Compliant Foil Bearings Used As Emergency Bearings

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ABSTRACT

A foil-magnetic hybrid bearing will provide an ideal solution for oil-free suspension of high-speed machinery such as air cycle machines, auxiliary power units and cryogenic turbo compressors. The hybrid bearing combines the advantages of high load capacity of compliant foil bearings at high speed and sufficient stiffness and damping characteristics control versatility and flexibility in load-sharing of active magnetic bearings at zero or low-speed operation. This paper is focused on the system modeling and prediction of both static and dynamic performances of compliant foil bearing first. A generalized method, which can be used for system modeling and dynamic analysis of a foil-magnetic hybrid bearing, was developed. The advancement of present research will improve the understanding of foil-magnetic hybrid bearings and meet the demands of the development of advanced bearing techniques in high power density, high speed, reliable, low maintenance and oil-free turbo machinery.

1 INTRODUCTION

It is possible that in the near future foil-magnetic hybrid bearing (FMHB) can be used in the field of advanced high-speed turbo machinery. The FMHB combines advantages of both compliant foil bearing (CFB) and active magnetic bearing (AMB) and brings a number of benefits as follows.

1) Safe operation at zero or extremely high speed (from 0 rev/min to 10×10^4 rev/min).
2) High load capacity due to function of load-sharing in FMHB.
3) Normal operation even in the case of magnetic bearing fail.
4) Potential for high temperature capacity and low power loss due to oil-free suspension.
5) Active control of both static and dynamic performances of system.

Although studies of two separated techniques, e.g., the CFB and AMB techniques all have a long history, investigation of the FMHB began only a few years ago, and available information is limited. Priori researches can be found by Heshmat[1] and Swanson, *et al* [2]. For purpose of engineering applications, a great deal more needs to be learned of the dynamic characteristics of the FMHB and dynamic performance of foil-magnetic hybrid bearing rotor systems (FMHBRS). There is a lack of existing literature regarding the stiffness and damping in the FMHB and the prediction of dynamic performance of the FMHBRS.

For the AMB alone, knowledge of its dynamic characteristics is sufficient , the dynamic behavior of a rotor supported by AMBs can be well predicted and measured in linear range [3,4]. As to the analysis of FMHBs, difficulty comes mainly from the CFB. In spite of a large number of papers on CFBs were published, most of them are limited to the investigation of static performance. As summarized by Dellacorte, "all of these applications relied on experimental build and test development sequence" [5]. Several models were presented for the prediction of the dynamic characteristics of compliant foil bearings, and excellent contributions were made by Ku and Heshmat [6-9].

In the model presented by Heshmat[6], compared with that presented by Walowit [10], improvements were achieved by taking into account all the interactive forces between the top foil and bumps or the bumps and the housing, and structural stiffness and damping were introduced to describe the characteristics of bump foil strips. But the attempt to separate the action of top-foil-bump strips from the whole foil bearing seems not very successful. Calculation of the stiffness and damping are dependent on journal motion according to [6], furthermore, solution of the damping brings people into a "limit cycle", e.g. non-linear rotor dynamic calculation. It is obvious that the stiffness and damping defined in [6] are not convenient in use and have little value for system dynamic analysis. In fact, as pointed by Howard[12], the so-called structural stiffness defined and measured in [6,9] cannot represent the dynamic stiffness but the steady-state one since the dynamic deflections of the top foil and bump foils were not taken into account. Neglecting of the dynamic deflection causes much information lost. The dynamic deflection of the top foil and the bumps results from the dynamic pressure of the gas film due to small perturbations of the journal and always plays an important role on both dynamic stiffness and damping, and it is always related to exiting frequency. Previous experiments showed that the dynamic characteristics of the CFB were frequency dependent [7], but further explanation was not found in literature, the neglect of the dynamic deformation is one of the factors. It's a pity that no knowledge of effects of dynamic deflection on dynamic characteristics in CFBs can be gained. It is necessary to further investigate the Mechanism of the dynamic stiffness and damping in CFBs by introducing the dynamic deformation into Reynold's equation and elastic equations, similar to the research done by Nillson [13, 14, 15].

