A 2½-D LINEAR MAGNETIC ANALYSIS FOR PARALLEL-AIRGAP RADIAL MAGNETIC BEARINGS

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ABSTRACT

This paper provides methods for investigating the design of a class of radial magnetic bearings in which axial MMF is provided in each one of a set of flat, uniform-thickness circular discs arranged in alternation such that every second disc is mechanically attached to the rotor and the interleaving discs are fixed to the stator. The axial MMF in the rotor discs is a sinusoidal function of angle and is taken to be independent of radius. Similarly, the axial MMF in the stator discs varies sinusoidally with respect to angle and is invariant with respect to radius. Provided that both the numbers of pole-pairs of rotor and stator discs differ by 1, a net transverse force can be produced.

Many questions arise with regard to the design of radial bearings of this configuration. A particular motivation behind these bearings is to achieve a high specific load capacity. This paper sets out a linear magnetic analysis for providing a first-cut approximate assessment of the force which can be generated from such bearings. The analysis proceeds using the assumption that lines of magnetic flux remain in concentric cylinders. Then for each radius, the problem of predicting the distribution of magnetic flux reduces to a 2D problem. This 2D problem is addressed using linear superposition. The flux field due to the MMF on the stator-discs is computed assuming that the rotor-discs contribute no MMF. Then the flux field due to the MMF on the rotor-discs is computed assuming that the stator-discs contribute no MMF. When these two fields are added, it is possible to compute Maxwell stresses at every position in the airgap. The net transverse components of force due to flux in a thin-walled cylindrical volume are computed and then integrated with respect to radius to produce a prediction of the total transverse flux in the bearing.

This paper investigates three different arrangements. In Class I: both rotor and stator discs are considered to be made up of permanent magnet materials and the necessary MMF is generated by the distribution of permanent magnet materials in the discs. Class I is a hypothetical case only since the net force produced between rotors is a known function of the relative rotation of the two components and cannot be controlled. In Class II: the rotor is considered to be made of permanent magnet and stator is wound. In Class III: both rotor and stator are wound. Results have been shown for the Class I & II in this paper.

INTRODUCTION

This paper addresses the magnetic design of a class of high specific load capacity magnetic bearings. Most magnetic bearings deploy magnetic normal stress directly in the production of the bearing force. Typically the ratio of maximum bearing force achievable divided by the total self-weight of the bearing for these designs is less than 100:1 and most usually it is more like 40:1 [1]. This ratio is called the *Specific Load Capacity*. A different concept for magnetic bearings has been put forward [2] in which the bearing force is generated as a result of the integration of magnetic shear stress over airgap area. This concept provides for the construction of magnetic bearings in which the same lines of magnetic flux can cut a large number of airgaps and can generate significant net shear forces at each one. Figure 1 illustrates schematically a set of circular discs comprising a bearing from this class. Such bearings are implicitly *radial* magnetic bearings since shear-forces at the airgaps between discs must necessarily produce resultant forces transverse to the rotor and stator axes. There are *Parallel-Airgap Serial Flux Magnetic Bearings* (PASFMBs) which can generate axial forces and these necessarily comprise interleavements of cylinders. The analysis presented in this paper is primarily directed at *Radial* PASFMBs for rotating machines and although some results can be carried over to the analysis of axial PASFMBs, this is not discussed explicitly.

