N92-27744

COMPUTER AIDED DESIGN OF DIGITAL CONTROLLER FOR RADIAL ACTIVE MAGNETIC BEARINGS *

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Abstract

A five degrees of freedom Active Magnetic Bearings (AMB) system is developed in Tsinghua University, which is controlled by digital controllers.

The model of the radial AMB system is linearized and the state equation is derived. Based on the state variables feedback theory, digital controllers are designed. The performance of the controllers are evaluated according to experimental results.

The Computer Aided Design (CAD) method is used to design controllers for magnetic bearings. The controllers are implemented with a digital signal processing (DSP) system. The control algorithms are realized with real-time programs. It is very easy to change the controller by changing or modifying the programs.

In order to identify the dynamic parameters of the controlled magnetic bearings system, a special experiment was carried out. Also, the on-line Recursive Least Squares (RLS) parameter identification method is studied. It can be realized with the digital controllers. On-line parameters identification is essential for the realization of adaptive controller.

Introduction

A magnetic bearings system with electromagnetic attractive force is an inherently open-loop unstable system. The uncontrolled magnetic force provides a negative stiffness to the magnetic bearings system, i.e., when the gap between rotor and bearing is reduced, the magnetic force will be increased. In this case, compensation to such a system is necessary. The AMB controllers can eliminate the negative stiffness and provide positive stiffness and damping to the AMB system. Therefore, the performance of the controllers is a decisive factor to the performance of the AMB system. Many papers have described design of the control system. [1, 2, 3, 4]

In order to design a practical AMB controller, which is easy to implement and has relatively good performance, some control algorithms are studied on a computer system.

Design of digital controllers can be divided into several steps. Firstly, the discrete model and state equation of the system are obtained according to the original system. Secondly, a controller is designed with the CAD method according to the selected control algorithm. Thirdly, the designed controller can be simulated with computer before implemented as a real-time control program. Lastly, the coefficients of the control programs are tuned to reduce the effects of deviations of the system model. And, if necessary, the system model is modified and the controller is redesigned.

In our magnetic bearings system, the rotor has five degrees of freedom need to control, one axial degree of freedom and four radial degrees of freedom. Generally, the axial AMB system can be considered as a single degree of freedom system, i.e. it is uncoupled with the radial AMB system. In our early work, digital and analog controllers for axial AMB had been studied. [5]

The axial AMB controllers are Single-Input

*Supported by the National Natural Science Foundation of China

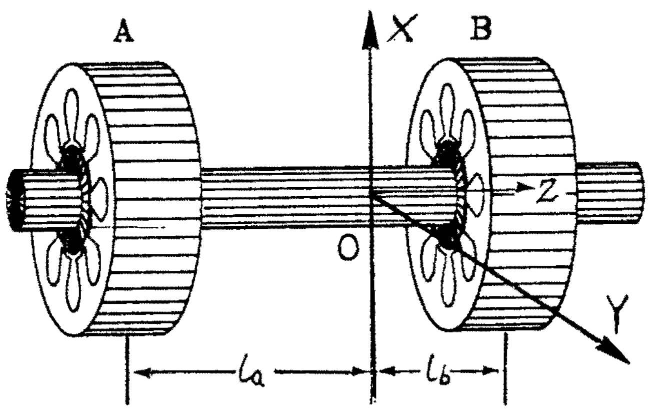

Figure 1: Structure of Radial Magnetic Bearings

Single-Output (SISO) controllers, which can be analyzed and designed with the classical control theory. However, it is difficult to design the Multi-Input Multi-Output (MIMO) controllers, as used for a radial AMB system, with the classical control theory.

In this paper, we focus our study on the radial AMB controllers with the state variables feedback theory.

Radial Bearings System

Figure 1 shows the schematic structure of radial magnetic bearings used in our experiment. The horizontal rotor is supported by two radial magnetic bearings, A and B. Each bearing controls two motion directions of rotor, x and y, in the fixed Cartesian coordinates. The origin of the coordinates, O is at the mass center of the rotor. The electromagnetic force of each direction is produced by two opposite magnets. The magnets in y direction provide an additional force to support the weight of the rotor.