The present research is mainly focused on the modeling of the compliant foil bearing. A generalized method was developed for the prediction of both static and dynamic performances of the CFB. Theoretical contributions can be further considered as the base for the prediction of the performance of the FMHBRS.

2 JOURNAL POSITION AND FOIL DEFORMATION IN CFB

A typical foil bearing design is shown in Fig.1. The foil bearing consists of top foil, bump foil and bearing housing. Position of a rotating journal at arbitrary time can be expressed as follows.

$$\varepsilon = \varepsilon_0 + E = \varepsilon_0 + E_0 e^{i\Omega T}$$

$$\theta = \theta_0 + \Theta = \theta_0 + \Theta_0 e^{i\Omega T} \qquad (1)$$

Where E_0 and Θ_0, both of which are defined as complex numbers, are disturbances of eccentricity and attitude angle respectively, Ω is frequency of disturbance, and T the dimensionless time, $i = \sqrt{-1}$.

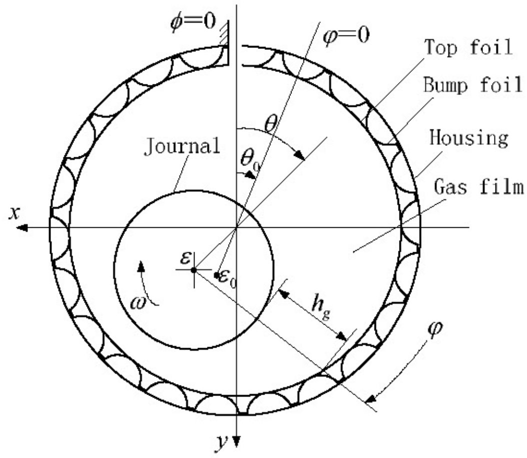

Fig.1 Foil bearing schematic

Gas film thickness H_g between the journal and undeformed top foil at arbitrary angular coordinate φ can be written as follows.

$$H_g = H_{g0} + H_{gd}$$

$$H_{g0} = 1 + \varepsilon_0 \cos\varphi$$

$$H_{gd} = H_{gd0} e^{i\Omega T} = (E_0 \cos\varphi + \Theta_0 \varepsilon_0 \sin\varphi) e^{i\Omega T} \qquad (2)$$

Where H_{g0} and H_{gd} are static and dynamic gas film thicknesses respectively and determined by the position of the journal and the geometrical profile of the top foil.

3 ELASTIC DEFORMATION OF TOP FOIL SUBJECTED TO FORCES

Forces acting on the top foil in a compliant foil bearing can be divided into two parts: the first is the gas film pressure $P(\phi, z)$ distributed on the surface of the top foil, and the second comes from the contact force $W(\phi, z)$ between the top foil and the bump foil strips. As

shown in Fig.2, the top foil in a journal bearing is generally leading-edge free and leakage-edge fixed.

Denote all the forces acting on domain A of the top foil be $\Pi(\phi, z)$, elastic deflection of the top foil at arbitrary point $g(\varphi, \lambda)$ due to the action of all the forces can be written as follows $P(\phi, z)$ and contact load $W(\phi, z)$ where A is the domain to be considered, and f_t the deflection coefficient of top foil. It is convenient to separate the gas film force while $W(\phi, z)$ depends mainly on the structure of from $\Pi(\phi, z)$ because of that $P(\phi, z)$ is related to gas-lubricated Reynolds equation, .

$$H_{\delta}(\varphi, \lambda) = \iint_A \Pi(\phi, z) f_t(\phi, z, \varphi, \lambda) d\phi dz \qquad (3)$$
$$((\varphi, \lambda) \in A; (\phi, z) \in A)$$

bump foil strips. The force distribution

$\Pi(\phi, z)$ can be written as

$$\Pi(\phi, z) = P(\phi, z) - W(\phi, z) \qquad (4)$$

Generally, the reaction force $W(\phi, z)$ due to contact differs from the gas film pressure $P(\phi, z)$ and takes place only on local contact area A_{tb}, and in most cases, contact between the top foil and bump foil is linear-contact; with increased load, contact taking place in top foil and bump foil maybe becomes area-contact. So, when the definition of contact load is in the whole domain A, $W(\phi, z)$ can be written as

$$W(\phi, z) = \begin{cases} W(\phi, z) & ((\phi, z) \in A_{tb}, A_{tb} \in A) \\ 0 & ((\phi, z) \in (A - A_{tb})) \end{cases} \qquad (5)$$

