Unfalsified PID Controller Design for Active Magnetic Actuators Based on Direct Measured Data and L_2 Gain Criterion

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Abstract— **In this paper, an iterative adaptive design method for practical PID controllers based on the unfalsification of the input-output data is considered. The proposed design method is plant model free and includes the procedure for the adjustment of the performance specification. The effectiveness of the proposed method is evaluated by experiments on a magnetic levitation system.**

I. INTRODUCTION

PID control method is effective for many actual plants including active magnetic actuator system, e.g. active magnetic bearing, magnetic levitation system. There are several methods for tuning PID gains. These conventional PID gain tuning methods are based, either implicitly or explicitly, on identifying approximate plant models. The mismatch and inaccuracy of plant models directly leads to the poor control performance. In particular, for active magnetic actuator system, in case the stabilizing controller is unknown, the tuning of the PID gains becomes much harder due to the instability of the system.

In this paper, we propose a new direct PID controller design method from measured data without using plant models. The proposed method applies the notion of "Unfalsification"[1]−[4] on the measured data and L_2 gain criterion. The notion and technique of "Unfalsification" enables us to determine whether candidate PID controller meets the desired performance specification or not without performing additional experiments. Our proposed method is one of the iterative adaptive control method.

II. PRELIMINARY

In this section, notations and the basic notions for unfalsified control are introduced for preparation.

Let denote a controller K, a plant P, a referecne input $\tilde{r}(t)$, a control input $u(t)$ and a measured output $y(t)$. Supposed standard feedback structure as follows;

$$y(t) = Pu(t), \qquad u(t) = K\{\tilde{r}(t) - y(t)\}. \tag{1}$$

For an unknown plant P, suppose a pair of measured input-output data is given by $(u_{\mathrm{data}}(t),\, y_{\mathrm{data}}(t))$.

Define a measured data set M_{data} as

$$M_{\mathrm{data}} := \{(y,u) \in \mathcal{Y} \times \mathcal{U} \ \mid\ (y_{\mathrm{data}}, u_{\mathrm{data}}) \in (y,u)\}, \tag{2}$$

where $\mathcal{U}$ and $\mathcal{Y}$ denote an adequate functional space. Introducing a fictitious reference input $r \in \mathcal{R}$, define measured information set P_{data} as

$$P_{\mathrm{data}} := \{(r,y,u) \in \mathcal{R} \times \mathcal{Y} \times \mathcal{U} \ \mid\ (y,u) \in M_{\mathrm{data}}\}. \tag{3}$$

Define an admissible control law K_{admiss} and a control specification T_{spec} as

$$K_{\mathrm{admiss}} \subset \mathcal{R} \times \mathcal{Y} \times \mathcal{U}, \tag{4}$$

$$T_{\mathrm{spec}} \subset \mathcal{R} \times \mathcal{Y} \times \mathcal{U}. \tag{5}$$

Now, the fundamental unfalsification theorem for unfalsified control method is described as follows;

Theorem 1 [Unfalsification Theorem][1]

K is unfalsified for P_{data} and T_{spec} if and only if

$$\forall (r_0, y_0, u_0) \in P_{\mathrm{data}} \cap K, \quad \exists (y_1, u_1)\ s.t.$$

$$(r_0, y_1, u_1) \in P_{\mathrm{data}} \cap K \cap T_{\mathrm{spec}} \tag{6}$$

holds. (Refer [1] for the proof.)

For the note of the theorem, we emphasize the following three points; 1) The theorem is necessary and sufficient condition without conservativeness. 2) In order to falsify a controller, there is no need to implement the controller. Controllers could be falsified based on arbitrary input-output data with other controllers. 3) Plant models are not needed.

III. PID CONTROLLER DESIGN BASED ON UNFALSIFICATION THEOREM

In this section, a PID controller design method based on the unfalsification theorem is proposed.

