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EXPERIMENTAL MEASUREMENT AND CALCULATION OF LOSSES IN PLANAR RADIAL MAGNETIC BEARINGS

M. E. F. Kasarda, Research Associate
P. E. Allaire, Professor
R. W. Hope, Research Associate
R. R. Humphris, Research Professor
University of Virginia
Charlottesville, VA

ABSTRACT

The loss mechanisms associated with magnetic bearings have yet to be adequately characterized or modeled analytically and thus pose a problem for the designer of magnetic bearings. This problem is particularly important for aerospace applications where low power consumption of components is critical. Also, losses are expected to be large for high speed operation. The iron losses in magnetic bearings can be divided into eddy current losses and hysteresis losses. While theoretical models for these losses exist for transformer and electric motor applications, they have not been verified for magnetic bearings. This paper presents the results from a low speed experimental test rig and compares them to calculated values from existing theory. Experimental data was taken over a range of 90 to 2,800 rpm for several bias currents, and two different pole configurations. With certain assumptions agreement between measured and calculated power losses was within 16% for a number of test configurations.

NOMENCLATURE

B_{max} = Maximum Magnetic Flux Density
d = Lamination Thickness
D = Windage Drag Force on Rotor
E_k = Rotor Kinetic Energy
f = Frequency (Hz)
I = Current
J = Rotor Polar Moment of Inertia
L = Rotor Lamination Length
P_e = Eddy Current Power Losses
P_{ha} = Hysteresis Power Loss Due to Alternating Flux
P_{hr} = Hysteresis Power Loss Due to Rotating Flux
P_I = Iron Power Losses
P_k = Kinetic Power Loss
P_W = Power Loss Due to Windage
R = Rotor Lamination Radius
U = Surface Velocity of Rotor Laminations
t = Time
ρ = Material Resistivity
ρ_g = Gas Density
η = Hysteresis Coefficient
ν = Gas Viscosity
ω = Rotor Angular Velocity (rad/sec)

INTRODUCTION

Magnetic bearings offer the designer of jet engines the ability to operate at much higher speeds than when conventional rolling element bearings are used. Because of the stiffness and damping properties of the magnetic bearings the squeeze film damper can be eliminated and the lubrication system is no longer necessary. In order to fully optimize the design of the engine the designer must account for losses in the bearings. While magnetic bearings do not have the kinds of losses associated with conventional bearing types, they do have several unique loss mechanisms that need to be quantified more accurately than they have been in the past.

The loss mechanisms in magnetic bearings may be classified in the categories of 1) coil losses, 2) iron losses, and 3) windage losses. The coil losses are the resistive losses due to current in the stator coils. They are relatively easily calculated by conventional I^2R formulas and are not discussed further here. The iron losses are the losses which occur in the stator and rotor of the magnetic bearings. Generally these have two components: eddy current losses and hysteresis losses. The rotor losses are the primary subject of this paper. The final category of losses is that of windage friction loss due to the gases or liquids surrounding the bearing rotor. While windage losses may be significant in some applications, they are a secondary subject of this paper.

The topic of magnetic core losses both in nonrotating devices, such as transformers, and rotating machines, such as electric motors, is a very complicated subject. Much of the early work on core losses, referred to as classical work in this paper, was done early in this century. It largely ignored the physics of the magnetic domain structure, but developed a series of relatively simple formulas for quantifying the losses. The iron losses were considered to be divided into two types: eddy current and hysteresis losses. Hysteresis losses were further sub-divided into alternating and rotational components. The more modern theory associates the losses with physical phenomena on a molecular level occurring with changing magnetism, called domain theory. All of the magnetic loss effects can be explained using the domain theory, but convenient formulas suitable for magnetic bearing design are not currently known to the authors.