Similar to the expression of the deformation of the top foil, deformation of the bump foil due to contact load $W(\phi, z)$ can be always expressed in the following form by using the deflection coefficient f_b of the bump foil.

$$H_{b\delta}(\varphi, \lambda) = \iint_A W(\phi, z) f_b(\phi, z, \varphi, \lambda) d\phi dz$$
$$= \iint_{A_{tb}} W(\phi, z) f_b(\phi, z, \varphi, \lambda) d\phi dz$$
$$((\varphi, \lambda) \in A; (\phi, z) \in A_{tb}) \qquad (6)$$

It must be pointed out that for the calculation of the deformation of bump foil, reacting forces N_i and friction forces F_i, as shown in Fig.3, also take important roles and must be also taken into account besides $W(\phi, z)$, consideration of the action of N_i and F_i can be dealt by using the method presented in reference [8], and details are neglected here for simplicity.

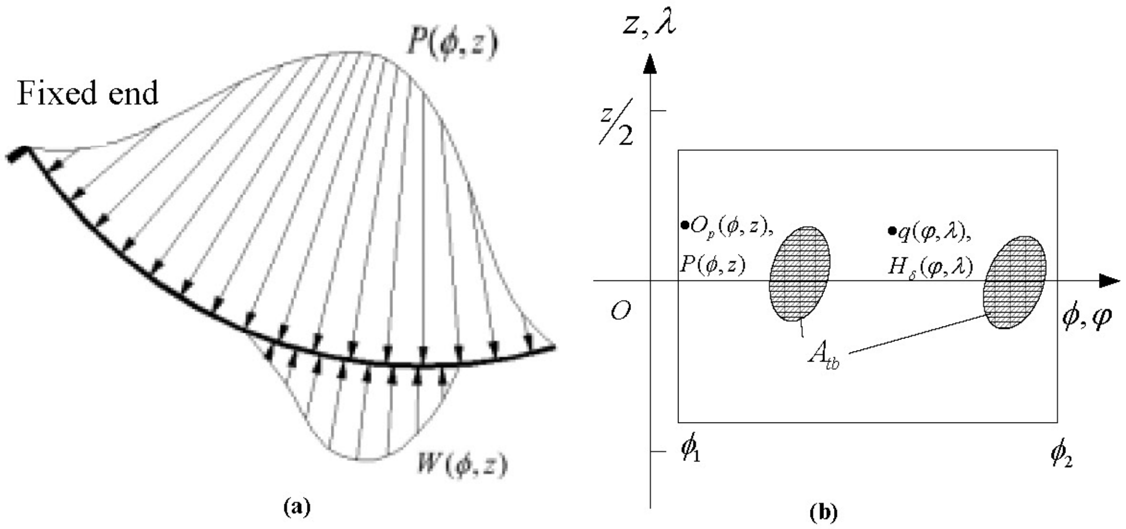

Fig.2 Forces acting on top foil
(a) Top foil subjected to forces (b) Coordinate system

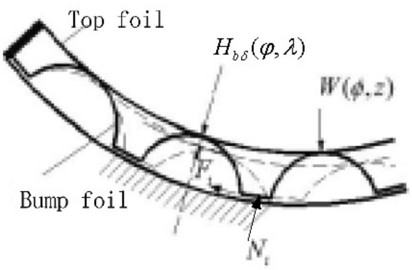

Fig.3 Deflection of bump foil subjected to forecs

Substitution of Eq.(4) into Eq.(3) yields

$$H_\delta(\varphi,\lambda)=\iint_A P(\phi,z)f_t(\phi,z,\varphi,\lambda)d\phi dz$$
$$-\iint_{A_{tb}} W(\phi,z)f_t(\phi,z,\varphi,\lambda)d\phi dz \qquad (7)$$

