VIBRATION DAMPING OF ELASTIC WAVES IN ELECTRICALLY CONDUCTING MEDIA SUBJECTED TO HIGH MAGNETIC FIELDS

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Abstract:

The propagation of vibrational energy in bulk, torsional, and flexural modes, in electrically conducting media can undergo strong attenuation if subjected to high magnetic fields in certain spatial arrangements. The reasons for this are induced Eddy currents which are generated by the volume elements of the media moving transversally to the magnetic field at acoustic velocities. In magnetic fields achievable with superconductors, the non-conservative (dissipative) forces are comparable to the elastic and inertial forces for most metals. Strong dissipation of vibrational energy in the form of heat takes place as a result. A simplified theory is presented based on engineering representations of electrodynamics, attenuation values for representative metals are calculated, and problems encountered in formulating a generalized theory based on electrodynamics of moving media are discussed. General applications as well as applications specific to maglev are discussed.

Introduction:

The interconnection of elastic and electromagnetic phenomena in media capable of supporting both, have with a few exceptions been only curiosities so far. Few applications have been developed so far, mainly because of the relative minuteness of the effects. The special cases of acoustic propagation and vibration in the presence of steady state magnetic fields, magneto-acoustics, however, did receive some attention and have been investigated on a few occasions.

In 1968, LILLEY and CARMICHAEL[1] at the University of Western Ontario, Canada, conducted laboratory experiments with standing elastic waves in a metal bar subjected to a magnetic field. The investigators reported slight damping effects which depended on the magnitude as well as the gradient of the field.

Later, in 1985, LEE[2] at the Stanford University Department of Mechanical Engineering analyzed electromagnetic damping together with thermoelastic damping of structures. Strong dependence on structural and geometrical configurations was found, as one would expect.

The major shortcoming in exploiting magneto-acoustic phenomena for practical applications was due to the limit of magnetic field strength which could be produced by practical means. This was exacerbated by the fact that the effects have a second order dependence on field strength, as will be shown later. There is always the possibility, of course, of using conventional (low temperature) superconducting technology to generate the required fields. The complexity of the cryogenic support systems and the cost of such systems could be justified only in the rarest of circumstances.

Recent advances in the technology of high temperature superconductors, will render magneto-acoustic effects much more relevant in the future. This was first recognized by HORWATH[3] in a paper presented to the 119th meeting of the Acoustical Society in May 1990. Magnetic fields of several Tesla and possibly tens of Tesla will soon be achievable with such high temperature superconducting materials at a low cryogenic overhead. Such materials have the further advantage of much higher critical fields and currents. Their only limitation at present is relatively low current densities.

In the high magnetic fields achievable with superconductors the non-conservative or dissipative forces in electrically conducting elastic media will become comparable to the elastic and inertial forces. As a result of this it will be possible to achieve significant direct dissipation of acoustic energy in metals, for instance, for both bulk waves and flexural waves. This will provide means for damping on a scale difficult to achieve before, and thus open new regimes for applications.

First order estimates of the damping effects will be presented in the following. The attenuation lengths of longitudinal waves in bulk materials subjected to high magnetic fields will be calculated. This will be followed by a similar determination of the transversal impedance component in electrically conducting plates, which are subjected to high magnetic fields. Only very simple special cases will be discussed, aimed mainly at introducing the concept.

Propagation of Longitudinal Waves in Bulk Material:

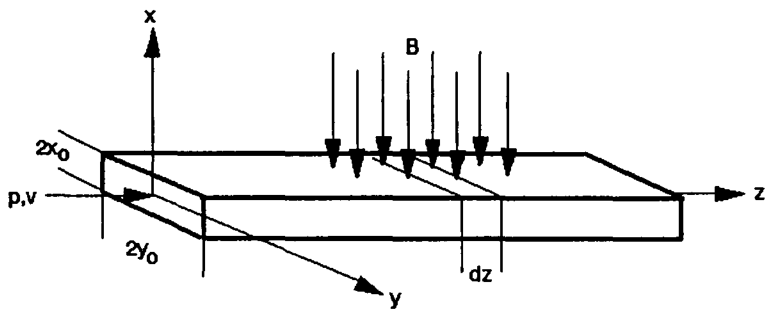

Figure 1

Figure 1 depicts a section of an electrically conducting slab, which is penetrated by a homogeneous magnetic field B, at normal incidence. A pressure disturbance, periodic or aperiodic, is applied at one end and propagates along the z-axis, through the magnetic field region, causing motion of the slab material at acoustic velocities. This produces a voltage in the y-direction in the volume element $4x_0 y_0 dz$

$$E = -d\Phi/dt = -2y_0 Bv \tag{1}$$

which in turn gives rise to a current in the same volume.