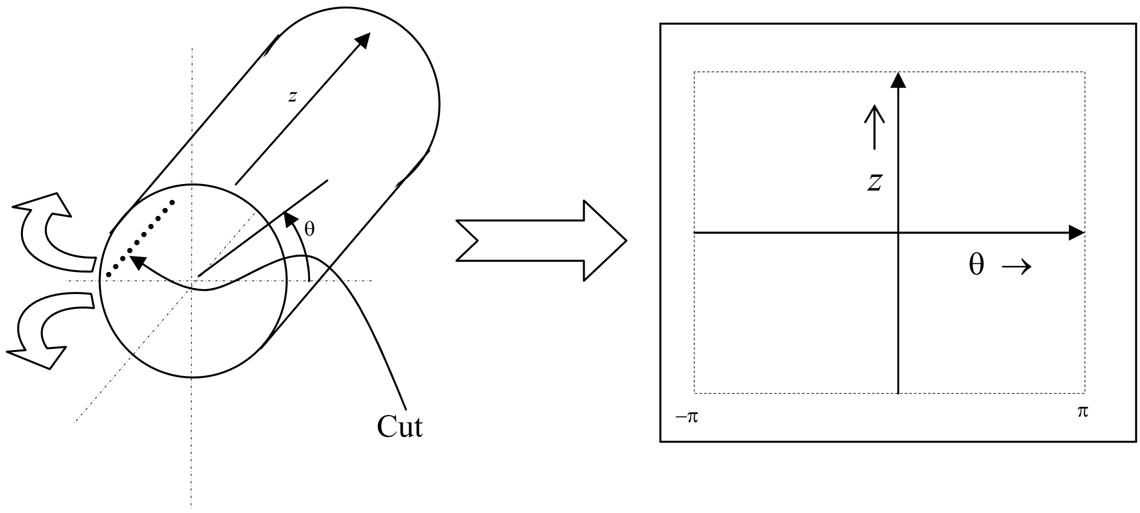


FIGURE 1. Performing 2D FEA on Axial MMF Radial A.M.Bs

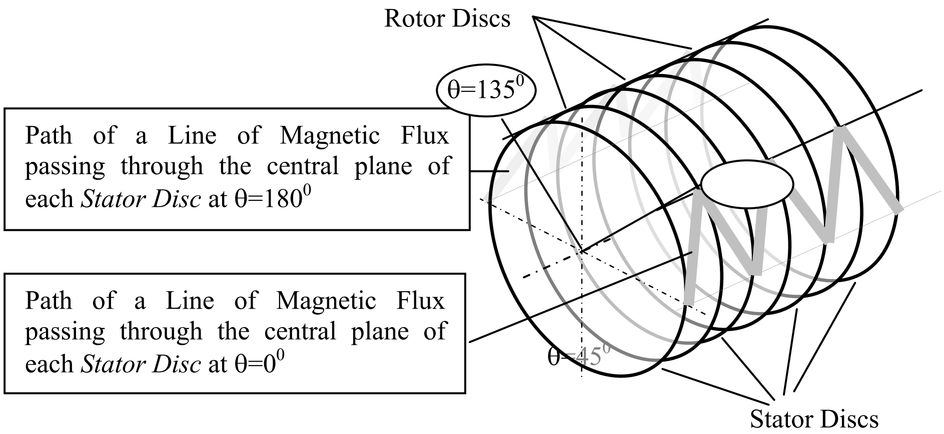


FIGURE 2: Axial MMF radial active magnetic bearing

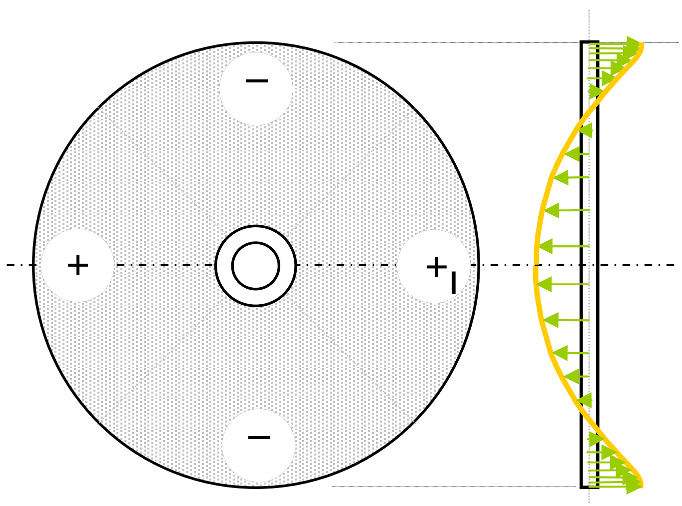


FIGURE 3: A 4-pole MMF field

The effective design of any PASFMB inevitably involves a full 3D finite element analysis at the final stages to address simultaneously mechanical, magnetic, heat-transfer and fluid-mechanical concerns. However, it is appropriate to have some simpler approximate analysis methods so that the designer can at least determine a design which will be sufficiently close to optimal for his purposes that only small adjustments are necessary in response to the findings of the 3D FEA.