Denote $H_\delta(\varphi,\lambda)=H_{p\delta}(\varphi,\lambda)-H_{w\delta}(\varphi,\lambda)$

where $H_{p\delta}(\varphi,\lambda)$ is the deformation of the top foil at point (φ,λ) due to the gas film pressure $P(\phi,z)$ alone, and $H_{w\delta}(\varphi,\lambda)$ the deformation of the top foil due to contact load $W(\phi,z)$.

$$H_{w\delta}(\varphi,\lambda)=\iint_{A_{tb}} W(\phi,z)f_t(\phi,z,\varphi,\lambda)d\phi dz$$
$$((\varphi,\lambda)\in A;(\phi,z)\in A_{tb})$$
$$H_{p\delta}(\varphi,\lambda)=\iint_A P(\phi,z)f_t(\phi,z,\varphi,\lambda)d\phi dz$$
$$((\varphi,\lambda)\in A;(\phi,z)\in A) \qquad (8)$$

Within the contact area A_{tb} between the top foil and the bump foil, the following deformation coordination condition must be satisfied.

$$H_{p\delta}(\varphi,\lambda)=H_{w\delta}(\varphi,\lambda)+H_{b\delta}(\varphi,\lambda)$$
$$((\varphi,\lambda)\in A_{tb}) \qquad (9\text{-}1)$$

or

$$\iint_A P(\phi,z)f_t(\phi,z,\varphi,\lambda)d\phi dz$$
$$=\iint_{A_{tb}} W(\phi,z)f_t(\phi,z,\varphi,\lambda)d\phi dz$$
$$+\iint_{A_{tb}} W(\phi,z)f_b(\phi,z,\varphi,\lambda)d\phi dz,((\varphi,\lambda)\in A_{tb}) \qquad (9\text{-}2)$$

On above equations, ϕ and z represent the coordinates of the source point, and φ and λ the coordinates of the field point to be considered.

So equation (9) in fact describes the constrained relation of the film pressure and the contact load in contact areas.

4 GENERAL SOLUTION OF ELASTO-AERODYNAMIC LUBRICATION

As well known, for a compliant foil journal bearing, the dimensionless Reynolds equation to describe a gas film lubrication problem has a general form as follows.

$$\frac{\partial}{\partial\varphi}(PH^3\frac{\partial P}{\partial\varphi})+\frac{\partial}{\partial\lambda}(PH^3\frac{\partial P}{\partial\lambda})=\Lambda\frac{\partial(PH)}{\partial\varphi}$$
$$+2\Lambda\frac{\partial(PH)}{\partial T} \qquad (10\text{-}1)$$

with the gas film thickness equation given by

$$H=H_g+H_\delta \qquad (10\text{-}2)$$

where H_{is} is the total dimensionless gas film thickness, and H_g the film thickness respect to rigid surface, H_δ the film thickness due to the deformation of top foil; P is the dimensionless gas film pressure, and Λ is the compressible number, $\Lambda=\frac{6\mu\omega}{P_a}\left(\frac{R}{C_0}\right)^2$. In the expression of Λ, μ is the air dynamic viscosity, ω the angular frequency, R the radius of the journal, C_0 the normal radial clearance, and P_a the ambient pressure.

4.1 Static Solution of Elasto-Aerodynamic Lubrication

In static state, the Reynolds equation is independent to time T, variables involved in Eq.(10) in static state are all denoted with subscript "0" for distinction. Thus equation (10) becomes

$$\frac{\partial}{\partial\varphi}(P_0H_0{}^3\frac{\partial P_0}{\partial\varphi})+\frac{\partial}{\partial\lambda}(P_0H_0{}^3\frac{\partial P_0}{\partial\lambda})=$$
$$\Lambda\frac{\partial(P_0H_0)}{\partial\varphi} \qquad (11\text{-}1)$$

The static film thickness H_0 consists of geometrical clearance H_{g0}, which only depends on the static equilibrium position of the journal and the undeformed profile of the top foil, and the clearance $H_{\delta 00}$ due to static deformation of the top foil.