In actual magnetic bearings, the relationship between magnetic force and control current and rotor position are nonlinear. However, when the rotor position is near the reference position, the relationship can be linearized as:

$$f = k_p x + k_i i \tag{1}$$

where,

k_p—position stiffness
k_i—current stiffness
x—displacement from reference position
i—control current

The total current in magnets can be divided into control current and bias current. The bias current makes the magnets work in the linear range of magnetizing curve. [6]

Taking state variables vector as,

$$\mathbf{X} = (x_a, x_b, y_a, y_b, \dot{x}_a, \dot{x}_b, \dot{y}_a, \dot{y}_b)^T$$

control variables vector as,

$$\mathbf{U} = (i_{xa}, i_{xb}, i_{ya}, i_{yb})^T$$

The state equation of the magnetic bearings system is,

$$\dot{\mathbf{X}} = \mathbf{AX} + \mathbf{BU}$$

where, state matrix,

$$\mathbf{A} = \begin{pmatrix} O & O & I & O \\ O & O & O & I \\ L & O & O & -M \\ O & L & M & O \end{pmatrix}$$

control matrix,

$$\mathbf{B} = \frac{k_i}{k_p} \begin{pmatrix} O & O \\ O & O \\ L & O \\ O & L \end{pmatrix}$$

The 2×2 submatrixes O, I are zero and identity matrixes, respectively, and,

$$\mathbf{L} = \frac{k_p}{m} \begin{pmatrix} 1 + \frac{m l_a^2}{J_x} & 1 - \frac{m l_a l_b}{J_x} \\ 1 - \frac{m l_a l_b}{J_x} & 1 + \frac{m l_b^2}{J_x} \end{pmatrix}$$

$$\mathbf{M} = \frac{J_z \Omega}{J_x (l_a + l_b)} \begin{pmatrix} l_a & -l_a \\ -l_b & l_b \end{pmatrix}$$

Generally, $J_z/J_x \ll 1$, when the rotor rotates at the low speed, submatrix **M** can be considered as zero matrix, i.e., the gyroscopic effects can be omitted.

The rotor positions in each direction are measured by four eddy-current displacement sensors mounted near radial bearings. In the linear range of the sensors, the outputs of sensors, which are produced by analog circuits, are DC voltages proportional to the displacement of the rotor in each direction. When the rotor is at reference positions in bearings, all the outputs of the sensors are zero. Therefore, the output equation of the magnetic bearings system is,

$$\mathbf{Y} = \mathbf{CX}$$

where,

$$\mathbf{Y} = (x_a, x_b, y_a, y_b)^T$$

output matrix,

$$\mathbf{C} = \begin{pmatrix} I & O & O & O \\ O & I & O & O \end{pmatrix}$$

Design of Control System

The CAD method is used to design digital optimal state variables feedback control system. [7]

For a continuous time system, the performance index to be minimized is,

$$J = \int_0^\infty [\mathbf{X}^T(t)\mathbf{Q}\mathbf{X}(t) + \mathbf{U}^T(t)\mathbf{R}\mathbf{U}(t)]dt$$

where, **Q** and **R** are the state weighting matrix and control weighting matrix, respectively.

Given sampling period T, the state equation can be discretized as,

$$\mathbf{X}(k) = \mathbf{F}\mathbf{X}(k-1) + \mathbf{G}\mathbf{U}(k-1)$$

where, **F** and **G** are the discrete state matrix and control matrix, respectively.

And the performance index can be discretized as,

$$J = \sum_{k=0}^{\infty}[\mathbf{X}^T(k)\mathbf{Q_1}\mathbf{X}(k) + 2\mathbf{X}^T(k)\mathbf{Q_3}\mathbf{U}(k) + \mathbf{U}^T(k)\mathbf{Q_2}\mathbf{U}(k)]$$