However, to a reasonable approximation, it may be assumed that any flux line remains on a cylinder of constant radius concentric with the axis of the bearing, definitely if distance between poles is short compared with radial span. With this assumption in place, it is possible to perform a 2D analysis by "developing the cylinder" onto a plane and imposing suitable periodicity boundary conditions at the "cut" edges as Fig. 1 indicates.

THE MULTI-POLE AXIAL-MMF RADIAL PASFMBS

The PASFMBs themselves divide into multiple classes depending on how the magnetic shear stresses in each of the airgaps are achieved. For example, one class is a class of reluctance bearings wherein the direction of the bearing force is determined completely by the relative misalignment of the rotor and stator axes. Such bearings are clearly passive bearings. The MMFs in the rotor and stator discs of these bearings are all homopolar (at least, with respect to angle).

The present paper is interested in high specific load capacity bearings in a different class which might be described as *Multi-Pole, Axial-MMF, Radial*. They are Axial-MMF because each disc in the bearing has a net axial MMF at every point (r, θ). It is this axial MMF which is responsible for generating the bearing forces. Any other components of MMF (due, for example, to coils not lying perfectly flat within the discs) are ignored here. The bearing of interest is *multi-pole* because the axial MMF varies with θ. In each disc, there is at least one complete reversal of axial MMF between $\theta = 0$ and $\theta = 2\pi$. It is assumed here that the axial MMF is independent of radius, r over a range of radii.

Every second disc in the stack is mechanically connected to a platform called the *stator*. The details of the mechanical connection are not relevant here but for the sake of visualizing the bearing, it is useful to think of every *stator-disc* being fixed at its outside diameter to the inside of a single cylinder which is held in position by exterior supports. The first and last of the *stator-discs* are different from the others insofar as they must provide for completion of the magnetic circuit. Every aspect of the completion of the magnetic circuit is ignored here.

The stator discs each have axial MMF, $M_S(r, \theta)$ given by

$$M_S(r, \theta) = K_S.\cos(N_S\theta + \phi_S) \quad \text{for all} \quad R_I < r < R_O \quad (1)$$

Where R_I and R_O are, respectively, the inner and outer radial limits of the working area of the stator discs. The stator discs all have constant thickness, $2t_S$. They are also all assumed to have an equivalent (non-directional) effective absolute permeability of μ_S. The phase angle, ϕ_S, remains always at 0 if the MMF in the stator discs is achieved by a distribution of permanent magnet material. More usually, however, it is expected that the MMF in the stator discs is achieved by a distribution of current-carrying conductors within the stator discs (very much like the motor windings in the disc of so-called *pie-shaped* motors such as [3]).

Every other disc in the stack (i.e. every disc that is not a *stator-disc*) is mechanically connected to a platform called the *rotor*. Again, the details of the mechanical connection are not relevant here but for the sake of visualizing the bearing, it is useful to think of every *rotor-disc* being fixed at their inside diameter to the outside of a single cylinder which is mechanically integral with the rotor of a rotating machine.

The rotor discs each have axial MMF, $M_R(r, \theta)$ given by

$$M_R(r, \theta) = K_R.\cos(N_R\theta + \phi_R) \quad \text{for all} \quad R_I < r < R_O \quad (2)$$

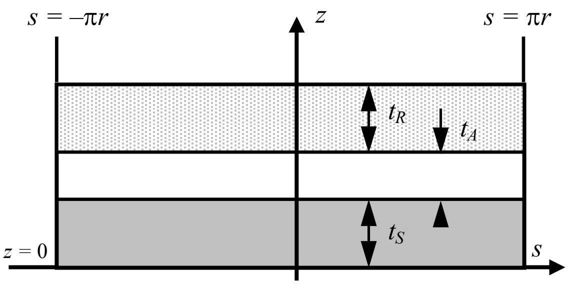


FIGURE 4: The 2-D region analyses

Accepting that there is only one axis in the bearing, it is assumed that all lines of magnetic flux remain at constant radius from the axis of the bearing.

This assumption decouples the task of predicting the full 3-D flux field into the problem of predicting the 2-D flux field in every cylinder of arbitrary radius r with $R_I < r < R_O$.

