DESIGN OF BEARINGS FOR ROTOR SYSTEMS BASED ON STABILITY

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ABSTRACT

Design of rotor systems incorporating stable behavior is of great importance to manufacturers of high speed centrifugal machinery since destabilizing mechanisms (from bearings, seals, aerodynamic cross–coupling, noncolocation effects from magnetic bearings, etc.) increase with machine efficiency and power density. A new method of designing bearing parameters (stiffness and damping coefficients or coefficients of the controller transfer function) is proposed, based on a numerical search in the parameter space. The feedback control law is based on a decentralized low–order controller structure, and the various design requirements are specified as constraints in the specification and parameter spaces. An algorithm is proposed for solving the problem as a sequence of constrained 'minimax' problems, with more and more constraints becoming active in each subsequent stage. This helps in moving the closed–loop eigenvalues into an acceptable region in the complex plane. The algorithm utilizes the method of feasible directions to solve the nonlinear constrained minimization problem at each stage. This methodology emphasizes the designer's interaction with the algorithm to generate acceptable designs by relaxing various constraints and changing initial guesses interactively. A design–oriented user interface is proposed to facilitate the interaction.

INTRODUCTION

Many rotor systems currently in use tend to operate at supercritical speeds, where multiple bending modes are likely to be excited during operation. Under these circumstances, safe and reliable operation under external excitations and internal loadings caused by sudden changes in machine dynamics such as blade loss is warranted, along with reduction of rotor amplitude due to mass imbalance and prevention of rotor instability [ref. 1]. Destabilizing mechanisms such as aerodynamic cross–couplings in turbomachinery, oil–film forces in journal bearings, seal forces, unsymmetric shafts, internal friction, and noncollocation effects in magnetic bearings are the known causes of instability in rotating machinery.

Design of rotating machinery incorporating stable behavior is one of the primary goals of the designer, and has been the subject of ongoing research. Sudden loss of stability may lead to large and uncontrollable rotor amplitude relative to the casing, causing rubs and severe damage leading to machine failure. As such, a designer would like to know during the design process whether a rotor will run stable during its operation, and the size of the stability threshold for the endangered modes of vibration. Moreover, a knowledge of the effect of known parameter variations on the stability characteristics is deemed useful. Using the available design tools, it is the task of the designer to achieve an acceptable steady–state rotor response and also maintain a specified minimum level of stability.

In recent years the development of magnetic bearings has enabled active control of rotor–bearing systems by suppressing the lateral vibration through feedback control. These bearings can be used either to replace oil–film bearings or in addition to them to provide enhanced vibration control and stability characteristics. Several studies have been made on the control of synchronous vibration and stability of rotating machinery employing these bearings. However, unlike conventional oil–film bearings where there is relatively less flexibility in choice of the bearing parameters (due to their inherent interdependence) and the nature of the feedback control law (which is implicit in the dynamics of the structure–oil film interaction), magnetic bearings offer the designer enormous flexibility to choose the nature of the feedback control and the parameters that govern the law. However, this flexibility cannot be put to effective use due to the limited availability of practical and implementable feedback design methods to exploit the range of available bearing parameters.

The goal of this paper is to bridge this gap and develop a practical and simple feedback control law for design of bearings for rotating machinery. A methodology has been developed based on a simple proportional derivative (PD) controller structure where the feedback control force is essentially a linear function of the "stiffness" and "damping" coefficients (related to the displacement and velocity of the shaft at the bearing location) and the objective is to find acceptable values of these coefficients that satisfy given design specifications and constraints. The formulation is then expanded to design low–order dynamic controllers and find the controller parameters that satisfy the required design criteria.

Research into control system design and optimization has been performed by numerous researchers, and has followed a number of separate paths depending on the final objective and methodology. Eigenvalue placement and eigenstructure assignment has been a particular area of interest, with many differing perspectives and techniques. However, a number of problems beset this method, namely

i) the choice of a desirable eigenstructure is not obvious,
ii) the inability to achieve such a structure by output feedback schemes,
iii) possible poor robustness, and
iv) a lack of a control magnitude penalty.

Controller design based on dynamic response optimization using a quadratic cost index to obtain an optimal control law is another alternative approach. Such optimization is classified under the title of linear quadratic regulator problem or LQR problem. A fair amount of research has been directed at this problem as applied to the control of mechanical structures, and simultaneous structural and control optimization. Recently, this method has been used to design controllers for rotor bearings systems [ref. 2,3]. A controller designed to minimize a weighted sum of the mean square output (system response) and mean square input (control forces), i.e., to minimize the performance index remedies many of the problems faced by eigenstructure assignment. Moreover, the availability of a closed–loop unique solution of the LQR problem via solution of the Ricatti equation is an attractive feature. The main problems of this method are

i) the resulting controller requires access to the full set of plant states, or to a state observer,
ii) the controller is of the same order as the model of the plant, which becomes difficult to design and implement for a structural system (e.g. a multi–level rotor–bearing system),
iii) actual design specifications must be translated into a choice of weighting matrices,
iv) a state–observer based controller may have poor stability margins and be very sensitive to modeling errors and parameter variations, and
v) the scalar quadratic cost function is often inadequate for the representation of certain design objectives.