$$H_0=H_{g0}+H_{\delta 00}$$

$$H_{g0} = 1 + \varepsilon_0 \cos\varphi$$

$$\begin{aligned} H_{\delta 00} &= \iint_A P_0(\phi, z) f_t(\phi, z, \varphi, \lambda) d\phi dz \\ &- \iint_{A_{tb}} W_{00}(\phi, z) f_t(\phi, z, \varphi, \lambda) d\phi dz \end{aligned} \quad (11\text{-}2)$$

Calculation of the static deformation $H_{\delta 00}$ is also concerned with the deformation of the bump foil besides the static contact load W_{00}. By using equation (9-2), the deformation coordination condition gives

$$H_{p\delta_0}(\varphi, \lambda) = H_{w\delta_0}(\varphi, \lambda) + H_{b\delta_0}(\varphi, \lambda)$$
$$((\varphi, \lambda) \in A_{tb})$$

or

$$\begin{aligned} &\iint_A P_0(\phi, z) f_t(\phi, z, \varphi, \lambda) d\phi dz = \\ &\iint_{A_{tb}} W_{00}(\phi, z) f_t(\phi, z, \varphi, \lambda) d\phi dz \\ &+ \iint_{A_{tb}} W_{00}(\phi, z) f_b(\phi, z, \varphi, \lambda) d\phi dz \end{aligned} \quad (11\text{-}3)$$

Corresponding to a set of given parameters of $(\varepsilon_0, \theta_0)$, variables P_0, W_{00} and $H_{\delta 00}$ can be obtained by simultaneously solving Eqs. (7), (8) and (11).

4.2 Dynamic Solution of Elasto-Aerodynamic Lubrication

In small perturbation state, the dynamic pressure and gas film thickness distribution can be written as

$$P = P_0 + Q_0 e^{i\Omega T}$$

$$W = W_{00} + \tilde{W}_0 e^{i\Omega T}$$

$$H = H_0 + \tilde{H} = H_0 + \tilde{H}_0 e^{i\Omega T} \quad (12)$$

By using Eqs.(10) and (11) and neglecting all the high-order terms of Q_0 and $\tilde{H}_0$, the dynamic Reynolds equation(10) can be simplified.

$$\frac{\partial}{\partial\varphi}(PH^3 \frac{\partial P}{\partial\varphi}) = \frac{\partial}{\partial\varphi}\left(P_0 H_0^3 \frac{\partial P_0}{\partial\varphi}\right)$$
$$+ \frac{\partial}{\partial\varphi}\left\{\left(H_0^3 \frac{\partial P_0}{\partial\varphi} Q_0 + 3H_0^2 P_0 \frac{\partial P_0}{\partial\varphi}\tilde{H}_0 + P_0 H_0^3 \frac{\partial Q_0}{\partial\varphi}\right) e^{i\Omega T}\right\}$$

$$\frac{\partial}{\partial\lambda}(PH^3 \frac{\partial P}{\partial\lambda}) = \frac{\partial}{\partial\lambda}\left(P_0 H_0^3 \frac{\partial P_0}{\partial\lambda}\right)$$
$$+ \frac{\partial}{\partial\lambda}\left\{\left(H_0^3 \frac{\partial P_0}{\partial\lambda} Q_0 + 3H_0^2 P_0 \frac{\partial P_0}{\partial\lambda}\tilde{H}_0 + P_0 H_0^3 \frac{\partial Q_0}{\partial\lambda}\right) e^{i\Omega T}\right\}$$

$$\Lambda \frac{\partial(PH)}{\partial\varphi} = \Lambda \left\{\frac{\partial(P_0 H_0)}{\partial\varphi} + \left[\frac{\partial}{\partial\varphi}(H_0 Q_0 + P_0 \tilde{H}_0)\right] e^{i\Omega T}\right\}$$

$$2\Lambda \frac{\partial(PH)}{\partial T} = 2i\Lambda\Omega(H_0 Q_0 + P_0 \tilde{H}_0) e^{i\Omega T}$$