A. Fictitious input and performance specification

Restrict the controller K to PID controllers as

$$u = \left(K_P + \frac{K_I}{s} + K_D s\right)(r - y), \tag{7}$$

where K_P, K_I and K_D are positive feedback gains. Hereafter, we employ the notation $K(K_P, K_I, K_D)$ as admissible controllers K_{admiss}, which imply the controllers in eqn.(7). The controller $K(K_P, K_I, K_D)$ is causal left-invertible, that is, fictitious reference r could be uniquely determined from input-output data u and y. The fictitious reference r is calculated by filtering operation as follows;

$$r = \frac{us}{K_D s^2 + K_P s + K_I} + y. \tag{8}$$

For $(r, y, u) \in L_{2e} \times L_{2e} \times L_{2e}$, the performance specification is defined as

$$||w_1 * (y - r)||^2_{L_2[0,\tau]} + ||w_2 * u||^2_{L_2[0,\tau]} \le ||r||^2_{L_2[0,\tau]}, \ \forall \tau, \tag{9}$$

where

$$\begin{aligned} ||w_1 * (y-r)||^2_{L_2[0,\tau]} &= \int_0^\tau [w_1(t-\tau) \cdot \{y(\tau) - r(\tau)\}]^2 dt \\ ||w_2 * u||^2_{L_2[0,\tau]}|| &= \int_0^\tau \{w_2(t-\tau) \cdot u(\tau)\}^2 dt \\ ||r||^2_{L_2[0,\tau]} &= \int_0^\tau \{r(t)\}^2 dt \end{aligned} \tag{10}$$

holds. In case the plant is time invariant, eqn. (9) becomes

$$\left\| \begin{bmatrix} W_1 S \\ W_2 K S \end{bmatrix} \right\|_\infty \le 1, \tag{11}$$

where $S := 1/(1 + PK)$. Suppose $w_1(t)$ and $w_2(t)$ are the impulse responses of the systems represented as minimum-phase transfer functions $W_1(s)$ and $W_2(s)$. Define a time domain function $\tilde{T}_{\text{spec}}$ as

$$\begin{aligned} &\tilde{T}_{\text{spec}}(r(t), y(t), u(t)) := \\ &\qquad |w_1 * \{y(t) - r(t)\}|^2 + |w_2 * u(t)|^2 - |r(t)|^2, \end{aligned} \tag{12}$$

and also define a sampling time of the input-output data by Δt. Eqn. (9) becomes

$$\int_0^{k\Delta t} \tilde{T}_{\text{spec}}(r(t), y(t), u(t)) \le 0, \ \ \forall k = 1, \cdots, n \tag{13}$$

where $n := \tau/\Delta t$.

B. Unfalsified PID controller design method

Suppose an initial controller set $\hat{K}$ is

$$\hat{K} := \{K_i(K_{Pi}, K_{Ii}, K_{Di}), \ \ i = 1, \cdots, m\}. \tag{14}$$

Based on the unfalsification theorem in II with fictitious reference and performance specification in III.A, a design procedure to select the controller to meet the performance specification is described as follows:

[Unfalsified PID Design Procedure 1]

1 Define $\mathcal{I} := \{1, 2, \cdots, m\}$, $\mathcal{K}_{\text{prev}} := \emptyset$.
2 Select an initial controller $K_E := K_i \in \hat{K}$.
3 while $\mathcal{I} \neq \emptyset$ and $K_E \notin \mathcal{K}_{\text{prev}}$
4   Carry out an experiment and obtain input-output data u_E and y_E with a controller K_E.
5   $\tilde{\mathcal{I}} := \emptyset$.
6   for $i \in \mathcal{I}$
7   Solve $r_i := \frac{su_E}{K_{Di}s^2 + K_{Pi}s + K_{Ii}} + y_E$.
8     $J(i) := 0$.
9   for $k = 1 : n$
10       $J(i) := J(i) + \int_{(k-1)\Delta t}^{k\Delta t} \tilde{T}_{\text{spec}}(r_i, y_E, u_E) dt$.
11   if $J(i) > 0$
12         $\tilde{\mathcal{I}} := \tilde{\mathcal{I}} \cup \{i\}$. Break for-loop.
13   endif
14   endfor
15   endfor
16   $\mathcal{I} := \mathcal{I} \backslash \tilde{\mathcal{I}}$.
17   $i_{\min} := \arg\min_{\mathcal{I}} J(i)$.
18   $\mathcal{K}_{\text{prev}} := \mathcal{K}_{\text{prev}} \cup \{K_E\}$.
19   $K_E := K_{i_{\min}}$.
20 endwhile