The flux-field in each cylinder (of radius r) is most easily visualized by making a *cut* in the cylinder along the generator $\theta = \pi = -\pi$ and then flattening-out the cut cylinder to produce a flux-plot in the coordinates s, z where

$$s = r\,\theta \quad (3)$$

Evidently, the flux field must be periodic w.r.t. θ since $\theta = -\pi$ represents precisely the same line in space as $\theta = \pi$ and $\theta = 3\pi$ etc.. It is quickly realized that the flux field is also periodic w.r.t. z with a period of $2(t_S + t_A + t_R)$ except very close to the ends of the stack of discs. This distance is termed as 'Zp'.

Here t_S, t_A and t_R are the half of the stator thickness, airgap thickness and half of the rotor thickness respectively. These end effects are ignored here.

The boundary conditions applicable are as follows:

- Full (2π) periodicity of flux in the s direction between the limits ($s = -\pi r$) and ($s = \pi r$)
- Normal flux conditions at the limits of z ($z = -(t_S + t_A + t_R)$) and ($z = (t_S + t_A + t_R)$)

The latter normal flux condition is a consequence of symmetry and periodicity. In fact, a simpler analysis is possible since the flux field is symmetric about every central plane of every stator disc and also about every central plane of every rotor disc. Hence the region which must actually analysed is that of Fig. 4 with the limits of z being ($z = 0$) and ($z = (t_S + t_A + t_R)$) and with normal flux conditions at these boundaries.

LINEAR ANALYSIS, SUPERPOSITION AND THE CURRENT DISTRIBUTION

There are three different areas in the region analysed. These are:

- a half-stator-disc ($-r\pi \le s \le r\pi$), ($0 \le z \le t_S$).
- an airgap ($-r\pi \le s \le r\pi$), ($t_A \le z \le (t_S + t_A)$).
- a half-rotor-disc ($-r\pi \le s \le r\pi$), ($(t_S + t_A) \le z \le (t_S + t_A + t_R)$).

There is a source of MMF in both the half-stator disc and the half-rotor disc. Since all three regions are treated here as being perfectly linear in magnetic characteristics, the total flux (vector) field $\boldsymbol{B}_T(s, z)$ will be treated as the sum of the rotor flux (vector) field, $\boldsymbol{B}_R(s, z)$, arising from the rotor excitation alone and the stator flux (vector) field $\boldsymbol{B}_S(s, z)$ arising from the stator excitation alone. Thus

$$\boldsymbol{B}_T(s, z) = \boldsymbol{B}_S(s, z) + \boldsymbol{B}_R(s, z) \qquad (4)$$

The problem of predicting $\boldsymbol{B}_T(s, z)$ splits into two problems of identical construction: prediction of $\boldsymbol{B}_R(s, z)$ and prediction of $\boldsymbol{B}_S(s, z)$.

For simplicity, the excitations in the rotor and stator half-discs are always considered to be derived from an equivalent current distribution. Within the stator half-disc, the current density is independent of z and it is denoted $J_S(r, s)$. Similarly, within the rotor half-disc, the current density is independent of z and this is denoted $J_R(r, s)$.

From a symmetry argument, it emerges that if the net stator MMF at every (r, s) is given by (1), then the current density, J_S, at every point in the half-stator disc must be given by

$$J_S(s) = \left(\frac{K_S N_S}{2t_S.r}\right).\sin\left(N_S\left(\frac{s}{r}\right) + \phi_S\right) \qquad (5)$$

The veracity of this equation can be confirmed from

$$\frac{\mathrm{d}M_S(r,s)}{\mathrm{d}s}.\Delta s = -2t_S.J_S(r,s).\Delta s \qquad (6)$$

THE AIRGAP SHEAR STRESS AND TOTAL FORCE

With $\boldsymbol{B}_S(s, z)$ and $\boldsymbol{B}_R(s, z)$ both known, the total flux, $\boldsymbol{B}_T(s, z)$, is found by vector addition at every point (s, z). The total force acting on a given rotor disc is the sum of two identical forces – one on each side. With the "horizontal" direction being defined by the directed line $\theta = 0$ and with the "vertical" direction being defined as $\theta = \pi/2$. The horizontal and vertical forces acting on each side of one rotor disc are given by