This paper does not address the domain theory of losses but employs classical formulas to calculate some of the losses for comparison to the experimental run down test results. Iron losses in magnetic bearings, rather than in transformers and electric motors, began to be reported in the literature approximately 10 years ago. Yoshima [1] discussed an eddy current effect in magnetic bearings using a detailed finite element model to calculate the magnetic flux in the bearing. The effect of the induced opposing magnetic forces was studied, but losses were not quantified. Higuchi, et. al [2], and Higuchi, et. al [3] gave short reports on experimental studies of rotating losses in magnetic bearings. Matsumura, et. al [4] presented a fourier analysis of the distribution of the magnetic field for an alternating pole arrangement (NSNSNSNS) and a paired pole arrangement (NNSSNNSS). The paper gave a theoretical prediction of higher losses in the alternating pole arrangement. Experimental results show only a slight difference in the losses. At high speeds the loss plots are nearly identical while at lower speeds the alternating arrangement yields slightly higher losses.

Ueyama and Fujimoto [5] evaluated power losses in a magnetic bearing supported spindle test rig. Iron and windage loss effects were studied for an eight pole radial magnetic bearing of conventional design. Deceleration studies gave the iron losses for four materials when the chamber was placed in a vacuum. A hysteresis coefficient and an eddy current coefficient were determined for the experimental results, but no comparison to theoretical predictions was presented. Measured and calculated windage losses were compared with good agreement for the turbulent flow calculation model. Matsumura and Hatake [6] conducted loss measurements using an eight pole radial magnetic bearing.

O'Kelly [7] has developed a model of nonlinear hysteresis BH effects in magnetic materials by employing a series of straight lines. Lin, et. al [8] formulated a numerical curve fitting BH routine for transformer coils and cores. Kasarda, et. al [9] presented measured losses with the journal at different eccentricities within the bearing stator and found this effect to be negligible for the test rotor under analysis.

The purpose of this paper is to measure the effect of both bias current and pole configuration on the power losses in a set of magnetic bearings and compare these results to theoretical predictions. Steady state bias currents directly affect the magnetic flux density so they are important. Two different pole configurations were studied: alternating poles (NSNSNSNS) vs. paired poles (NNSSNNSS). The measured results were then compared to calculated losses due to 1) eddy currents, 2) hysteresis, and 3) windage losses, from several existing theories. Calculations based upon iron losses in transformers and windage losses in annular clearances were employed.

EXPERIMENTAL PROCEDURE

A magnetic bearing test rig [10], illustrated in Figure 1, was set up for measurements of rotor run down time. The rotor consists of a 12.7 mm (0.50 inch) diameter shaft with three attached masses. The midspan disk measures 73.2 mm (2.88 inches) in diameter and 25.4 mm (1.0 inches) in length. The two outboard disks are the bearing journals, and measure 58.4 mm (2.3 inches) in diameter and 25.4 mm (1.0 inches) in length.

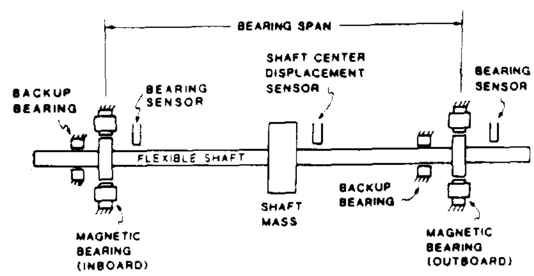

FIGURE 1 - Test Rotor Configuration

The magnetic bearings are eight pole radial bearings, shown in Figure 2. The leg width is 12.7 mm (0.50 inch) for each pole. The stators are constructed of solid soft magnetic iron and the journals are constructed of 0.18 mm (0.007 inch) laminations of non-oriented 3% silicon iron. The bearings have a 1.0 mm (0.039 inch) gap. The feedback system used an analog PD control with a linear power amplifier.