One approach to overcome some of the above mentioned problems has been the development of suboptimal low–order compensators (static or dynamic fixed–order) by prespecifying the feedback controller structure but retaining the quadratic cost function adopted in LQR design. The design of such compensators, often called output feedback compensators since they attempt to find a control based on the system output directly involves a numerical parameter optimization of the controller structure [ref. 4–7]. A general solution to the output feedback stabilization problem is not available, and consequently several numerical schemes have been developed. However, the main problems involved with low–order output feedback controller design has been

i) existence of multiple local optimal encountered during minimization,
ii) inability to characterize the stability margin properties directly into the problem, and
iii) inadequacy of the scalar quadratic index to represent the different system performance objectives.

DESIGN VIA PARAMETER OPTIMIZATION

The objective of this paper is to develop a design procedure for a decentralized low–order controller for rotor–bearing systems to achieve certain specifications regarding the stability of the closed loop system. A low–order decentralized controller for rotor systems would consist of a bearing with simple dynamics, that is a simple relationship between the shaft displacement and the bearing force. Decentralized or local control is defined as a control mechanism where only local state information is available, and can be regarded as a particular form of constrained output feedback where certain elements of the feedback transfer matrix are constrained to be zero [ref. 8–12]. In the context of a rotor system, the essential features of such a concept are

i) the force at the bearing is dependent only on the measurements at the sensor location

ii) the relationship between the bearing force and the local measurements is a low–order transfer function.

One of the simplest examples of such a scheme is selecting the "stiffness" and "damping" coefficients for a PD controller in order to control rotor response and stability.

The rotor system or the plant is represented as a second–order matrix differential equation, which is a fair approximation of the continuous system for modeling purposes if the number of degrees of freedom chosen to represent the plant is large enough. This can be readily converted to a first–order or state–space form as

$$\begin{aligned} \dot{x} &= A\,x + Bu \\ y &= Cx \end{aligned} \tag{1}$$

For plants with high system order, a model reduction may be deemed necessary to improve the computational efficiency in subsequent numerical optimization, and also since the finite–element model is unable to represent the high frequency dynamic behavior and the bandwidth of the controller is limited by practical considerations. A reduced order model can be constructed by using the modal truncation method, dynamic condensation method, the singular perturbation method and the internal balancing method. The modal truncation method has been effectively used to produce reduced order plant models using both the undamped (free–free) modes and damped modes as basis vectors and is adopted for our research.

The magnetic bearings are represented as a low–order static or dynamic controller, the state–space description of which is given as

$$\begin{aligned} \dot{x}_c &= A_c x_c + B_c y \\ u &= C_c x_c + D_c y \end{aligned} \tag{2}$$

In general, the order of the controller is chosen a priori to the design process. A lower bound on the controller order required to satisfy the design objectives is presently unavailable and needs further theoretical considerations.

The state–space description of the closed loop system including the plant and the controller states is

$$\begin{aligned} \begin{bmatrix} \dot{x} \\ \dot{x}_c \end{bmatrix} &= \begin{bmatrix} A + BD_cC & BC_c \\ B_cC & A_c \end{bmatrix} \begin{bmatrix} x \\ x_c \end{bmatrix} \\ &= \left\{ \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} B & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} D_c & C_c \\ B_c & A_c \end{bmatrix} \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix} \right\} \begin{bmatrix} x \\ x_c \end{bmatrix} \end{aligned} \tag{3}$$

or, $\dot{\tilde{x}} = [A_0 + \tilde{B}\, K\, \tilde{C}]\tilde{x} = A_{CL}\, \tilde{x}$

The problem has been converted to a static output feedback form, and the goal is to find the controller K to satisfy the specified design requirements. For the purpose of optimization, it is prudent to convert (A_c,B_c,C_c,D_c) to a canonical form that minimizes the number of free parameters. In this paper, we have adopted the controller canonical form, where each controller is represented as

$$A_c = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ -\beta_0 & -\beta_1 & -\beta_2 & \dots & -\beta_{n-1} \end{bmatrix} \qquad B_c = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \tag{4}$$

$$C_c = [\alpha_0\ \alpha_1\ \alpha_2 \cdots \alpha_{n-1}] \qquad D_c = \delta_0$$

and the corresponding transfer function is

$$\begin{aligned} G(s) &= C_c(sI - A_c)^{-1} B_c + D_c \\ &= \frac{\alpha_{n-1} s^{n-1} + \alpha_{n-2} s^{n-2} + \dots + \alpha_1 s + \alpha_0}{s^n + \beta_{n-1} s^{n-1} + \beta_{n-2} s^{n-2} + \dots + \beta_1 s + \beta_0} + \delta_0 \end{aligned} \tag{5}$$

The problem of finding the controller matrix K is now translated into finding the vector of design parameters

$$z = [\alpha_{n-1} \cdots \alpha_1\, \alpha_0\, \beta_{n-1} \cdots \beta_1\, \beta_0\, \delta_0]$$

for each controller, which satisfies the the closed–loop design criteria.