Due to the stopping criterion in Step3, the procedure is terminated when the all of the candidate controllers are falsified or the same controller is selected again. In case that the all controllers are falsified, i.e.$\mathcal{I} = \emptyset$, it is necessary to restart the procedure after enlarging the candidate controller set $\hat{K}$ or changing the performance specification.

C. Renewal of the weighting function

When the actual performance, i.e. (u_E, y_E), of the selected controller in Step 19 is poor, the cost function $J(i)$ should be renewed. The mismatch between the performance specification $J(i)$ and the actual necessary property directly leads to the poor performance. The renewal of $J(j)$ means the renewal of the weighting function w_1 and w_2 in eqn. (9).

In order to renew the performance specification, identifying the plant $P_E(s)$ from the measured input-output data $u_E(t)$ and $y_E(t)$, calculate the sensitivity function $S_E = 1/(1 + P_E K_E)$ including implemented controller K_E and identified plant P_E.

Based on this result, we can renew the frequency weighting function $W_1(s)$ and $W_2(s)$ as $\tilde{W}_1(s)$, $\tilde{W}_2(s)$ respectively as follows;

$$\left\| \begin{bmatrix} \tilde{W}_1(s) \cdot S_E \\ \tilde{W}_2(s) \cdot K S_E \end{bmatrix} \right\|_\infty \ge 1 \tag{15}$$

$$||W_1(s)\tilde{W}_1(s)^{-1}||_\infty \le 1 \tag{16}$$

$$||W_2(s)\tilde{W}_2(s)^{-1}||_\infty \le 1 \tag{17}$$

In this renewal process, we should note the influence of discretizing time Δt, the number of data and the identifying method to the frequency property of P_E. Also note to avoid the excessive renewal. Suppose the impulse responses of the renewed frequency weighting functions $\tilde{W}_1(s)$ and $W_2(s)$ are $\tilde{w}_1$ and $\tilde{w}_2$ respectively. After renewing w_1, w_2 in eqn. (12) by $\tilde{w}_1, \tilde{w}_2$, carry out the unfalsified PID controller design procedure.

The overall controller design procedure including renewal of the weighting function is described as follows;

[Unfalsified PID Design Procedure 2]

1 Step1$\sim$Step4 in [Design Procedure 1]
2 If [the experimental performance of (u_E, y_E) is poor]
3 Renew $W_i(s)$ as $\tilde{W}_i(s)$ with eqs.(15)$\sim$(17).
4 Renew $\tilde{T}_{\text{spec}}$ by replacing w_i of $\tilde{w}_i$ $(i = 1, 2)$.
5 endif
6 Step5$\sim$Step20 in [Design Procedure 1]

D. Convergence of the design procedure

Step16 in [Design Procedure 1] implies the element number of the index set $\mathcal{I}$ decreases uniquely. This property enables us to shape up the candidate controller set $\hat{K}$.

Concerning to [Design Procedure 2] including the weighting function renewal, eqns.(16) and (17) in Step3 guarantees the performance specification becomes harder. This fact implies the following inclusion relationship holds;

$$\mathcal{I}_{\text{rnw}} \subset \mathcal{I}, \tag{18}$$

where $\mathcal{I}_{\text{rnw}}$ indicate the renewed index set of $\mathcal{I}$. Due to this inclusion relationship, the unique decrease of the element number of the index set $\mathcal{I}$ is also guaranteed in [Design Procedure 2].