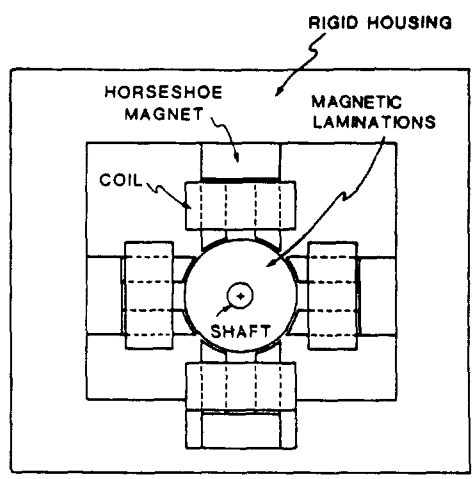

FIGURE 2 - Radial Magnetic Bearing Configuration

The rotor was decoupled from the electric motor, usually employed to drive the rig. This avoids the friction effects in the motor rolling element bearings. For each test run, a cord was wrapped around the rotor shaft and then pulled off at a rapid rate. The rotor was thus accelerated to a speed somewhat above 2,800 rpm. Then the rotor was allowed to decelerate and measurements of speed vs. time were taken from 2,800 rpm to 90 rpm. Real time run down tests were made and data recorded on tape. From this data, decay rates were calculated and the corresponding total power loss, due to the conversion of kinetic energy of rotation into heat, determined.

No magnetic thrust bearing was used in the system, but reluctance centering due to end effects in the magnetic bearings was sufficient to keep the rotor axially centered. Small axial transients immediately present after a cord pull were manually damped out.

The kinetic energy of the rotor due to rotation is

$$E_k = \frac{1}{2} J \omega^2 \tag{1}$$

This kinetic energy was gradually converted to heat as the rotor decelerated. The power loss is the time derivative of the energy

$$P_k = \frac{\partial E_k}{\partial t} = \frac{\partial}{\partial t}\left(\frac{1}{2} J\omega^2\right) = J\omega \frac{\partial \omega}{\partial t} \tag{2}$$

The speed vs. time measurements were fit with a least squares polynomial of order 2

$$\omega = b_0 + b_1 t + b_2 t^2 \tag{3}$$

and the time derivative evaluated analytically. Time can be treated as a parameter and eliminated. Thus the change in energy vs. speed was calculated from Eq. (2).

EXPERIMENTAL RESULTS

A series of rotor run-down measurements was taken for two parameter studies: bias current effects and pole configuration effects.

Bias Current Effects

Steady state currents, called bias currents, are supplied to all coils in the bearings. Generally, the bias current allows control currents to add or subtract from the bias current and control the rotor dynamic forces. If there was no steady state load due to weight, all bias currents would be set equal and there would be no net steady state radial force acting on the journal. However, there is a vertical load due to the rotor weight so the upper magnets were set at a higher bias current than the lower vertical magnets. The side magnets were set at an equal intermediate bias current. The bias currents in each magnet can be varied within a certain range. Thus, the rotor laminations see a significant flux density change due to alternating N and S pole faces as they rotate. In between the pole faces the flux density is nearly zero. In these tests, the bias currents were set at three different levels.

Figure 3 shows a plot of decay times for the rotor when the bearing was configured as a paired pole (NNSSNNSS) for three bias currents: 1.4, 1.6, and 1.8 amps/bearing axis. It is clear from this data that increasing the bias current increases the decay rate, indicating that the losses are higher. The total run-down time from 2,800 rpm to 90 rpm ranged from a high of approximately 558 sec for the 1.4 amp bias current setting to a low of 413 sec for the 1.8 amp bias current setting.

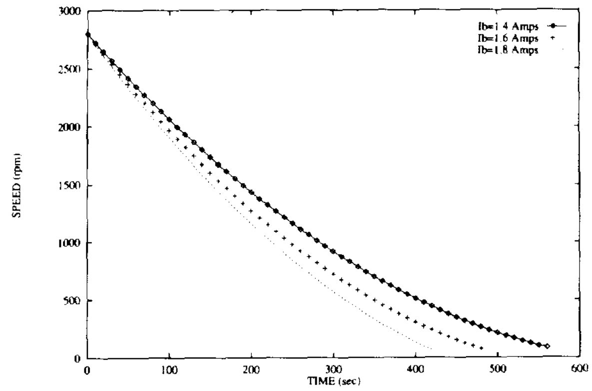

FIGURE 3 - Rotor Deceleration (Speed vs. Time), Paired Pole Configuration (NNSSNNSS)

Pole Configuration Effects

The power losses due to different pole winding patterns were examined. The bearings were configured with both alternating poles (NSNSNSNS) and paired poles (NNSSNNSS). Matsumura and Hataka [6] have measured data which indicated that the two configurations gave nearly the same losses for high speeds, ~ 3000 RPM, but somewhat different losses for lower speeds. The purpose of this test was to compare a different set of magnetic bearing results to their results, as well as to compare calculated losses to measured losses.