The controller canonical form may not be the best suited for representing the transfer function, since it leads to numerical ill–conditioning of the augmented system matrix (A_{CL}). A diagonal or Jordan canonical form with real 2 x 2 block representations of the complex eigenvalue pairs may be a more suitable alternative.

One of the main issues of controller design is the specification of the objective function for minimization. Selection of an appropriate objective function based on stability of the closed–loop system is a primary aspect of this paper. General trend in control system design has been a minimization of the quadratic performance measure (LQR problem) based on weighted state and control cost. However, such an objective function does not give a direct handle on the closed–loop eigenvalue locations and the stability of the system.

To overcome this limitation, we present performance measures and constraints defined in terms of the eigensolution of the closed–loop system. The performance index should ideally measure a weighted sum of the stability margins of the individual eigenvalues. This leads to the formulation of a nonlinear optimization problem and the feedback parameters will be obtained by numerical search over the parameter space.

For rotor systems, stability is often measured in terms of the logarithmic decrement of the damped eigenvalues defined as $\delta_i = -\frac{2\pi p_i}{\omega_i}$ where p_i, ω_i are the real and imaginary parts of the i^{th} damped mode. For most designs of rotor systems, it may suffice to have a minimum log decrement δ_L for modes below a certain lower cut–off frequency ω_L, and another value of log decrement δ_U for modes above another upper cut–off frequency δ_U, and a minimum $\delta_B(\omega_i)$ for modes in between. Also, it may be prescribed that no damped eigenvalue may be within a specified envelope around the operating speed of the machine (typically 10% above and below the running speed). These requirements translate into moving the closed–loop eigenvalues into an *acceptable region* of the complex plane. Attention is focused upon the eigenvalues that are the farthest outside the acceptable region and control effort is spent trying to bring them into the acceptable region [ref. 13]. Mathematically, the objective can be formulated as the minimization of an Acceptability Function A, which is only required to be continuous and differentiable (almost everywhere) and have a value zero in the acceptable region and positive everywhere else.

$$\min A(z)$$

subject to the constraints

$$g_j(z) \leq 0 \qquad (6)$$

where z = vector of design parameters

The Acceptability Function is *not* a performance index. It merely indicates if a solution is acceptable or not, and can be chosen at will by the designer to facilitate the minimization. Controller design will proceed based on a numerical search in the design parameter space, and a reduction in the value of A will occur at every iteration unless a local minimum of A is reached. Thus, unless this occurs first, A will be eventually reduced to zero, yielding an acceptable solution to the problem. Convergence to a local (non–zero) minimum of A would call for restarting the search from a new initial guess, or relax the constraints, or even a change of the acceptability function to get out of the local minimum. Repeated failure to reduce A to zero will indicate the absence of an acceptable solution for the designer specified values.

The design problem requires the satisfaction of a set of specifications. Often, finding an acceptable solution considering all the specifications simultaneously as constraints may become too costly from a computational point of view. This led to the idea of solving the design problem as a sequence of constrained minimization phases [ref. 14]. The order in which these phases occur in the sequence depend on the designer, though the 'harder' or more important constraints are put in as the initial phases. For our case, the optimization proceeds in four phases, with each phase consisting of a constrained (or unconstrained) minimization problem.

Phase I – Satisfaction of stability requirements

$$p_i(z) = \mathrm{Re}\lambda_i(z) \leq 0 \quad \text{where } \lambda_i = \text{ith eigenvalue}$$

The optimization problem is

$$\min \left\{ \sum_{i=1}^{m} \max\,[0, p_i(z)] \right\} \tag{7}$$

subject to no constraints

Phase II – Satisfaction of lower and upper bounds on the design parameters

$$z_{jlow} \leq z_j \leq z_{jup}$$

The optimization problem is

$$\min \left\{ \sum_{j=1}^{l} \max\,[0, (z_{jlow} - z_j), (z_j - z_{jup})] \right\} \tag{8}$$

subject to $p_i \leq 0$ Stability constraints

Phase III – Satisfaction of acceptability region requirements

$$\delta_i(z) \geq \delta_{i_{spec}}$$

The optimization problem is

$$\min \left\{ \sum_{i=1}^{m} \max\,[0, \delta_{i_{spec}} - \delta_i)] \right\} \tag{9}$$

subject to $p_i \leq 0$ Stability constraints

$z_{jlow} \leq z_j \leq z_{jup}$ Design parameter constraints

Phase IV – Satisfaction of operating speed requirements

$$\omega_i \geq \alpha_U \Omega \qquad\qquad \text{where} \quad \omega_i = i^{th} \text{ damped frequency}$$
$$\omega_i \geq \alpha_L \Omega \qquad\qquad \Omega = \text{operating speed}$$
$$\alpha_U, \alpha_L = \text{ratios defining the operating speed envelope.}$$