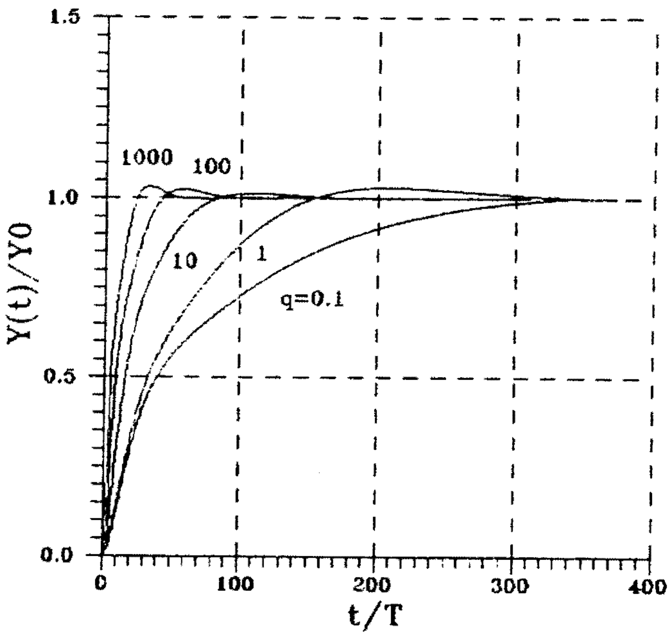

Figure 2: Simulation Results of Step Response

where, $\mathbf{Q_1}, \mathbf{Q_2}$ and $\mathbf{Q_3}$ are the discrete state weighting matrix, control weighting matrix and cross weighting matrix, respectively.

Given the continuous state equation, weighting matrixes **Q** and **R**, and sampling period T, a computer program is available to compute the optimal state variables feedback gain matrix **K** for digital controller. The choices of weighting matrixes **Q** and **R** have decisive effects to the dynamics of a magnetic bearings system.

We choose the matrixes,

$$\mathbf{Q} = diag(q, q, q, q, 0, 0, 0, 0),$$
$$\mathbf{R} = \mathbf{I}$$

where, $q > 0$ and **I** is a 4×4 identity matrix. These choices mean that only the squares of displacements and control currents are weighted in the performance index J.

The simulation of step response of the magnetic bearings system controlled by controllers with different values of q are shown in Figure 2.

The simulation results shows that a big q corresponds fast response of the system.

Identification of System Parameters

The optimal state variables feedback control theory is very suitable for designing AMB control

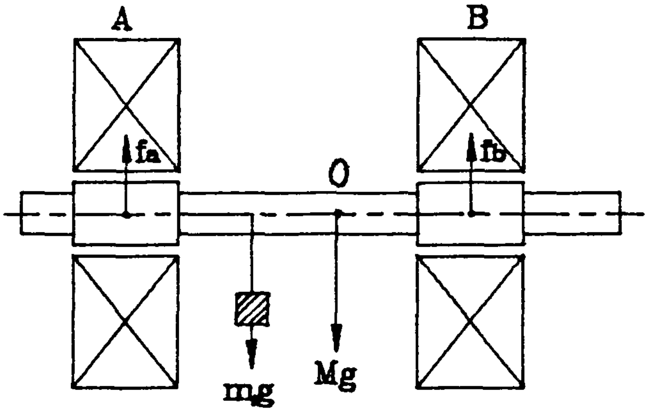

Figure 3: Parameters Identification Experimental System

system from the given system model. However, if the parameters of the system are not accurate enough, the designed control system will not be optimal or even make controlled system unstable.

Because the AMB system is open-loop unstable and nonlinear, some important parameters of system, such as position stiffness k_p and current stiffness k_i, are strongly dependent on operating point of AMB system. It is difficult to measure the system parameters accurately from an open-loop system. However, when a simple controller, like PD controller, can suspend the rotor stably, the parameters can be measured from the close-loop system.

Figure 3 shows a schematic experimental system. The horizontal rotor, with rotation speed $\Omega = 0$, is suspended stably. Setting this working state as the operating point of the magnetic bearings system, the reference position of rotor in bearings, the currents in bearing coils can be measured and the bearing forces can also be decided according to the structure of magnetic bearings and the rotor.

When a small weight m_1 is loaded to the rotor, it will have small displacements from the reference position. The displacements of rotor and the currents in bearings can be measured directly. The changes of bearing forces can also be decided. Changing the weight to $m_2 (m_2 \neq m_1)$, another set of data can be obtained in the same way. According to the linear assumption, bearing forces are expressed as Eq. (1). Therefore, a set of equations is obtained. Consequently, the k_p and k_i can be obtained by solving these equations.