Moreover, by the condition $K_E \notin \mathcal{K}_{\text{prev}}$, the convergence to a final controller in the finite iteration number without procedure loop is guaranteed.

IV. Experiments

In this section, the effectiveness of our proposed method is evaluated by the experiments on a magnetic levitation system.

A. Magnetic levitation system

The mass of the levitated steel ball in our magnetic levitation system is 4.2 [kg], and the diameter is 100[mm]. The objective of experiments is to design a PID controller levitating the steel ball stably. Concerning to the general context of the research on unfalsified control method, there exists no reports of the experiments on the actual unstable system. The experimental results in this section can contribute to the general theory of unfalsified control method.

The input to the magnetic levitation system is the operation voltage u[V] of the amplifier driving the electromagnetic coil. The output of the system is the gap y[m] between the electromagnetic coil and the steel ball. The gap between the initial starting point of the steel ball and the electromagnetic coil, i.e. maximal gap, is 1×10^2[m].

The property of the magnetic levitation system nonlinearly varies according to the objective gap of the levitated steel ball. In order to evaluate the effectiveness to the nonlinearity of the plant, we carried out many experiments on the various objective gap and confirmed our proposed method can achieve the stable levitation. In this section, we shows the results on the most difficult case in the context of the nonlinearity in our experiments, that is, the objective gap is 2×10^{-3}[m].

B. Controller design

Due to the hardware limitation, the discretizing time is set to 2×10^{-3}[s] and the number of each data is set to 3000. Concerning to the criterion for the renewal of the weighting functions in Step2 of [Design Procedure 2], we renewed the weighting function when the system became unstable. That is, in case the best controller according to the performance specification cannot even stabilize the system, we judge that the performance specification is not adequate and renew the weighting function.

Based on the consideration for the physical meaning of the PID controller, e.g. steady current on the objective gap, the candidate PID controller $K(K_P, K_I, K_D)$ set is defined as follows;

$$\begin{aligned} K_P &= \{1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0\}, & (19)\\ K_I &= \{0.1, 1.0, 8.0, 15, 22, 29, 36, 43\}, & (20)\\ K_D &= \{0.01, 0.05, 0.09, 0.13, 0.17, 0.21, 0.25\}. & (21)\end{aligned}$$

The number of the element in the initial candidate controller set is 392. We carried out the experiments on the various initial implemented controllers. In this section, we however shows a result on the initial controller $K(4.0, 40, 0.225)$ as the difficult case for converging to the final controller.

Concerning to the performance specification, first derived the complementally sensitivity function considering the influence of the physical parameters variation. Then, derive the sensitivity function considering the consistency to the complementally sensitivity function. As the result, the performance specification $W_1(s)$ and $W_2(s)$ are set to

$$W_{11}(s) := W_1(s) = \frac{0.6s + 0.0002}{s} \tag{22}$$

$$W_2(s) = \frac{1.868 \times 10^6 s^3 + 2.105 \times 10^{10} s^2 + 5.564 \times 10^{12} s + 4.161 \times 10^{12}}{2.228 \times 10^6 s^3 + 6.687 \times 10^9 s^2 + 5.027 \times 10^{12} s + 1.103 \times 10^{13}}. \tag{23}$$

Table.I shows the number of iterations, controllers, stability ,the number of unfalsified controller and weighting function. The time responses with each implemented controllers are in Fig.1~Fig.6. Fig.1 indicates the initial controller $K(4, 40, 0.225)$ is unstable. Carrying out the first iteration of the [Unfalsified PID Design Procedure 2] using the measured input-output data by $K(4, 40, 0.225)$, the number of the unfalsified controllers becomes 97, i.e. 295 controllers are falsified. The controller $K(1, 1, 0.001)$ takes the best value of the cost function and is selected to the next implemented controller.