Figure 3, from the previous section, showed the run-down times for the bearings in the paired pole (NNSSNNSS) configuration. A second series of tests was run for the same bearing and bias settings with the alternating pole (NSNSNSNS) configuration. A comparison of these results is shown in Figure 4, for a bias current of 1.6 amps/axis. This data is presented in semi-log form to better demonstrate the differences at lower frequencies. It shows that there is essentially no difference between the decay rates in the higher frequency range and some minor, but decreasing, differences in the lower speed range, where the alternating pole configuration decays somewhat faster. The trends in these results agree well with results from Matsumura and Hatake [6].

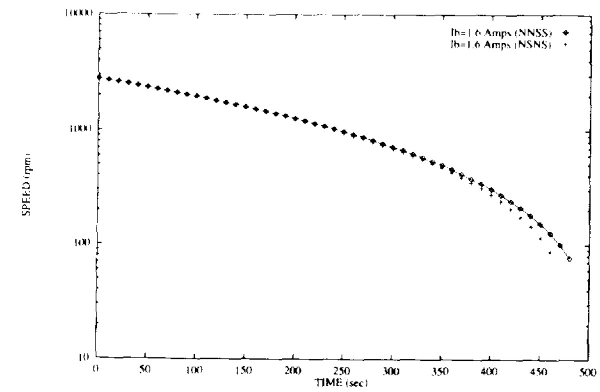

FIGURE 4 - Rotor Deceleration (Speed vs. Time), Paired Pole Configuration (NNSSNNSS) vs. Alternating Pole Configuration (NSNSNSNS)

THEORETICAL LOSS MODEL

Iron losses are the sum of two components: eddy current and both alternating and rotational hysteresis losses, which can be expressed as

$$P_I = P_e + P_{ha} + P_{hr} \quad \textbf{(4)}$$

The individual formulas for these components are given in Appendices A and B.

It is difficult to quantify some of the input parameters to these formulas for the core losses. For example, the effective rotor lamination volume to be employed is not immediately obvious. Therefore it is probably impossible to calculate without a complete finite element analysis, and thus an estimate was made for this study. The numerical value may possibly be different for the hysteresis loss calculations as compared to the eddy current analysis. This is due to differences in the magnetic flux patterns in the rotor. Also the frequency to be employed is subject to much discussion, as is noted in the references. It depends upon the number of flux paths, number of pole faces, etc. The proper frequency might be $2f$, $4f$, or $16f$ (where f is the rotational frequency in Hz) for different loss formulas for the eight pole bearing.

An initial attempt is made in this paper to determine relatively accurate measures of the rotor properties to provide the best loss estimates for magnetic bearings by comparing theoretical formulas to measured data. A windage loss formula, based upon turbulent boundary layer theory, is given in Appendix C. This was employed to calculate the windage loss for the experimental geometry.

COMPARISON OF MEASURED AND CALCULATED LOSSES

The theoretical loss models were used to predict losses based upon several choices of frequencies and effective volumes. Table 1 presents the various calculated results and the measured data for a bias current of 1.6 amps per bearing axis.

All of the calculations in Table 1 were carried out using the equations from Appendices A, B, and C. The predicted power losses are given by

$$P_T = 2\,P_e + 2\,P_{ha} + 2\,P_{hr} + P_W \quad \textbf{(5)}$$

The factors of 2 are due to the two rotor lamination stacks, one for each bearing. P_W is the total windage loss for the rotor.

As noted earlier in this work, factors which are not easy to determine are the effective volume of magnetic material and the appropriate frequency components. For the iron losses, the volume used was based upon inspection of a finite element calculation of the magnetic flux penetration depth into a typical rotor lamination stack. All of the loss calculations were thus made with an estimate of effective volume equal to 23% of the total journal lamination volume. As seen in Table 1, the error is relatively small with this choice of effective volume.