Another close-loop measurement method, called Recursive Least Squares (RLS) parameters identification method, is also studied. This method can be realized on-line with a digital controller and the difference equations of an AMB system can be obtained directly.

Consider a single degree of freedom rotor, the motion equation,

$$m\ddot{x} = k_p x + k_i i$$

can be transformed to a difference equation,

$$x(k) = a_1 x(k-1) + a_2 x(k-2) + b_1 i(k-d) + \eta(k)$$

where,

$$a_1 = 2 + \frac{k_p T^2}{m} \tag{2}$$

$$b_1 = \frac{k_i T^2}{m} \tag{3}$$

$a_2 = -1$
$d = 2$ — delay number of sampling period
$\eta(k)$ — random noise at sampling time kT
T — sampling period

The predicted system output is,

$$\begin{aligned}\hat{x}(k) &= \alpha_1 x(k-1) - x(k-2) + \beta_1 i(k-d) + \eta(k \\ &= -x(k-2) + \mathbf{V}^T(k-1)\theta(k-1) + \eta(k)\end{aligned}$$

where, the input and output vector,

$$\mathbf{V}(k-1) = [x(k-1), i(k-d)]^T$$

predicted parameters vector,

$$\theta(k-1) = [\alpha_1, \beta_1]^T$$

From the control inputs i and system outputs x, at kth sampling period, the predicted system parameters are,

$$\theta(k) = \theta(k-1) + \mathbf{N}(k)[x(k) - \hat{x}(k)]$$

where, $x(k)$ is the rotor displacement measured at kTth time.

And the matrix

$$\mathbf{N}(k) = \frac{\mathbf{P}(k-1)\mathbf{V}(k-1)}{1 + \mathbf{V}^T(k-1)\mathbf{P}(k-1)\mathbf{V}(k-1)}$$

the positive definite covariance matrix

$$\mathbf{P}[k] = \mathbf{I} - \mathbf{N}[k]\mathbf{V}^T[k-1]\mathbf{P}[k-1]$$

The initial iteration values are,

$$\mathbf{P}(0) = \alpha\mathbf{I}, \quad \alpha = 5 \sim 10^4$$

$$\theta(0) = 0$$

The simulation results of RLS parameters identification with a PD controller are shown in Figure 4. The predicted system parameters converge to the real system parameters stably. i.e.,

$$\alpha_1 = a_1, \quad \beta_1 = b1$$

From the identified parameters α_1, β_1 and Eq. (2) and (3), the parameters k_p, k_i can be computed.

It is the advantage of the on-line parameters identification that the difference equation of the system is obtained directly, and according to the difference equation an on-line control output can be calculated. Based on this idea, the adaptive controller can be designed.

Implementation of Digital Control System

As we know, the radial AMB system has 8 state variables and 4 control variables. The feedback gain matrix is a 4×8 matrix. When the optimal control algorithm is directly realized in a real-time program, at least 32 multiplications and additions have to be done within an interrupt period. And, counting in the calculation of observing state variables, the control program will be very large.

In our test-rig, a TMS32010 DSP system was used to implement the control algorithm.

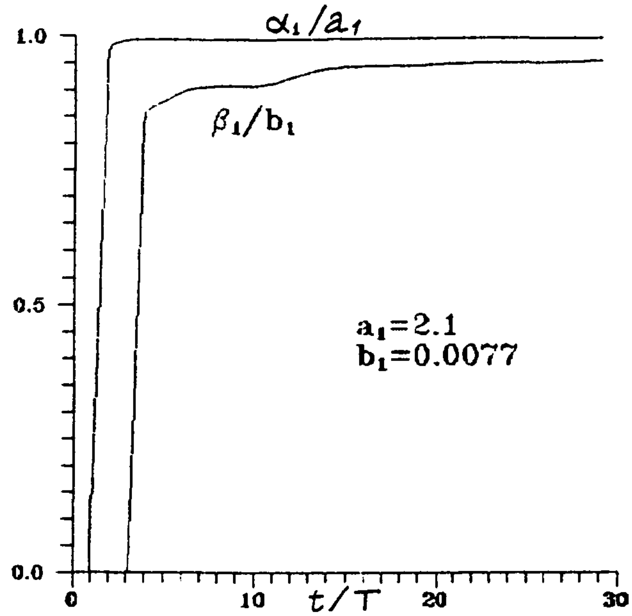

Figure 4: Simulation of On-line Parameters Identification with PD Digital Controller