Fig.2 shows the controller $K(1, 1, 0.001)$ is also unstable. In this case, due to the Step2 in the [Unfalsified PID Design Procedure 2], the weighting function is renewed. Identifying ARX model based on the input-output data with the controller $K(1, 1, 0.001)$, derive the transfer function model of the plant as $P_{ID}(s)$. The sensitivity function S_1 with $K(1, 1, 0.001)$ and P_{ID} is obtained by

$$S_1 := \frac{1}{1 + P_{ID}K(1, 1, 0.001)} \qquad (24)$$

$$= \frac{s^5 + 1074s^4 + 1.61 \times 10^6 s^3 + 5.17910^7 s^2 - 5.827 \times 10^8 s}{s^5 + 1074s^4 + 1.61 \times 10^6 s^3 + 5.179 \times 10^7 s^2 - 5.817 \times 10^8 s + 1.03 \times 10^6}. \qquad (25)$$

Based on eqs.(15) and (17), renew the weighting function $W_{11}(s)$ by

$$W_{12}(s) = \frac{0.6s + 0.0015}{s}. \qquad (26)$$

Fig.7 shows the frequency responses of $W_{11}(s)$, $S_1(s)$ and $W_{12}(s)$. Replacing w_1 by the impulse response w_{12} of $W_{12}(s)$, the unfalsification procedure with the measured input-output data by $K(4, 40.0.225)$ is carried out. As the result, the number of unfalsified controllers becomes 96. A controller $K(1.5, 1, 0.050)$ takes the best value of the cost function. Fig.3 shows the time response of the controller $K(1.5, 1, 0.050)$ and implies that the controller stabilizes the magnetic levitation system.

In the same manner, the controllers $K(2, 1, 0.17)$, $K(1, 1, 0.13)$ and $K(1.5, 1, 0.17)$ are obtained. Fig.Fig.3 ~Fig.6 shows the time responses of each controllers. Fig.5 implies the closed loop system with the controller $K(1, 1, 0.13)$ is unstable. Then, the weighting function W_{12} is renewed as $W_{13} = (0.6s + 0.0017)/s$. Fig.8 shows the frequency responses of weighting functions.

The final controller $K(1.5, 1, 0.17)$ is unfalsified by the input-output data with itself and takes the best value of the cost function. Then, the design procedure is terminated due to the stopping criterion $K_E \notin \mathcal{K}_{\text{prev}}$.

V. Conclusion

Based on the unfalsification theorem, an iterative direct design procedure for PID controller with the input-output data was proposed. The proposed procedure includes the renewal step of the performance specification, which is effective in case that the controllers to be falsified are obvious. On the unstable magnetic levitation system, the effectiveness of the proposed method was confirmed. On the various initial controllers, the proposed method can achieve stabilization of the system.

The proposed method enables us to design the PID controllers directly from the measured input-output data including unstable time response. The method can avoid the influence of the plant model quality to the controller design due to the plant model free property. The flexibility in the controller structure is also practically profitable. Further research in this area is needed.

TABLE I

Experimental results

	(K_P, K_I, K_D)	Stability	Quantity	$W_i(s)$
1	$(4, 40, 0.225)$	Unstable	(392) → 97	W_{11}, W_2
2	$(1, 1, 0.001)$	Unstable	–	–
3	$(4, 40, 0.225)$	Unstable	96	W_{12}, W_2
4	$(1.5, 1, 0.050)$	Stable	82	W_{12}, W_2
5	$(2, 1, 0.17)$	Stable	7	W_{12}, W_2
6	$(1, 1, 0.13)$	Unstable	–	–
7	$(2, 1, 0.17)$	Stable	6	W_{13}, W_2
8	$(1.5, 1, 0.17)$	Stable	6	W_{13}, W_2
9	$(1.5, 1, 0.17)$	Stable	–	–