Calculations were also carried out using the assumption of flux penetrating to a depth equal to the width of a leg. This yields an effective volume of 63% of the total volume, resulting in values which are significantly above the measured values, and are not presented.

Iron losses were evaluated with particular frequencies as well. From an examination of the flux-time patterns in each type of loss, it was believed for the eight pole bearing that the eddy current loss was dominated by a $16f$ component, the alternating hysteresis loss was dominated by a $4f$ component, and the rotating hysteresis was dominated by a $2f$ component (where f is the rotational frequency of the rotor). These values were employed in determining the results shown in Table 1.

A breakdown of the components of the calculated losses is shown in Figure 5 for the case when the bias current equals 1.6 amps per bearing axis. For the relatively low speeds of this experiment the dominant mechanisms are initially the rotating and alternating hysteresis losses. At higher speeds the windage and eddy current losses will dominate.

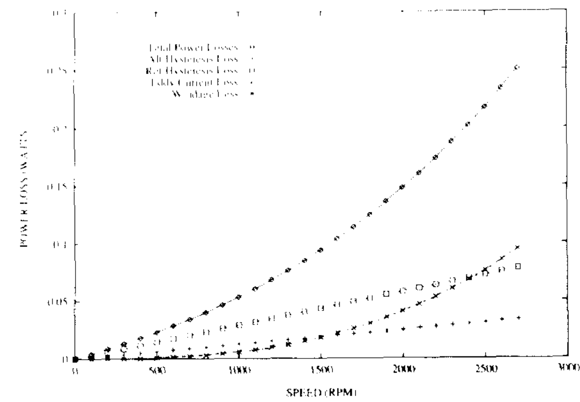

FIGURE 5 - Calculated Power Loss Components vs. Speed, Paired Pole Configuration (NNSSNNSS) with Bias Current at 1.6 Amps

Figures 6, 7, and 8 show a comparison of the measured power loss for the paired pole configuration (NNSSNNSS) as determined from run-down data by the use of Eq. (2) along with a predicted total power loss for bias currents of 1.4, 1.6, and 1.8 amps/bearing axis in the bearings, respectively. For all three bias current settings the calculated losses were within 16% of the measured losses.

As shown in Figures 6-8 there is good overall correlation between the total calculated and measured power losses in the paired pole (NNSSNNSS) magnetic bearings with the calculated losses ranging between 84% and 101% of the measured power losses.

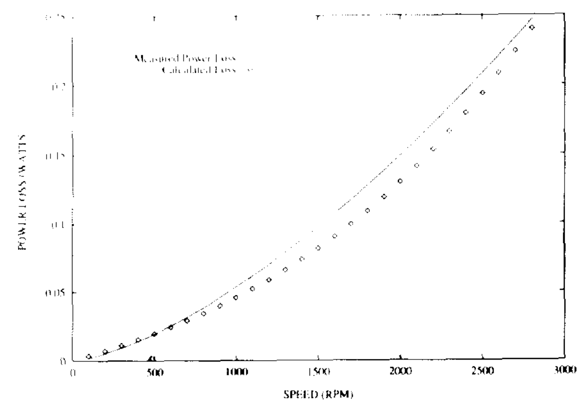

FIGURE 6 - Measured and Calculated Power Loss vs. Speed, Paired Pole Configuration (NNSSNNSS) with Bias Current at 1.4 Amps

Similar calculations were made for the case when the bearings are in the alternating pole (NSNSNSNS) configuration. However, the effective volume for this case is approximately 10% greater than the value used in the paired pole calculations. This is due to the fact that with the alternating pole arrangement, flux emanating from one pole will be attracted towards the opposite pole on the adjacent magnet pair. Therefore, the flux paths bulge towards the adjacent magnet pair and traverse a larger volume, as compared to the paired pole configuration. A comparison of measured and calculated power losses for the alternating pole (NSNSNSNS) configuration for a bias current of 1.6 amps is shown in Table 2. This comparison demonstrates good correlation between the calculated and measured power losses for the alternating pole (NSNSNSNS) configuration.