TMS32010 can do a multiplication or addition in a 200ns instruction cycle. However, the limitation of in-chip data RAM makes it difficult to run the optimal control program, which needs many data RAM for controller coefficients. To simplify the structure of the control system, a decentralized controller is used, which is obtained by simply omitting all the coupling elements in the optimal feedback gain matrix.

For our magnetic bearings system, at low rotation speed of rotor, the coupling elements in optimal feedback gain matrix $\mathbf{K}$ are very small compared with the uncoupling elements. Therefore, it is believable that this simplification will not cause large degradation of system performance compared with the optimal central control. Figure 5 shows the step response of radial magnetic bearings controlled by the decentralized controller and the optimal central controller. And this result has also been verified in some other papers. [8]

To obtain velocity of the rotor, which can not be measured directly, the digital differentiators are used. A modified differentiators algorithm is,

$$u_d(k) = \frac{1}{6T}[x(k) + 3x(k-1) - 3x(k-2) - x(k-3)]$$

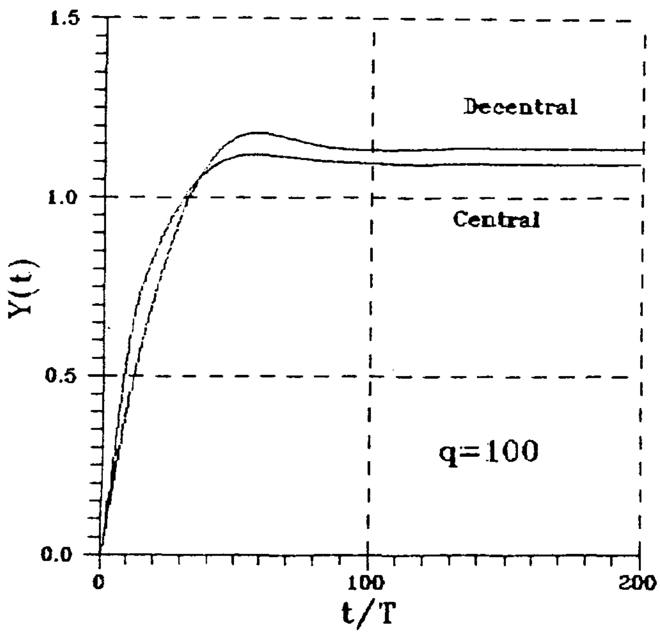

Figure 5: Comparsion of Central and Decentralized Controllers

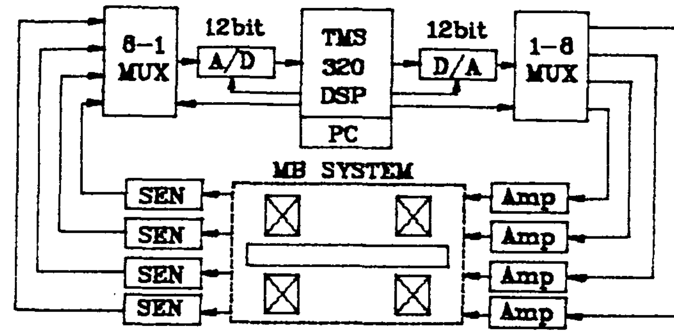

Figure 6: Block Diagram of Control System

where,

T—sampling period
$x(k)$—input displacement at sampling time kT

This algorithm can reduce the effects of input noise to system performance.