The general Reynolds equation in dynamic state for compressible gas film lubrication problem becomes

$$\begin{aligned} &\frac{\partial}{\partial\varphi}(P_0 H_0^3 \frac{\partial Q_0}{\partial\varphi}) + \frac{\partial}{\partial\lambda}(P_0 H_0^3 \frac{\partial Q_0}{\partial\lambda}) + \frac{\partial}{\partial\varphi}(H_0^3 \frac{\partial P_0}{\partial\varphi} Q_0) \\ &+ \frac{\partial}{\partial\lambda}(H_0^3 \frac{\partial P_0}{\partial\lambda} Q_0) + \frac{\partial}{\partial\varphi}(3H_0^2 P_0 \frac{\partial P_0}{\partial\varphi}\tilde{H}_0) \\ &+ \frac{\partial}{\partial\lambda}(3H_0^2 P_0 \frac{\partial P_0}{\partial\lambda}\tilde{H}_0) = \Lambda \frac{\partial}{\partial\varphi}(H_0 Q_0 + P_0 \tilde{H}_0) \\ &+ i2\Lambda\Omega(H_0 Q_0 + P_0 \tilde{H}_0) \end{aligned} \quad (13)$$

where H_0 and P_0 are all static variables known, and the dynamic gas film thickness $\tilde{H}_0$ and pressure $\tilde{Q}_0$ are complex variables to be found, both of them are functions of perturbations E_0 and Θ_0.

Let the dynamic film thickness $\tilde{H}_0$ be

$$\tilde{H}_0 = H_{gd0} + H_{\delta 0} \quad (14)$$

The increment of the film thickness between the journal and undeformed top foil due to small perturbation of the journal has the form as follows.

$$H_{gd0} = (E_0 \cos\varphi + \Theta_0 \varepsilon_0 \sin\varphi) \quad (15)$$

The dynamic deformation of the top foil due to force disturbance can be obtained by using Eq.(7). It follows that

$$H_\delta = H_{\delta 00} + H_{\delta 0} e^{i\Omega T} = \iint (P_0 + Q_0 e^{i\Omega T}) f_t(\phi, z, \varphi, \lambda) d\phi dz$$
$$- \iint (W_{00} + \tilde{W}_0 e^{i\Omega T}) f_t(\phi, z, \varphi, \lambda) d\phi dz$$

and

$$\begin{aligned} H_{\delta 0} &= \iint_A Q_0(\phi, z) f_t(\phi, z, \varphi, \lambda) d\phi dz \\ &- \iint_{A_{tb}} \tilde{W}_0(\phi, z) f_t(\phi, z, \varphi, \lambda) d\phi dz \end{aligned} \quad (16)$$

Similarly for the bump foil, the dynamic deformation due to force disturbance becomes

$$H_{b\delta 0} = \iint_{A_b} \tilde{W}_0(\phi, z) f_b(\phi, z, \varphi, \lambda) d\phi dz \quad (17)$$

and dynamic state, the corresponding deformation coordination condition gives

$$\begin{aligned} &\iint_A Q_0(\phi, z) f_t(\phi, z, \varphi, \lambda) d\phi dz \\ &= \iint_{A_b} \tilde{W}_0(\phi, z) f_t(\phi, z, \varphi, \lambda) d\phi dz \\ &+ \iint_{A_b} \tilde{W}_0(\phi, z) f_b(\phi, z, \varphi, \lambda) d\phi dz \end{aligned} \quad (18)$$

In above equations, all the variables H_{gd0}, $H_{\delta 0}$, $H_{\delta 0}$ and $\tilde{W}_0$ are complex numbers.

5 DYNAMIC STIFFNESS AND DAMPING COEFFICIENTS OF CFB

In Eq.(13), perturbation parameters E_0 and Θ_0 are involved. The partial derivative method can be used to calculate the dynamic stiffness and damping coefficients directly [12]. Let $Q_E = \frac{\partial Q_0}{\partial E}$; $Q_\theta = \frac{1}{\varepsilon_0}\frac{\partial Q_0}{\partial \Theta_0}$; $W_E = \frac{\partial \tilde{W}_0}{\partial E}$; $W_\theta = \frac{1}{\varepsilon_0}\frac{\partial \tilde{W}_0}{\partial \Theta_0}$; $H_E = \frac{\partial \tilde{H}_0}{\partial E}$; $H_\theta = \frac{1}{\varepsilon_0}\frac{\partial \tilde{H}_0}{\partial \Theta_0}$; Derivation of Eq.(13) yields the partial differential equations concerning variables Q_E and Q_θ.