The optimization problem is

$$\min \left\{ \sum_{i=1}^{m} \min \left[(\alpha_U \Omega - \omega_i), (\omega_i - \alpha_L \Omega) \right] \right\} \tag{10}$$

subject to $p_i \leq 0$ — Stability constraints

$z_{jlow} \leq z_j \leq z_{up}$ — Design parameter constraints

$\delta_i(z) \geq \delta_{i_{spec}}$ — Acceptability region constraints

Constrained numerical search schemes can be used to minimize the objective/acceptability function within each stage. Sequential unconstrained minimization techniques (SUMT) using penalty function, the method of feasible directions, or the generalized reduced gradient method may be used [ref. 15, 16]. We have adopted the method of feasible directions as our numerical search strategy, which starts from a feasible point and proceeds by iteratively searching along feasible directions. If no constraints are violated, different methods like conjugate gradient (Fletcher–Reeves), variable metric (Davidson Fletcher Powell, Broyden Fletcher Goldfarb Shanno) or nongradient (Powell) may be employed within the feasible directions method. The objective and the constraint functions are evaluated at each iteration and within the unidimensional line search for finding a new estimate of the controller parameters, while the gradient information for the objective function and the 'active' or violated constraints is calculated at the end of each iteration.

APPLICATION EXAMPLE

The rotor system chosen to illustrate the design methodology is a uniform symmetric beam 50 inches in length, and 4 inches in diameter. The rotor has been modeled by 11 mass stations, and the order of the system is 44 (each mass station or node is associated with four degrees of freedom, two translational and two rotational). The rotor is supported at two ends by magnetic bearings represented as two low–order decentralized controllers Fig. (1). For this example, the controllers are implemented as second–order strictly proper transfer functions, with the displacement and velocity at the two ends as the outputs and the control forces at the bearing locations as the input. The initial guess for the transfer functions are

$$G_1(s) = \frac{10^{12}s + 10^{14}}{s^2 + 10^6 + 10^{12}} = \frac{10^{12}(s + 100)}{\{s + 10^5(5 + i\ 8.6603)\}\{s + 10^5(5 - i\ 8.6603\}}$$

$$G_2(s) = G_1(s) \tag{11}$$

The initial design parameter vector consists of the numerator and denominator coefficients of the transfer functions.

$$z^0 = [10^{12}\ \ 10^{14}\ \ 10^{6}\ \ 10^{12}\ \vdots\ 10^{12}\ \ 10^{14}\ \ 10^{6}\ \ 10^{12}]^T$$

The pole–zero locations and the bode plot of the transfer function for this initial guess of the design parameters is shown in Fig. (2). The corresponding closed loop eigenvalues are listed in Table 1.

The specifications for the design are laid out as follows

I. Stability of the system must be insured. This implies

$$\text{Re}\ \lambda_i(z) \leq 0 \quad \text{or,} \quad p_i \leq 0$$

II. Upper and lower bounds on the coefficients of the transfer function have been fixed. For this case, the requirements are

$$1 \leq z_j \leq 1 \times 10^{30}$$

III. The acceptability region for the logarithmic decrement δ has been established

A minimum log dec $\delta_L = 1.5$ for modes below a lower cut–off frequency $\omega_L = 800$ rad/sec.
A minimum log dec $\delta_U = 0.01$ for modes above an upper cut–off frequency $\omega_U = 30{,}000$ rad/sec.
A minimum $\delta_B(\omega_i)$ given by a straight line interpolation between δ_L and δ_U for modes with frequency $\omega_L \leq \omega_i \leq \omega_U$.

IV. No avoidance of operating speed envelope has been requested.

The vector of design parameters is subjected to the optimization procedure as described in the previous section. A variable metric method (BFGS) is adopted, and the optimization terminated after 11 iterations, yielding the final design vector.

$$z^* = [1.266 \times 10^{13}\ \ 1.76877 \times 10^{13}\ \ 2.3968 \times 10^{6}\ \ 4.9862 \times 10^{10}\ \vdots\ 1.266 \times 10^{13}\ \ 1.76877 \times 10^{13}\ \ 2.3968 \times 10^{6}\ \ 4.9862 \times 10^{10}]^T$$