The block diagram of control system is shown in Figure 6. The digital control system consists of displacement sensors, switching power amplifiers, DSP system and a host computer. The control program can be loaded to the memory of the DSP. The structure and parameters of the controllers can be changed easily in the memory. The flowchart of the real-time control program is shown in Figure 7. In this digital system, only one A/D converter and one D/A converter are used. Four input and output ports are switched by two multiplexers. This simplification is economic use of hardware, but increases the time of interrupt service program.

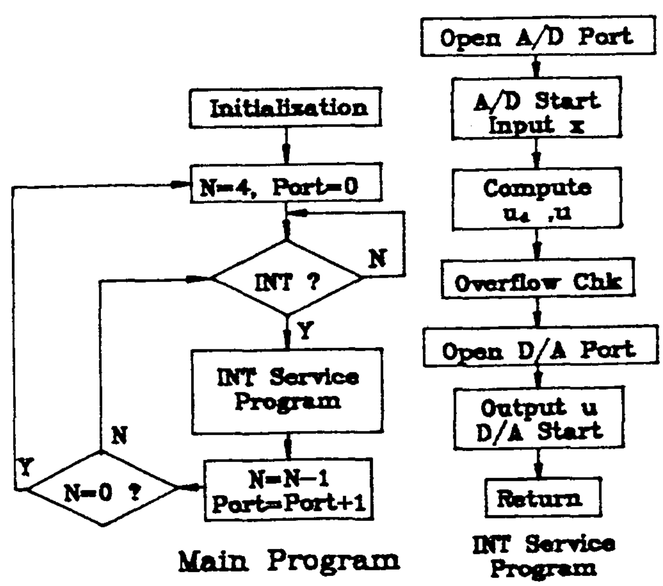

Figure 7: Flowchart of the Real-time Control Program

Experimental Results

According to the designed controller, the feedback parameters are tuned. The step responses of the radial magnetic bearings with different feedback parameters are measured. In fact, when the velocities of the rotor are obtained by differentiators, the decentralized radial AMB con-

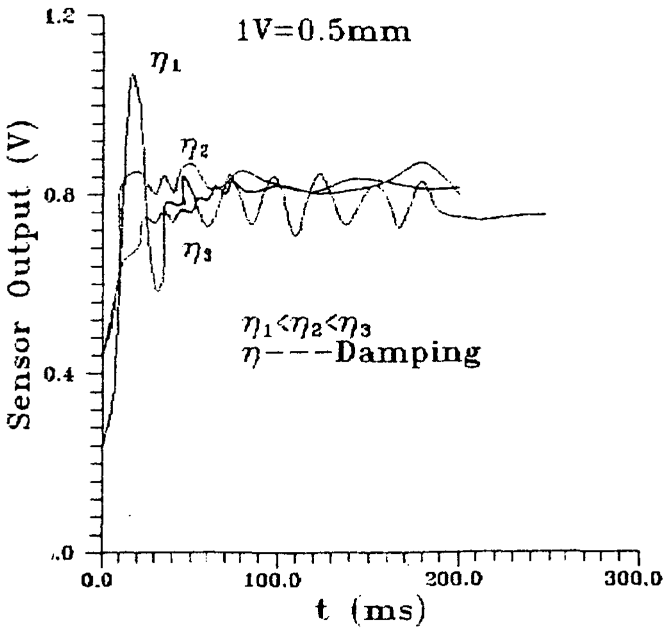

Figure 8: Step Response of the Radial AMB System

troller can also be considered as four PD controllers. From the view of PD controller. changing of gains of proportional and differential paths means changing of stiffness and damping of system. Figure 8 shows a set of step responses curves with different damping η and same stiffness.

Conclusions

Computer Aided Design method is very suitable for the design of radial magnetic bearings control system. The designed controller can be simulated with the simulation program before implemented as a practical AMB controller. And, the effects of controller coefficients to the system performance can be revealed by simulation. For example, increasing of position weights in a performance index will increase the speed of system response. With the CAD method, the development cycle of AMB controller is shorter.

The experimental results show that the simple decentralized state variables feedback digital controllers also have relatively good performance. This control system has simple structure and is easy to implement in a digital control system.

When the static rotor is suspended stably, the close-loop measurement can be carried out to identify the system parameters. The online parameters identification is an advanced parameters identification method. Based on this method, an adaptive controller can be realized.

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