$$\begin{aligned} &\frac{\partial}{\partial \varphi}(P_0 H_0^3 \frac{\partial Q_E}{\partial \varphi}) + \frac{\partial}{\partial \lambda}(P_0 H_0^3 \frac{\partial Q_E}{\partial \lambda}) + \frac{\partial}{\partial \varphi}(H_0^3 \frac{\partial P_0}{\partial \varphi} Q_E) \\ &+ \frac{\partial}{\partial \lambda}(H_0^3 \frac{\partial P_0}{\partial \lambda} Q_E) + 3\Lambda \frac{1}{H_0}\frac{\partial (P_0 H_0)}{\partial \varphi} H_E \\ &+ 3H_0^3 P_0 \left[\frac{\partial P_0}{\partial \varphi}\frac{\partial}{\partial \varphi}\left(\frac{1}{H_0} H_E\right) + \frac{\partial P_0}{\partial \lambda}\frac{\partial}{\partial \lambda}\left(\frac{1}{H_0} H_E\right)\right] \\ &= \Lambda \frac{\partial}{\partial \varphi}(H_0 Q_E + P_0 H_E) + i2\Lambda\Omega(H_0 Q_E + P_0 H_E) \quad (19) \end{aligned}$$

$$H_E = \cos\varphi + \frac{\partial H_{\delta 0}}{\partial E} \quad (20\text{-}1)$$

$$\begin{aligned} \frac{\partial H_{\delta 0}}{\partial E} &= \iint_A Q_E(\phi, z) f_t(\phi, z, \varphi, \lambda) d\phi dz \\ &- \iint_{A_b} W_E(\phi, z) f_t(\phi, z, \varphi, \lambda) d\phi dz \end{aligned} \quad (20-2)$$

$$\begin{aligned} &\iint_A Q_E(\phi, z) f_t(\phi, z, \varphi, \lambda) d\phi dz \\ &= \iint_{A_b} W_E(\phi, z) f_t(\phi, z, \varphi, \lambda) d\phi dz \\ &+ \iint_{A_b} W_E(\phi, z) f_b(\phi, z, \varphi, \lambda) d\phi dz \end{aligned} \quad (20\text{-}3)$$

Q_E can be obtained by simultaneously solving equations (19) and (20).

Similarly, by using Eq.(13), equation concerning variable Q_θ can be expressed as follows.

$$\begin{aligned} &\frac{\partial}{\partial \varphi}(P_0 H_0^3 \frac{\partial Q_\theta}{\partial \varphi}) + \frac{\partial}{\partial \lambda}(P_0 H_0^3 \frac{\partial Q_\theta}{\partial \lambda}) + \frac{\partial}{\partial \varphi}(H_0^3 \frac{\partial P_0}{\partial \varphi} Q_\theta) \\ &+ \frac{\partial}{\partial \lambda}(H_0^3 \frac{\partial P_0}{\partial \lambda} Q_\theta) + 3\Lambda \frac{1}{H_0}\frac{\partial (P_0 H_0)}{\partial \varphi} H_\theta \\ &+ 3H_0^3 P_0 \left[\frac{\partial P_0}{\partial \varphi}\frac{\partial}{\partial \varphi}\left(\frac{1}{H_0} H_\theta\right) + \frac{\partial P_0}{\partial \lambda}\frac{\partial}{\partial \lambda}\left(\frac{1}{H_0} H_\theta\right)\right] \\ &= \Lambda \frac{\partial}{\partial \varphi}(H_0 Q_\theta + P_0 H_\theta) + i2\Lambda\Omega(H_0 Q_\theta + P_0 H_\theta) \quad (21) \end{aligned}$$

$$H_\theta = \sin\varphi + \frac{1}{\varepsilon}\frac{\partial H_{\delta 0}}{\partial \Theta_0} \quad (22\text{-}1)$$

$$\begin{aligned} \frac{1}{\varepsilon}\frac{\partial H_{\delta 0}}{\partial \Theta_0} &= \iint_A Q_\theta(\phi, z) f_t(\phi, z, \varphi, \lambda) d\phi dz \\ &- \iint_{A_b} W_\theta(\phi, z) f_t(\phi, z, \varphi, \lambda) d\phi dz \end{aligned} \quad (22-2)$$

$$\begin{aligned} &\iint_A Q_\theta(\phi, z) f_t(\phi, z, \varphi, \lambda) d\phi dz \\ &= \iint_{A_b} W_\theta(\phi, z) f_t(\phi, z, \varphi, \lambda) d\phi dz \\ &+ \iint_{A_b} W_\theta(\phi, z) f_b(\phi, z, \varphi, \lambda) d\phi dz \end{aligned} \quad (22-3)$$