$$F_H = \int_{R_I}^{R_O}\int_{-\pi}^{\pi}\left(\tau_{rz}(r,\theta).\cos(\theta) - \tau_{\theta z}(r,\theta).\sin(\theta)\right).d\theta.r.dr \qquad (7)\ \&\ (8)$$

$$F_V = \int_{R_I}^{R_O}\int_{-\pi}^{\pi}\left(\tau_{rz}(r,\theta).\sin(\theta) + \tau_{\theta z}(r,\theta).\cos(\theta)\right).d\theta.r.dr$$

where $\tau_{rz}(r,\theta)$ and $\tau_{\theta z}(r,\theta)$ are magnetic shear stresses in the airgap. The assumption that flux remains in cylinders means that in any plane of constant θ cutting through the magnetic bearing, the flux lines cross the airgap perfectly normal to the rotor and stator discs. For the purposes of computing force, therefore, $\tau_{rz}(r,\theta)$ is not considered to contribute and attention is focused explicitly on $\tau_{\theta z}(r,\theta)$. This stress is computed as

$$\tau_{\theta z} = (B_s B_z)/\mu_0 \qquad (9)$$

where B_s and B_z are the components of $\boldsymbol{B}_T(s, z)$ at the center of the airgap ($z = (t_S + t_A/2)$).

Once the horizontal and vertical forces acting on each side of one rotor disc are known, the Mean effective magnetic shear stress (MEMSS) is calculated by dividing net force by the airgap area. Finally the specific load capacity is calculated.

RESULTS AND DISCUSSIONS

Based on the method of achieving MMF in the rotor disc and in the stator disc this type of bearing can be divided into three different classes. In class I: both rotor and stator discs are considered to be made up of permanent magnet and the necessary MMF is achieved by the distribution of permanent magnet materials in the discs. Class I is a hypothetical design only since the net force produced between rotors is a known function of the relative rotation of the two components and cannot be controlled. In class II: the rotor is made of permanent magnet and the required MMF is achieved by the distribution of permanent magnet materials and the stator is wound and required MMF is achieved by the distribution of current carrying conductors. In class III: the rotor and the stator are wound and the required MMF is achieved by the distribution of current carrying conductors. In this paper results have been shown for the Class I and II.

Class I:

In the present analysis of this class of bearing the design parameters are airgap thickness, inside radius of the stator/or rotor, outside radius of the stator/or rotor, stator disc thickness, rotor disc thickness, and the number of pole pairs in the rotor/or stator disc.

Figure 5 shows the MEMSS versus (*Zp*/IPD) for different *Zp* value when the airgap thickness is 0.4 mm. The term *Zp* is already defined. IPD is the inter pole distance of the stator pole and calculated as

$$\mathbf{IPD} = \frac{2\pi}{2N_s} R_{mean} \tag{10}$$

where N_S is the number of stator pole pairs and R_{mean} is the mean radius of the bearing.

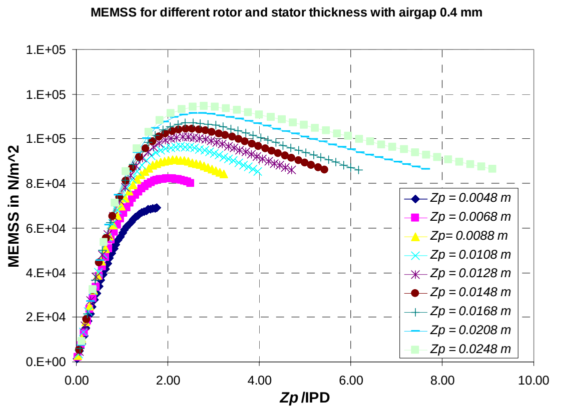


FIGURE 5: Mean Effective Magnetic Shear Stress (MEMSS) for different stator and rotor thickness when the airgap thickness is 0.4 mm; inside and outside radii of the stator/or rotor are100 mm and 160 mm respectively.