$$I = E/R = -2\sigma x_0 Bvdz \tag{2}$$

It is tacitly assumed here, for simplicity, that the current path is closed through the fringe regions outside the magnetic field with essentially zero resistance. The current flowing in this volume element of the slab experiences a Lorentz Force, which is counteracting the driving disturbance.

$$dF = 2IBy_0 = -4\sigma x_0 y_0 vB^2 dz \tag{3}$$

From this the pressure is obtained as

$$dp = dF/dA = dF/4x_o y_o = -\sigma v B^2 dz \quad (4)$$

For elastic wave propagation in bulk material the pressure is related to the velocity by

$$v = p/(\delta E)^{1/2} \quad (5)$$

where δ is the density and E the bulk modulus of the material. Using this relationship the differential equation is obtained from equation (4)

$$dp/p = -\sigma B^2 dz/\delta c \quad (6)$$

The attenuation length thus follows as

$$z_{ab} = (\delta E)^{1/2}/\sigma B^2 \quad (7)$$

The attenuation is a function of the usual material parameters, such as density, sound velocity, and electrical conductivity, as expected, and has an inverse square dependence on the magnetic field.

Table 1 below presents the attenuation length calculated by the above methodology for different metals subjected to a magnetic field of 10 Tesla.

ATTENUATION LENGTH FOR BULK WAVES
IN VARIOUS METALS

(Magnetic Field = 10 Tesla)

Material	Electrical Conductivity $[Ohm^{-1} m^{-1}]$	Density $[kgm^{3}]$	Bulk Modulus $[Nm^{-2}]$	Attenuation Length [m]
Aluminum	3.77×10^{7}	2690	7.18×10^{10}	3.68×10^{-3}
Copper	5.85×10^{7}	8930	1.23×10^{11}	5.67×10^{-3}
Steel	6.21×10^{6}	7650	2.12×10^{11}	6.48×10^{-2}

Table 1

Propagation of Torsional (Shear) Waves:

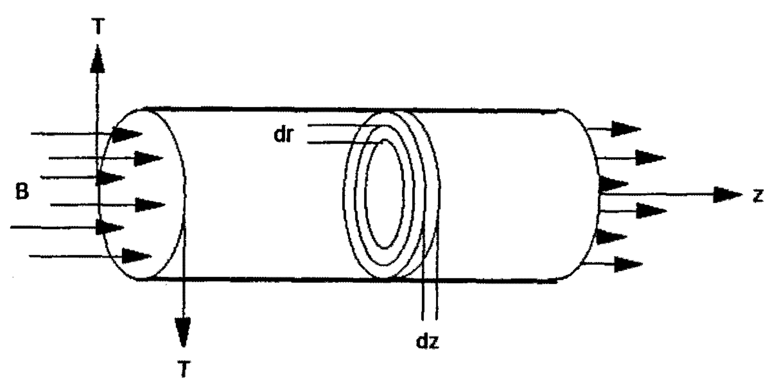

Figure 2

The calculations proceed in a similar fashion as in the above case. Figure 2 shows a section of an electrically conducting rod which is penetrated by an axial magnetic field B. A torque disturbance is applied to the rod at one end and propagates along the longitudinal axis, producing a radial voltage in the annular volume element of the rod section $2\pi rdrdz$

$$E = -d\Phi/dt = -Brwdr \tag{8}$$

The current in this volume element is

$$I = E/R = -2\pi\sigma wBdz \tag{9}$$

The Lorentz force acting on this current element generates a counter torque, which per unit cross sectional area is

$$dT = -\sigma wB^2 dz \tag{10}$$

For torsional waves propagation this torque per unit area is related to the angular velocity by

$$w = T/(\delta G)^{1/2} \tag{11}$$

where G is the shear modulus of the material. Using this relationship together with equation (10) the differential equation follows

$$dT/T = -\sigma B^2 dz/(\delta G)^{1/2} \tag{12}$$

which yields the attenuation length for torsional waves

$$z_{at} = (\delta G)^{1/2}/\sigma B^2 \tag{13}$$

Again, as expected, the attenuation is a function of the same material parameters as above, and is inversely proportional to the square of the magnetic field.

Table 2 lists attenuation lengths for torsional waves in different metals in a field of 10 Tesla.