Translated into the transfer function form, the resultant controllers are

$$G_1(s) = \frac{1.266 \times 10^{13}\ s + 1.76877 \times 10^{13}}{s^2 + 2.3968 \times 10^{6}\ s + 4.9862 \times 10^{10}} = \frac{1.266 \times 10^{13}\ (s+1.3971)}{(s+2.3758 \times 10^{6})(s+0.0210 \times 10^{6})}$$

$$G_2(s) = G_1(s) \tag{12}$$

It is to be noted that the symmetry is retained though it was not imposed explicitly during the optimization. The pole–zero locations and the bode plot of the transfer function for the resultant decentralized controller are shown in Fig. (3). Even though the structure of the bode

TABLE 1

PHASE III Minimization of Acceptability Region Violation With Stability and Box Constraints

EIGEN- VALUE NO.	REAL PART (1/SEC)	IMAG PART (RAD/S)	LOG DECREMENT	ACCEPTABILITY REQUIREMENT	DIFFERENCE
1	–2.1705	20.6084	.6617	2.0000	1.3383
2	–6.3899	35.4201	1.1335	2.0000	.8665
3	–8.1298	1757.6281	.0291	1.9347	1.9057
4	–7.3796	4747.3243	.0098	1.7310	1.7212
5	–6.3471	9132.3508	.0044	1.4321	1.4278
6	–5.0613	14826.8471	.0021	1.0441	1.0419
7	–3.6631	21760.9279	.0011	.5715	.5704
8	–2.3489	29791.4218	.0005	.0242	.0237
9	–1.2859	38526.9425	.0002	.0100	.0098
10	– .5496	47027.6262	.0001	.0100	.0099
11	– .1330	5 571.5344	.0000	.0100	.0100
12	–499978.2707	866012.8513	3.6275	.0100	.0000
13	–499978.2707	866012.8513	3.6275	.0100	.0000

ACCEPTABILITY MINIMIZATION FN. = 8.92519436

TABLE 2

PHASE III Minimization of Acceptability Region Violation with Stability and Box Constraints

EIGEN- VALUE NO.	REAL PART (1/SEC)	IMAG PART (RAD/S)	LOG DECREMENT	ACCEPTABILITY REQUIREMENT	DIFFERENCE
1	–1.3946	.0000			
2	–1.4032	.0000			
3	–244.3605	789.4629	1.9448	2.0000	.0552
4	–986.1444	3383.5133	1.8313	1.8239	.0000
5	–1991.3497	8470.3625	1.4772	1.4773	.0001
6	–7441.7080	7350.6089	6.3611	1.5536	.0000
7	–7789.6355	7302.0838	6.7027	1.5569	.0000
8	–1370.6901	15194.6896	.5668	1.0190	.4522
9	–667.8307	22222.5295	.1888	.5400	.3512
10	–270.1718	30097.9415	.0564	.0100	.0000
11	–93.3636	38678.1632	.0152	.0100	.0000
12	–26.8946	47084.2560	.0036	.0100	.0064
13	–4.9641	53583.9048	.0006	.0100	.0094
14	–2375944.7970	.0000			
15	–2375986.9465	.0000			

ACCEPTABILITY MINIMIZATION FN. = .87453036 plots remains similar (essentially a PD type structure for the dynamic range of the plant), some loop shaping has occurred leading to an improved design. The corresponding closed–loop eigenvalues are listed in Table 2. The results indicate an appreciable improvement in meeting the specifications over the initial guess, though all the specifications have not been met fully.

A design oriented user interface is extremely important for engineering design optimizations like these, and is currently under development. Ideally, the information at the end of each iteration process should be available graphically to the user, and control must be transferred to the user to enable him/her to change various program variables. Fig. (4) shows the graphical display at the beginning of the design process, for the initial guess of the design vector. The bottom half of the screen shows the acceptability regions for the closed–loop eigenvalues in terms of logarithmic decrement and the real and imaginary parts of the eigenvalues. The top half displays the upper and lower bounds on the design parameters and their values at the initial guess. The corresponding display at the end of the optimization process is shown in Fig. (5), clearly displaying the results of the particular optimization run.

CONCLUSIONS

A method has been presented for the design of low–order decentralized controllers for rotor systems by parameter optimization. The controller has been represented in terms of a control canonical form, to reduce the number of free parameters or design variables. Instead of minimizing a performance index, the method emphasizes satisfying a set of specifications laid down by the designer through a sequence of constrained minimization problems. The proposed methodology has been illustrated by means of an example, and a graphical user interface is currently being developed. Although the method shares the problems of other parameter optimization techniques such as providing a good initial guess and not guaranteeing a solution if one exists, the reduced complexity and flexibility of the controller structure and the ability to handle different design constraints directly make it a very viable alternative to other design methods.

