functions of $\mathbf{r}(t)$ and $\text{sign}(\dot{\mathbf{r}})$ as defined before.

SUMMARY

It has been proposed that an actively-controlled 6 DOF magnetic servo levitation device can be employed to characterize the force interaction between permanent magnets and superconductors. When this device is used in force measurement mode, it would be possible to measure static superconductor forces and torques in all 6 DOF and summarize them in matrix form. By monitoring the change in the transient or frequency response of the magnetic servo levitation system when superconductor forces are applied, the dynamics superconductor–magnet interaction, in the form of damping and elastance characteristics, can be quantified and summarized in matrix form.

Then, given the static force matrix, $f_{sc}(t)$, and the damping and elastance matrices, \mathbf{d}_{sc} and \mathbf{k}_{sc} , an engineer designing superconductor levitation application can treat the superconductor–magnet combination as a well-defined "mechanical" element with known response characteristics. A complicated system can be designed with several of these elements and the total system's response can be calculated using principles of superposition and unification of coordinate systems. This should take much of the guesswork out of designing a superconductor magnetic levitation system and allow computer simulation to be used instead of trial-and-error prototype construction. Additionally, extensive knowledge of static and dynamic force characteristics may help to verify or create theoretical models concerning the physical principles of superconducting phenomena.

REFERENCES

1. Takahata, R.; Ueyama, H.; and Yotsuya, T.: Load Carrying Capacity of Superconducting Magnetic Bearings. *Electromagnetic Forces and Applications*. Elsevier Science Publishers B.V., 1992.
2. Takahata, Ryoichi; and Yotsuya, Tsutom: Rotational Decay Torques of Superconducting Magnetic Bearing. *IEEE Transactions on Magnetics*, vol. 27, no. 2, March 1991, pp. 2423-2426.
3. Weinberger, B.R.; Lynds, L.; and Hull, J.R.: Magnetic Bearings Using High-Temperature Superconductors: Some Practical Considerations. *Superconductor Science Technology*, vol. 3, 1990, pp. 381-388.
4. McMichael, C.K.; Ma, K.B.; Lamb, M.A.; Lin, M.W.; Chow, L.; Meng, R.L.; Hor, P.H.; and Chu, W.K.: Practical Adaption in Bulk Superconducting Magnetic Bearing Applications. *Applied Physics Letters*, vol. 60, no. 15, April 13, 1992, pp. 1893-1895.
5. Chang, Pei-Zen: Mechanics of Superconducting Magnetic Bearings. Ph.D. Dissertation, Cornell University, January, 1991.
6. Torng, Terry; and Chen, Q.Y.: Magnetic Forces for Type II Superconductors in a Levitation Field. *Journal of Applied Physics*, vol 73, no. 3, February 1, 1993, pp. 1198-1204.
7. Weeks, David E.: A Torsion Balance to Measure Hysteretic Levitation Forces in High Tc Superconductors. *Review of Scientific Instruments*, vol. 61, no. 1, January, 1990, pp. 197-200.
8. Weinberger, B.R.; Lynds L.; Van Valzah, J.; Eaton, H.E.; Hull, J.R.; Mulcahy, T.M.; and Basinger, S.A.: Characterization of Composite High Temperature Superconductors for Magnetic Bearing Applications. *IEEE Transactions on Magnetics*, vol. 27, no. 2, March 1991, pp. 2415-2422.
9. Ouseph, P.J.: Effect of an External Force on Levitation of a Magnet Over a Superconductor. *Applied Physics A*, vol. 50, 1990, pp. 361-364.
10. Unsworth, Joseph; Du, Jia; Crosby, Bryan J.; and Macfarlane, John C.: Magnetic Levitation Force Measurement on High Tc Superconducting Ceramic/Polymer Composites. *IEEE Transactions on Magnetics*, vol. 29, no. 1, January 1993, pp. 108-112.
11. Riise, A.B.; Johansen, T.H.; Bratsberg, H.; and Yang, Z.J.: Logarithmic Relaxation in the Levitation Force in a Magnet-High Tc Superconductor System. *Applied Physics Letters*, vol. 60., no. 18, May 4, 1992, pp. 2294-2296.
12. Moon, F.C.; Yanoviak, M.M.; and Ware, R.: Hysteretic Levitation Forces in Superconducting Ceramics. *Applied Physics Letters*, vol. 52, no. 18, May 2, 1988.
13. Hollis, R.L.; A.P. Allan; and Salcudean, S.; and : A Six Degree-of-Freedom Magnetically Levitated Variable Compliance Fine Motion Wrist. *Preprint of the 4th ISRR*, 1987, pp. 241-249.

14. Fujiwara, Shigeki; and Higuchi, Toshiro: Miniaturized and Lightened 6-Axis Controlled Magnetically Levitated Intelligent Hand, Dai 9 Nippon Robotto Gakkai Gakujutu Kouenkai, November 27-29, 1991, pp. 403-404.
15. Tsuda, Masahiro: Magnetic Servo Levitation and Its Applications. Ph.D. Dissertation, University of Tokyo.
16. Tsuda, Masahiro; and Higuchi, Toshiro: Design and Control of Magnetic Servo Levitation. *Report of the Institute of Industrial Science, The University of Tokyo*, vol. 37, no. 2, serial no. 236, March, 1992.
17. Phillips, Charles L.; and Harbor, Royce D.: *Feedback Control Systems*. Prentice-Hall, Inc., 1988.
18. Boyce, William, E.; and DiPrima, Richard C.: *Elementary Differential Equations and Boundary Value Problems*. John Wiley & Sons, Inc., 1986.
19. Ishigaki, H.; Itoh, M.; Hida, A.; Endo, H.; Oya, T.; Ohyama, T.; and Minemoto, T.: Measurement of Repulsive Force of High T_c Materials due to Meissner Effect and its Two Dimensional Distribution. *IEEE Transactions on Magnetics*, vol. 27, no. 2, March 1991, pp. 2427-2430.