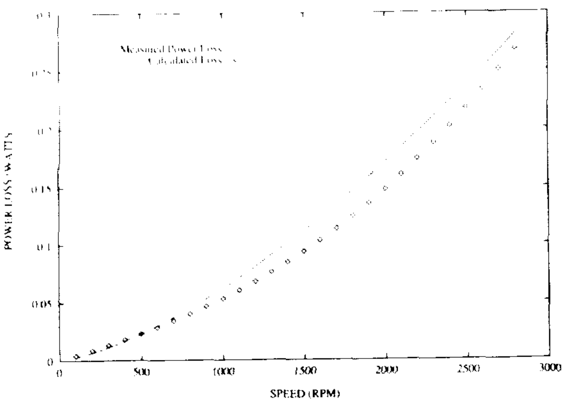

FIGURE 7 - Measured and Calculated Power Loss vs. Speed, Paired Pole Configuration (NNSSNNSS) with Bias Current at 1.6 Amps

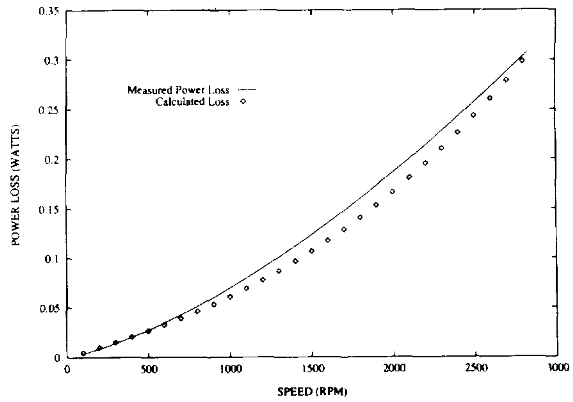

FIGURE 8 - Measured and Calculated Power Loss vs. Speed, Paired Pole Configuration (NNSSNNSS) with Bias Current at 1.8 Amps

LOSS CATEGORY	2500 RPM	1500 RPM	500 RPM
EDDY CURRENT	.0377	.0136	.0015
HYSTERESIS/ ALTERNATING	.0311	.0187	.0062
HYSTERESIS/ ROTATING	.0724	.0434	.0145
WINDAGE	.0763	.0183	.0008
TOTAL CALCULATED	.2175	.0940	.0230
MEASURED	.2376	.1121	.0231
% CALCULATED ERROR	8%	16%	<1%

TABLE 1 - Calculated and Measured Power Loss (watts) vs. Speed in Paired Pole (NNSSNNSS) Magnetic Bearing With Bias Current at 1.6 amps.

LOSS CATEGORY	2500 RPM	1500 RPM	500 RPM
EDDY CURRENT	.0415	.0150	.0017
HYSTERESIS/ ALTERNATING	.0342	.0206	.0068
HYSTERESIS/ ROTATING	.0796	.0477	.0160
WINDAGE	.0763	.0183	.0008
TOTAL CALCULATED	.2316	.1016	.0253
MEASURED	.2395	.1137	.0241
% CALCULATED ERROR	3%	11%	-5%

TABLE 2 - Calculated and Measured Power Loss (watts) vs. Speed in Alternating Pole (NSNSNSNS) Magnetic Bearings with Bias Current at 1.6 amps.

CONCLUSIONS

Power loss measurements were made, based on the deceleration rate of a rotor supported in magnetic bearings, for the variation of bias currents, and pole configurations. Measured losses were compared to calculated losses based on classical theory from transformer and electric motor analyses and showed good agreement, within 16%. Results of this study indicate that the proper application of classical power loss equations may adequately predict losses in magnetic bearings at low speeds. Future work includes high speed measurements in a vacuum chamber, further finite element analyses, and a more analytical method for determining the effective volumes and frequencies to use in the analysis.