ACKNOWLEDGEMENT

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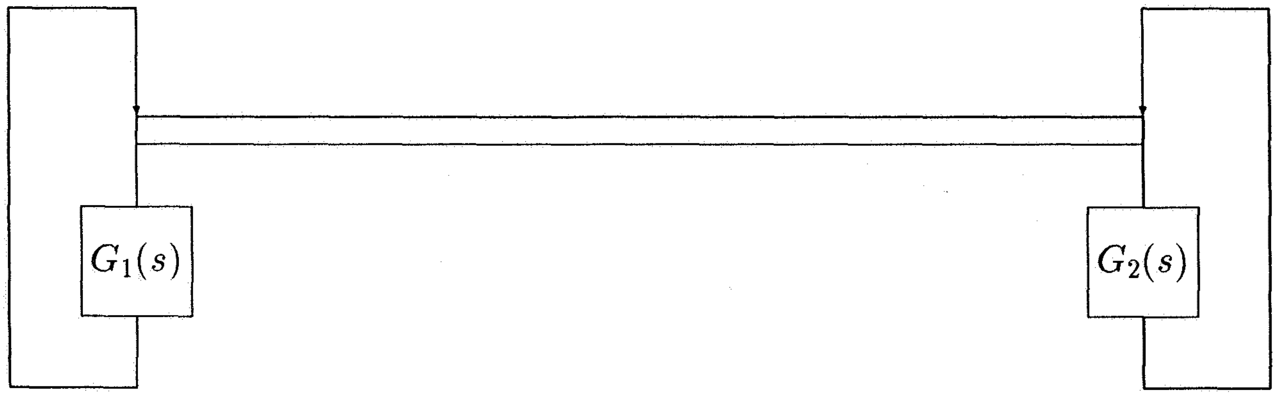

Fig. 1. Rotor supported on two magnetic bearings (low–order decentralized controllers)

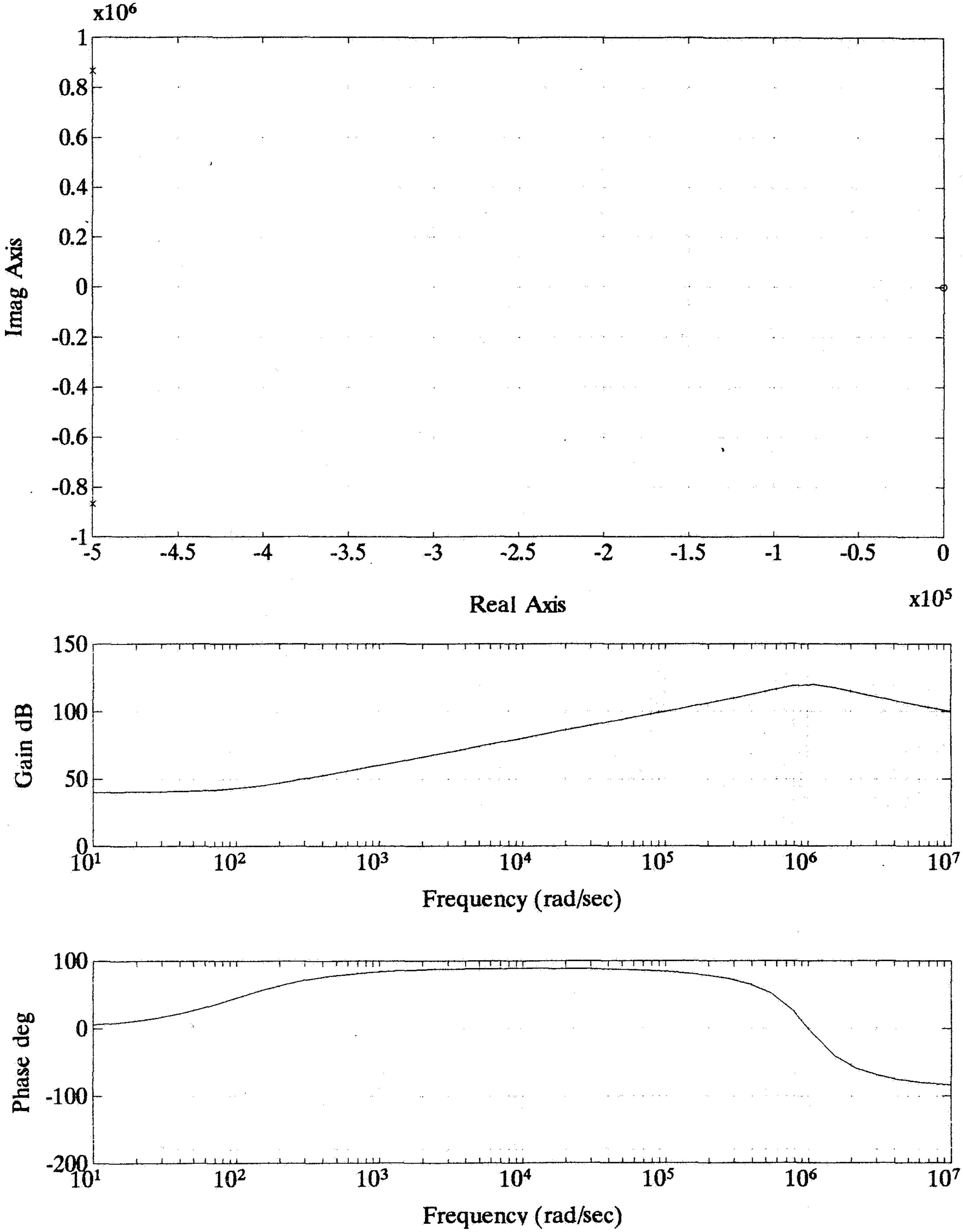

Fig. 2. Pole–zero locations and bode plot of the controller transfer function for initial guess design.