ATTENUATION LENGTH FOR TORSIONAL WAVES IN VARIOUS METALS

(Magnetic Field = 10 Tesla)

Material	Electrical Conductivity $[Ohm^{-1}m^{-1}]$	Density $[kgm^{3}]$	Shear Modulus $[Nm^{-2}]$	Attenuation Length [m]
Aluminum	3.77×10^{7}	2690	2.69×10^{10}	2.25×10^{-3}
Copper	5.85×10^{7}	8930	4.55×10^{10}	3.44×10^{-3}
Steel	6.21×10^{6}	7650	7.95×10^{11}	3.97×10^{-2}

Table 2

Transversal Damping Impedance in Plates:

Flexural waves are more difficult to treat in this context because of their highly dispersive nature, which precludes the use of a simple relationship between pressure and velocity. A somewhat different approach is taken therefore, calculating a damping impedance rather than an attenuation length.

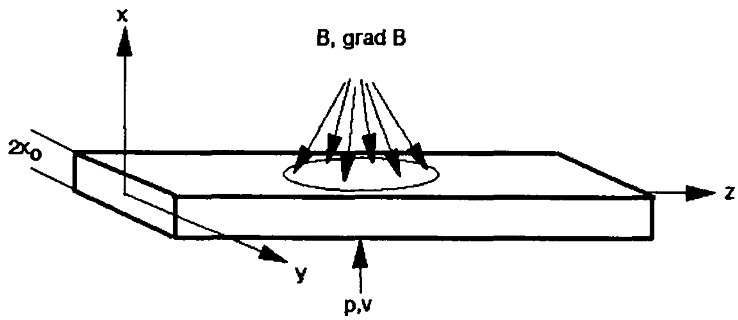

Figure 3

Figure 3 depicts a section of an electrically conducting plate is now subjected to an inhomogeneous magnetic field B, with a strong gradient grad B. Both field and gradient are perpendicular to the plate. It is further assumed that the field region is contained within a circle of the radius Ra. A pressure disturbance is applied from the opposite side of the plate, causing the plate material to move in the direction of the field gradient, with an acoustic velocity typical for a flexural vibration mode. Circular currents are induced in annular elements of the material with the volume $4\pi r x_o dr$ as a result of the changing magnetic flux caused by this motion.

The voltage induced is obtained from Faraday's Law of Induction as

$$E = -d\Phi/dt = -\iint dB/dx \; dx/dt \; dA \tag{14}$$

or

$$E = \iint v_x \; grad_x B \; dA \tag{15}$$

The current element is obtained from Ohm's Law as

$$dI = E\sigma x_o\, dr/\pi r \quad (16)$$

performing the integration in (15) and substituting the result into this the current element becomes

$$dI = \sigma r x_o\, dr v_x\, grad_x B \quad (17)$$

which allows determination of the Lorentz force element

$$dF = 2\pi r dI B_{yz} \quad (18)$$

and by substituting (17) into (18) and integrating, the total Lorentz force acting on the circular region is obtained as

$$F = \int 2\sigma\pi x_o\, r^2\, v_x\, B_{yz}\, grad_x B\ dr \quad (19)$$

which becomes

$$F = 2/3\ \sigma\pi x_o\, R_a{}^3\, v_x\, B_{yz}\, grad_x B \quad (20)$$

Finally, considering that the pressure p = F/A, and the transversal damping impedance z_t = p/v, the latter becomes

$$z_t = 2/3\ \sigma x_o\, R_a\, B_{yz}\, grad_x B \quad (21)$$

This transversal damping impedance is again a function of electrical conductivity, thickness of the plate, radius of the field region, and is proportional to the product of the magnetic field and its gradient.

Table 3 below presents the transversal damping impedances for the same materials. Assumed is a thickness of the plate of one centimeter, an active area of one square meter, and a magnetic field of 10 Tesla with a gradient of 10 Tesla per meter. For these conditions the damping impedances for metals are very high.