ACKNOWLEDGMENTS

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APPENDIX A - EDDY CURRENT LOSSES

A formula for the power loss due to eddy currents has been developed by Golding and others [11-14] by integrating ρI^2 over the volume of the magnetic material, where ρ is the material resistivity and I is the current due to the induced emf from the alternating flux. The loss formula for one rotor is

$$P_e = \frac{10^{-16}\pi^2 d^2 B_{\max}^2 f^2}{6\rho}\left(watts / cm^3\right) \tag{6}$$

Here d is the lamination thickness in cm, B_{max} is the flux density in gauss, f is the frequency in Hz, and ρ is in Ω-cm. This relation assumes that the permeability of the material is constant and that the lamination thickness is less than the penetration depth of the eddy currents (skin effects are neglected).

Eddy currents are dominated by the time rate of change in the magnetic field. Thus it is felt that the number of pole face edges that the rotor laminations pass per sec is the proper value for f, 16 times the rotation frequency for this case.

APPENDIX B - HYSTERESIS LOSSES

As noted in the text of the paper, hysteresis losses can be considered as two types: alternating and rotational. Alternating hysteresis loss occurs in iron when the magnetic field alternates at some frequency, from a maximum positive value to a maximum negative value and the loss is proportional to the enclosed area of the hysteretic loop. Rotational hysteresis occurs when the magnitude of the field is relatively constant but its direction changes with respect to the material in which losses are occurring.

The alternating component of the hysteresis losses in a magnetic material is due to the effects of traversing a complete cycle of the BH curve. This occurs in the rotor of a magnetic bearing as it passes the differently polarized pole faces. The kinetic energy of rotor rotation is converted into heat generation. The loss per cycle for one rotor lamination stack is given by the formula from Steinmetz [16]

$$P_{ha} = 10^{-7}\,\eta f B_{max}^{k} \left(watts/cm^{3} \right) \qquad \textbf{(7)}$$

Here the hysteresis coefficient η has a value of approximately 0.00046 for a good grade of silicon iron and the frequency f is in Hz. Also B_{max} is in gauss and k has the value of approximately 1.6 for flux densities in the range of 1500 and 12,000 gauss. It is felt that the proper frequency to use in this formula is the alternating frequency near the surface of the laminations, $4f$ for this eight pole bearing.

The rotational component of the hysteresis loss occurs when the magnitude of the magnetic field is relatively constant but its direction is changed within the rotating magnetic material. This situation occurs below the rotor surface in a magnetic bearing. This loss due to a rotational variation of the flux is different from the loss due to the alternating flux. Experimental curves have been generated [11,15]. For example, interpolation from a curve for a transformer silicon steel, at a flux density of 2000 gauss used in this work, gives, for one rotor lamination stack

$$P_{hr} = 3.60 x 10^{-5} \left(watts/cm^{3} - Hz \right) \qquad \textbf{(8)}$$

This value is then multiplied by the effective volume and frequency to obtain a power loss in watts. It is felt that the dominant frequency for this relation is based upon the number of magnetic field rotations experienced by the rotor per sec, 2 times the rotation frequency in this case.

APPENDIX C - WINDAGE LOSSES

Ueyama and Fujimoto [5] achieved good correlation to experimental data of windage loss calculations using a turbulent flow model. Therefore, windage losses on the rotor were calculated based upon the drag force on a turbulent boundary layer as developed by Von Karman and presented in Granger

[17]. For a fully turbulent boundary layer on a flat plate of length 2πR and width L, the drag force is

$$D = .072\left(\rho_g 2\pi R U^2 L\right)\left(\frac{\nu}{2\pi R U}\right)^{0.2} \qquad \textbf{(9)}$$

The surface speed, U, is given as Rω, so this expression becomes

$$F_D = 0.144\pi\rho_g R^3 L\left(\omega^2\right)\left(\frac{\nu}{2\pi R^2 \omega}\right)^{0.2} \qquad \textbf{(10)}$$

The windage power loss for each disk is

$$P_{w_1} = F_d U = .144\rho_g L\pi R^4 \omega^3\left(\frac{\nu}{2\pi R^2 \omega}\right)^{0.2} \qquad \textbf{(11)}$$

and the total power loss is the sum of the losses on the two bearing journals and the midspan disk

$$P_w = 2P_{Brgs} + P_{midspan\ disk} \qquad \textbf{(12)}$$