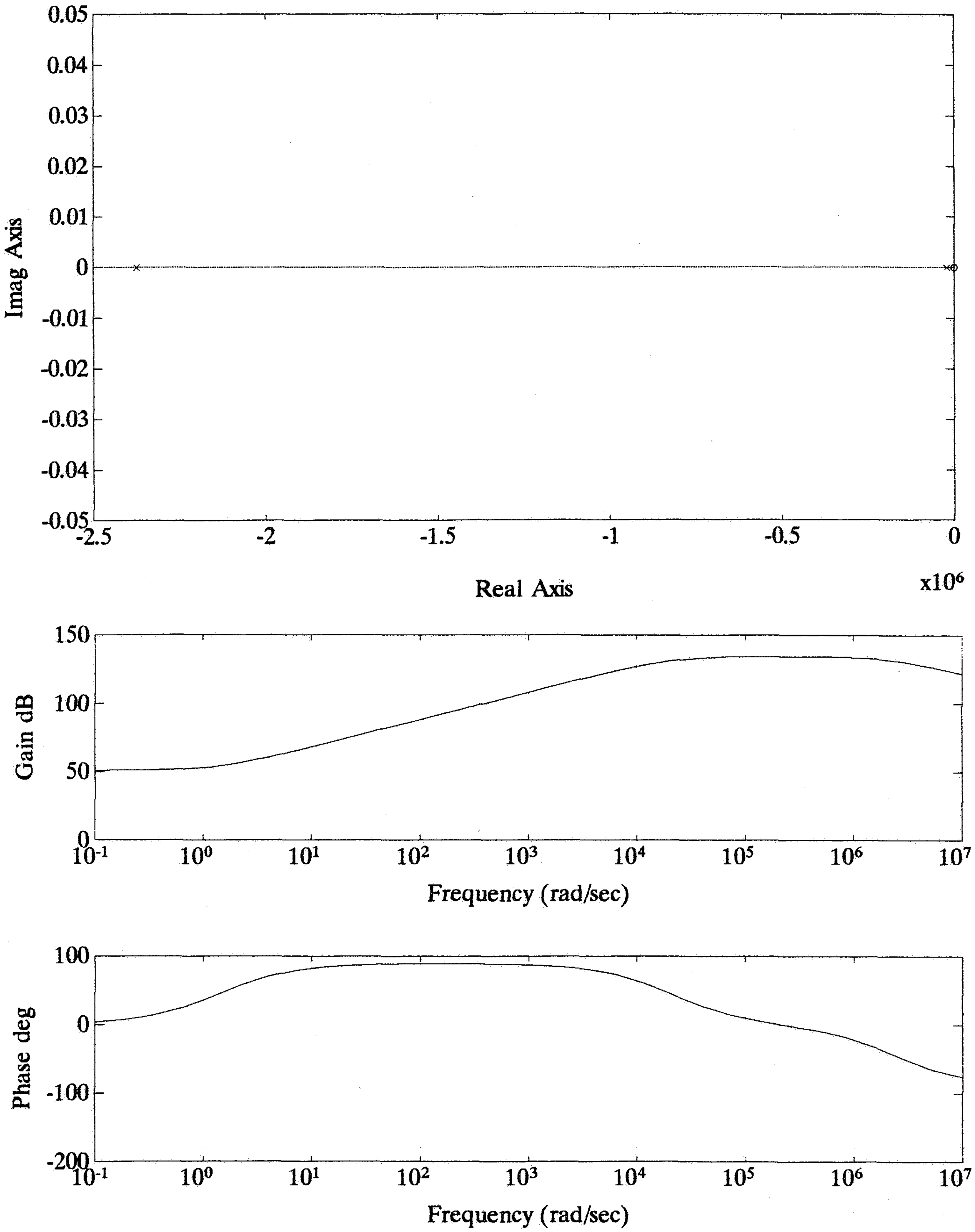

Fig. 3. Pole–zero locations and bode plot of the controller transfer function for optimized design.