By simultaneously solving Eqs.(21) and (22), the distribution of Q_θ can be obtained.

When both Q_E and Q_θ are solved, the dynamic stiffness and damping coefficients for a compliant foil bearing can be calculated according to the formulas below.

$$\begin{aligned} -\iint_A Q_E \cos\phi d\phi d\lambda &= K_{y\varepsilon} + i\Omega D_{y\varepsilon} \\ -\iint_A Q_E \sin\phi d\phi d\lambda &= K_{x\varepsilon} + i\Omega D_{x\varepsilon} \\ -\iint_A Q_\theta \cos\phi d\phi d\lambda &= K_{y\theta} + i\Omega D_{y\theta} \\ -\iint_A Q_\theta \sin\phi d\phi d\lambda &= K_{x\theta} + i\Omega D_{x\theta} \end{aligned} \quad (23)$$

These coefficients defined in formulas (23) can be conveniently transfered to that in the Cartesian coordinate system by using a transfer matrix A_T.

$$\begin{bmatrix} K_{xx} \\ K_{xy} \end{bmatrix} = \begin{bmatrix} \sin\theta_0 & \cos\theta_0 \\ \cos\theta_0 & -\sin\theta_0 \end{bmatrix} \begin{bmatrix} K_{x\varepsilon} \\ K_{x\theta} \end{bmatrix} = [A_T] \begin{bmatrix} K_{x\varepsilon} \\ K_{x\theta} \end{bmatrix}$$

$$\begin{bmatrix} K_{yx} \\ K_{yy} \end{bmatrix} = [A_T] \begin{bmatrix} K_{y\varepsilon} \\ K_{y\theta} \end{bmatrix}$$

$$\begin{bmatrix} D_{xx} \\ D_{xy} \end{bmatrix} = [A_T] \begin{bmatrix} D_{x\varepsilon} \\ D_{x\theta} \end{bmatrix} \qquad (24)$$

$$\begin{bmatrix} D_{yx} \\ D_{yy} \end{bmatrix} = [A_T] \begin{bmatrix} D_{y\varepsilon} \\ D_{y\theta} \end{bmatrix}$$

The coefficients $K_{ij}, D_{ij}\,(i,j = x,y)$ defined in formula (24) exactly describe the dynamic characteristics of a compliant foil bearing in small perturbation state.

6 CONCLUSIONS

1) For a compliant foil bearing, general equations of elastic deformation of the top foil and bump foil and compressible gas-lubricated Reynolds equations are deduced, and all the effects of static or dynamic deformations and journal perturbations on the performance of foil bearing are taken into account. These equations are generally available in both static and dynamic states.
2) Connections among the journal perturbation, dynamic gas film thickness and pressure and the surface dynamic deformations of the top foil and the bump foil strips are satisfactorily expressed by introducing terms of dynamic deformations in Reynolds equation and elastic deformation equations, and corresponding theory and method are provided for the solution of a completely aero-elasto coupling lubrication problem of compliant foil bearings.
3) Calculation of dynamic stiffness and damping coefficients for a compliant foil bearing can be completed by solving dynamic Reynolds equation and dynamic deformation equations of flexible structures simultaneously. In small perturbation state, by using the partial derivative method, these rotordynamic coefficients can be obtained conveniently, and these coefficients are related to the parameters of the static operating points of the bearing and the frequency of perturbations only. Theory and methods presented in this paper provide necessary conditions for the dynamic analysis and performance prediction of a rotor system supported by compliant foil bearings in linear range.
4) The present model of the CFB provides a theoretical method for the design of the FMHB.

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