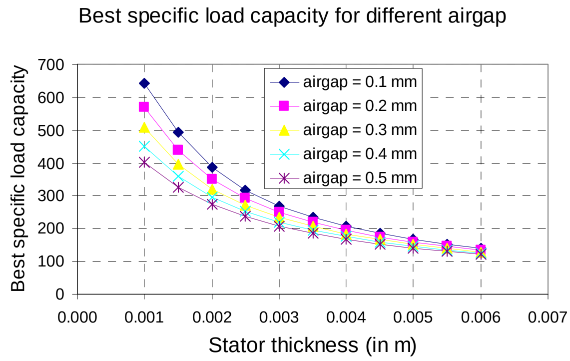
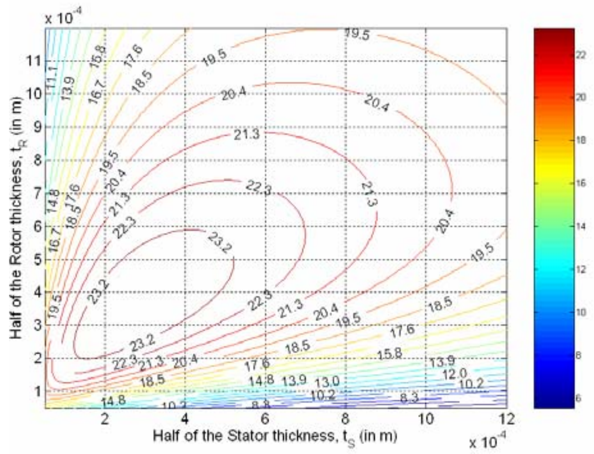


FIGURE 6: Specific load capacity for different airgap thicknesses; inside and outside radii of the stator/or rotor are100 mm and 160 mm respectively

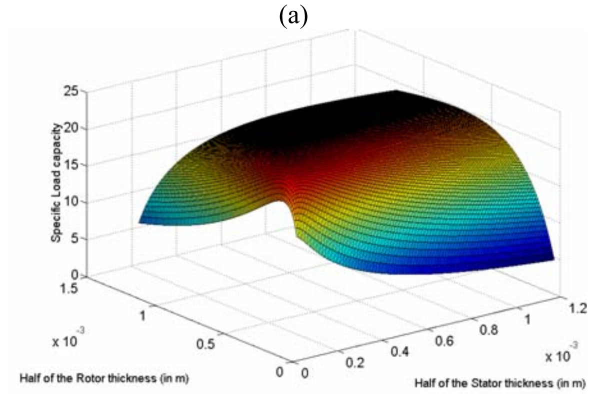
Figure 6 shows the specific load capacity against the stator thicknesses. Stator disc thickness is kept equal to the rotor disc thickness while calculating the specific load capacity. Specific load capacity is shown for five different airgaps ranging from 0.1mm to 0.5 mm. Maximum specific load capacity is 640:1 and obtained when the airgap is 0.1 mm and stator thickness is 1 mm.

Class II:

In the class II design, the required MMF is achieved in the stator disc by the distribution of current carrying conductors. In the analysis of this class of bearing we define a term called reference power dissipation density (Reference PDD) in the stator disc.



(a)



(b)

FIGURE 7(a) and 7(b): Optimum region of specific load capacity. Airgap thickness, t_A = 0.4 mm N_R = 50, N_S = 51, S (Fraction of Cu in the Stator Disc) = 0.7, R_I = 160 mm, R_O = 200 mm.

The reference power dissipation density is calculated as $J_{Sref}{}^2 (2t_{Sref})\rho_{cu}$. Where J_{Sref} is the reference current density in the stator coil, t_{Sref} is the half of the stator disc used as reference while calculating the reference power dissipation density (PDD) and ρ_{cu} is the resistivity of copper. The J_{Sref} and $2t_S$ are considered as 10 A/mm^2 and 10 mm respectively. Specific load capacities are calculated for different value of power dissipation densities.

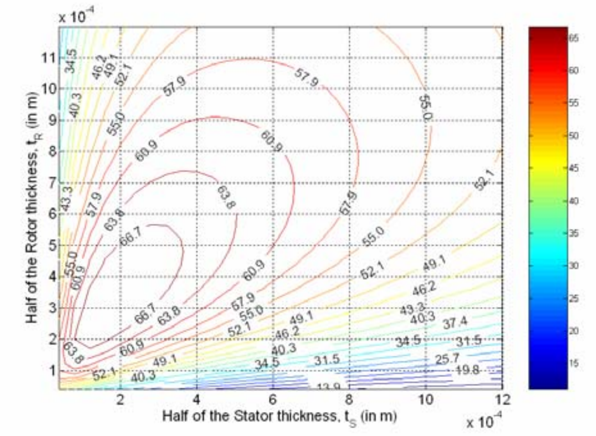
There will be some percentage of iron in the stator disc to drive the most of the flux. The optimum value of the fraction of copper in the stator disc is found to be 0.7 in this case. This fraction is denoted by *s*. The current density for any stator thickness in the analysis is calculated as