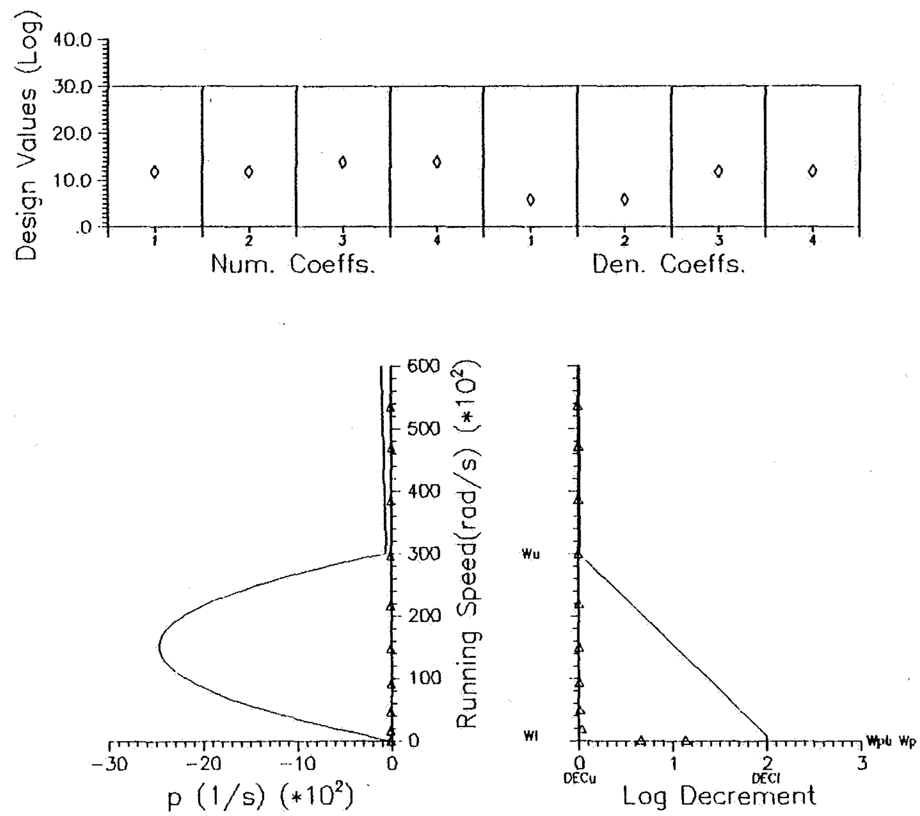

Fig. 4. Graphical display of design specifications, closed–loop eigenvalues and design parameters for initial guess design.

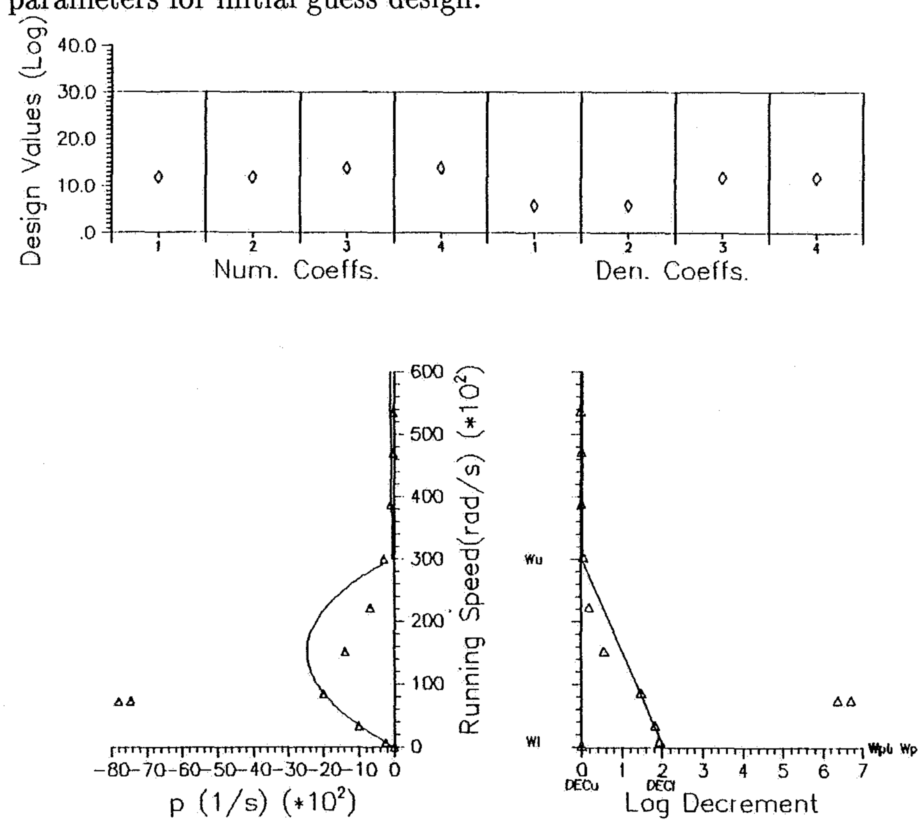

Fig. 5. Graphical display of design specifications, closed–loop eigenvalues and design parameters for optimized design.