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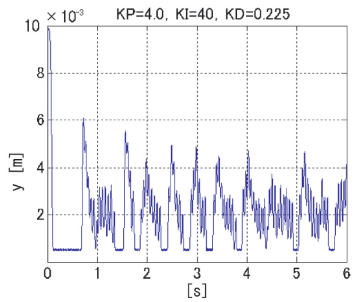

Fig. 1. Output response of closed-loop system

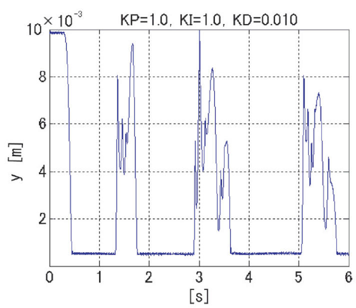

Fig. 2. Output response of closed-loop system

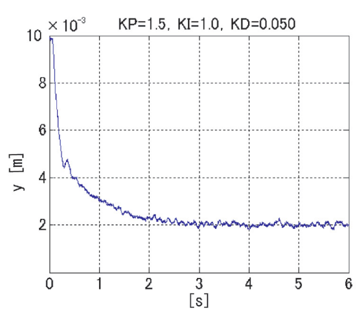

Fig. 3. Output response of closed-loop system

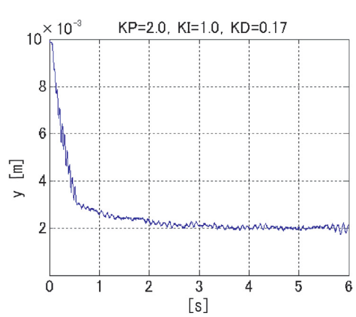

Fig. 4. Output response of closed-loop system

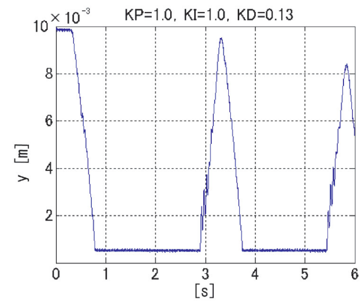

Fig. 5. Output response of closed-loop system

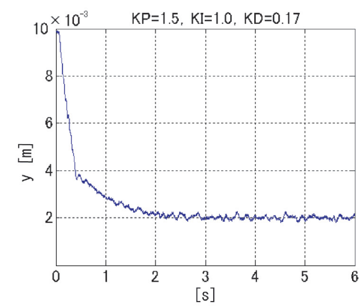

Fig. 6. Output response of closed-loop system

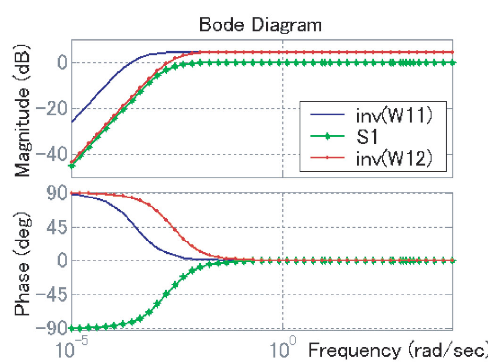

Fig. 7. Bode diagram of weighting fuctions

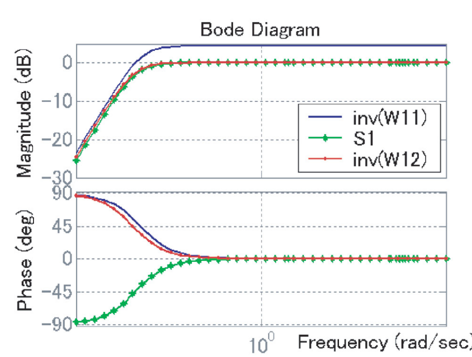

Fig. 8. Bode diagram of weighting fuctions