TRANSVERSAL IMPEDANCE
FOR VARIOUS METALS

(Area = $1m^2$, Thickness = 10^{-2} m,
Field = 10 T, Gradient = $10Tm^{-1}$)

Material	Electrical Conductivity $[Ohm^{-1} m^{-1}]$	Transversal Impedance $[Nm^{-3} sec]$
Aluminum	3.77×10^7	7.09×10^6
Copper	5.85×10^7	1.10×10^7
Steel	6.21×10^6	1.17×10^6

Table 3

Attempts to Formulate a Generalized Theory:

The above considerations were based on very elementary electromagnetic theory, and furthermore made use of engineering representations applied to very simple, special cases. It would be desirable to have a more rigorous and generalized theory, describing all possible interactions between electromagnetic fields and sound fields in electrically conducting media. In the attempt to construct such a theory it became immediately apparent, however, that it will be very complicated at best, if not entirely impossible to develop. The reasons for this are the following:

(1) The theory would have to be based on the Electrodynamics of Moving Media, or Relativistic Electrodynamics. The Maxwell

Equations, which are the cornerstone of most electrodynamic phenomena, do not apply in this case. They have no provision for moving media, and are hence not compatible with the description of the Lorentz Force, which is, the result of a moving conductor. Relativistic Electrodynamics have much more complicated formulations than the Maxwell Equations.

(2) Electrodynamics, relativistic or not, in general deals with time variant, non-stationary phenomena, and therefore by necessity is represented in time domain. Acoustic and elastic phenomena, on the other hand, because of their stationary nature are better described in the spectral domain. This lack of very basic compatibility is another shortcoming.

For the above reasons it is believed that investigations of even slightly more complicated geometries than discussed above will have to resort to numerical methods, based on engineering formulations of electromagnetic principles. Such approaches are useful and effective, but in the view of the author do not provide the same degree of insight in a general sense, and are also not as elegant from a physicists point of view.

General Applications:

Very interesting application possibilities will become practical when high magnetic fields generated with high temperature superconductors become routinely available. A few of such possibilities are outlined in the following:

(1) Vibration Isolation: The most obvious is, of course, vibration isolation. Because of the short attenuation length in metals, it will be possible to dissipate energy directly in vibration mounts such as springs. This will minimize both transmitted vibrations to the foundation, as well as the buildup of vibrational amplitudes at the source. The vibrational energy will, of course, be dissipated as heat in the regions subjected to the magnetic field, and provisions for heat removal will be required.

(2) Selective Mode Damping: In the most general case, the dissipation of acoustic energy will be anisotropic, depending on the relative orientations of the magnetic and acoustic vector variables. This characteristic can be exploited for the selective damping of vibration and propagation modes, and also for the suppression or enhancement of mode conversions.

(3) Anechoic Structures: The impedance figures for plates indicate that it may be possible to match the characteristic impedance of water. Under such conditions, plates and other structures submerged in water may be rendered anechoic.

(4) Suppressed Radiation: The same is true for vibrating plates radiating acoustic energy. It would be possible to change the impedance of such plates either locally, or globally, to suppress

selected radiating modes, or possibly even vibration in its entirety.

(5) Structural Hardening: Finally, it is foreseen that the response of plates to impinging transient pressure waves, such as shock waves, for example, could be altered significantly, providing much greater stiffness and resistance to such phenomena.

Specific Applications to Magnetic Suspension:

The utilization of the phenomena discussed to various magnetic suspension applications can result in considerable synergies particularly when very high magnetic fields are involved. Substantial damping forces can be obtained for various magnetic suspension applications if design configurations are chosen which enhance the discussed effects.

One increasingly important system concept based on magnetic suspension is the magnetically levitated train, or maglev. Maglevs are subjected to vibrations induced by irregularities of the tracks, by fluctuations of the magnetic propulsion and levitation forces, and by various transient aerodynamic phenomena such as wind gusts, entry and exit of tunnels, and other passing maglev trains. The vibrations induced by these various sources will have to be attenuated in the interest of ride quality. Attenuation has to be accomplished mainly by the suspension, may it be the primary suspension (levitation), the secondary, or a combination of the two. Electromagnetic vibration damping is a natural choice for maglevs because of the high magnetic fields used for levitation.

The values for damping impedances of plates presented in Table 2 are comparable to the stiffnesses of electrodynamic (EDS) maglev suspensions. This type of maglev system also utilizes superconducting magnets for primary levitation. The magnetic fields are of sufficient magnitude to allow the construction of simple and efficient electrodynamic vibration isolators using the same superconducting magnets which provide the primary levitation.

Conclusions:

It is envisioned that this area of magneto-acoustic interactions will play an important role in vibration damping in the future, leading to many interesting and important applications, including the ones outlined above. Practical means for generating the necessary magnetic fields are being developed at a rapid pace, since the discovery of high temperature superconducting materials. The fields of acoustics and structural vibrations will undoubtedly benefit from these developments, which are driven by a vast commercial potential encompassing all fields of electrical technologies.

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