$$J_s = \sqrt{\frac{\text{Reference PDD x s}}{2\rho_{cu} t_S}} \tag{11}$$

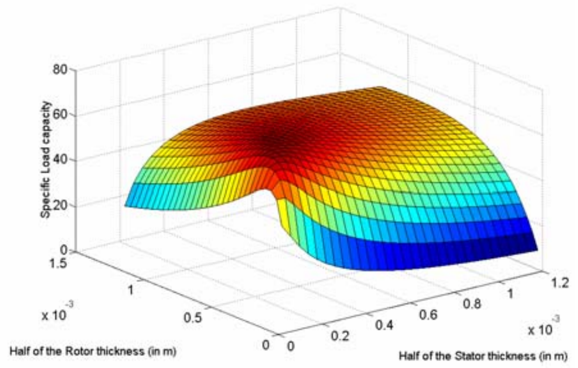
where *s* is the fraction of the copper in the stator disc and t_S is the half of the stator thickness.

In the present analysis of this class of bearing the design parameters are power dissipation density, airgap thickness, inside radius of the stator/or rotor, outside radius of the stator/or rotor, stator disc thickness, rotor disc thickness, fraction of copper in the stator disc and the number of pole pairs in the rotor/or stator disc.

Specific load capacity is analysed for different combinations of the design parameters covering a wide range of space for all the parameters. The optimum value of the fraction of copper in the stator disc is found to be 0.7 and the specific load capacity is optimum once the number of rotor pole pairs reaches 50. Specific load capacities are shown in figure 7(a) and 7(b) with varying stator and rotor disc thickness, when $N_R = 50$ ($N_S = 51$) and S = 0.7. The airgap thickness is 0.4 mm and power dissipation density is equal to the reference PDD. The maximum specific load capacity in this case is 23:1.



(a)



(b)

FIGURE 8(a) and 8(b): Optimum region of specific load capacity. Airgap thickness, t_A = 0.4 mm N_R = 50, N_S = 51, S (fraction of Cu in the Stator Disc) = 0.7, R_I = 160 mm, R_O = 200 mm.

For the same airgap, the number of rotor pole pairs and the fraction of copper in the stator disc, figure 8(a) and 8(b) show the variation of specific load capacity with stator and rotor disc thickness. The power dissipation density is equal to 10 times the reference PDD for this case. The maximum specific load capacity in this case is 66:1.

The specific load capacity versus power dissipation density is shown in figure 9.

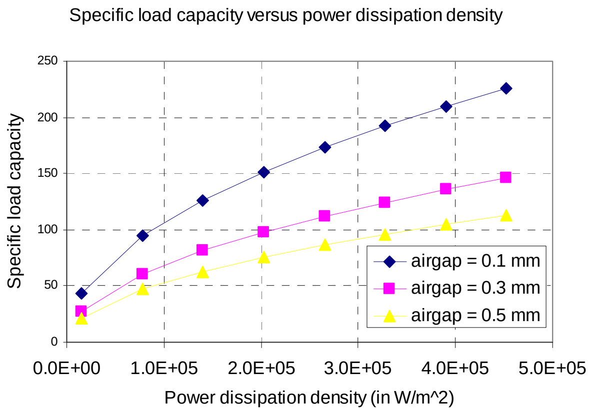


FIGURE 9: Variation of specific load capacity with power dissipation density for different airgap thicknesses

CONCLUSIONS

This paper describes the magnetic design of a Multi-Pole, Axial-MMF, Radial magnetic bearing. It investigates the optimum designs of class I, where rotor and stator discs are made up of permanent magnet and class II, where rotor is made of permanent and stator is wound, for maximum specific load capacity. Work is ongoing but the indications so far are that SLC values will be low.

ACKNOWLEDGEMENT

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