NEW SYSTEM IDENTIFICATION SCHEME OF AMB ROTOR SYSTEM CONSIDERING SENSOR AND ACTUATOR DYNAMICS

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ABSTRACT

This paper presents an improved identification scheme of AMB rotor systems including sensor and actuator dynamics. First, this paper discusses the necessity and synthesis of the fictitious proportional feedback gain (FPFG). The identification error due to over-parametrization of the MIMO system via a SIMO modeling is minimized using an optimal FPFG. Second, the identification procedure becomes simple and efficient through estimating system poles and zeros simultaneously. Third, more improvement in the identification is achieved through separate parameterization of the additional dynamics. The identification performance is verified with several simulations. Finally, the proposed identification scheme is compared with previous identification methods using experimental data.

INTRODUCTION

AMB systems have been widely applied to exploit their unique advantages, including non-contact, lubricant-free operation, high rotational speed, and flexibility of the bearing characteristics. AMB rotor systems always require feedback control of the magnetic force. However, the dependence of magnetic force on the control variables is intrinsically nonlinear so an approximate linearized model, valid near an operating point, is used in design of the associated controller. As a result of the approximate nature of the model, identification of the actual system is essential for a high performance controller.

Figure 1 shows the SISO pole-zero map of a flexible AMB rotor system. The flexible AMB rotor system contains three different dynamics: *additional dynamics* (real poles and zeros far from the imaginary axis, typically due to AMB electronics), *rigid modes* (real poles symmetric to the imaginary axis due to magnetic effects) and *flexible modes* (lightly damped complex poles and zeros symmetric to the real axis, primarily due to structural flexibility). In particular, it is very difficult to identify rigid and flexible modes together when the first flexible mode is low in frequency. Moreover, the additional dynamics makes it more difficult to identify rigid modes.

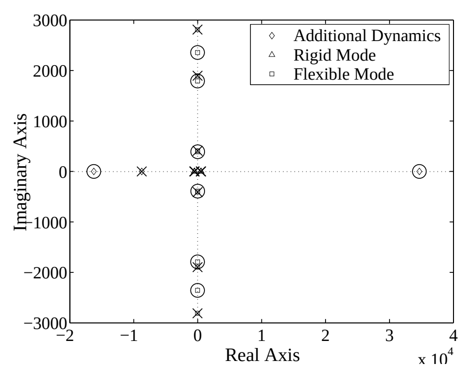


Figure 1: Pole-zero map of a AMB rotor system

Gahler et al. [1] introduced a FPFG and identified poles and zeroes of the closed-loop system sequentially; the resulting SIMO model were then converted to a MIMO model via singular value decomposition (SVD). However, the paper provided no discussion of criteria for choosing the FPFG that might affect identification results significantly. Moreover, it is very hard to obtain a good match in low frequency responses (especially off-diagonal elements) since some individual good curve fittings are lost due to the SVD procedure. Ahn et al. [2] performed identification of a MIMO AMB rigid rotor system through a MIMO parametrization. Although separate identification procedure revealed that additional dynamics greatly affect the identification performance, this method is is only effective for relatively rigid rotors where the rigid modes are easily separated from flexible modes. In addition, there was no clear criterion in selecting the frequency point where rigid modes are split from flexible modes.

This paper presents an improved identification scheme for AMB rotor systems considering sensor and actuator dynamics. The proposed identification scheme has the following advantages: First, the identification error due to over-parametrization of the MIMO system via a SIMO modeling is minimized using an optimal FPFG. Second, the identification procedure becomes simple and efficient through estimating system poles and zeros simultaneously. Third, the additional dynamics are parameterized separately considering their characteristics.

IDENTIFICATION OF AN AMB ROTOR SYSTEM

Frequency domain MIMO identification

A commonly used frequency domain identification procedure for a MIMO system is as follows: (a) measurement of ETFE (empirical transfer function estimate) from input and output signals during operation (b) estimation of SIMO polynomial transfer function through curve-fitting [3, 4]. (c) The desired order model is obtained with SVD based on prior knowledge of the system [1].

The SIMO model of $m \times n$ transfer function matrix for the second procedure is shown in (1). The residual error depends on the selection of model order and the order is generally chosen as the lowest order without significant degradation of residual error.

$$G(s) = \sum_{r=1}^{n} \frac{R_r}{s^2 + 2\delta_r \omega_r s + \omega_r^2} = \frac{[N(s)]}{d(s)} \tag{1}$$

Here the dyadic product R_r is $\varphi_r \cdot \psi_r^T$ and $[N(s)]$ is numerator polynomial matrix.

It is necessary to transform the curve-fitted SIMO model into a MIMO model. The residual R_r can be calculated approximately from

$$R_r \simeq (s^2 + 2\delta_r \omega_r s + \omega_r^2) \cdot G(s)|_{s=-\delta_r \omega_r \pm j\omega_r \sqrt{1-\delta_r^2}} \tag{2}$$

The rank of the calculated residual is usually not "1" because of the over-parametrization of the MIMO system via the SIMO model. Therefore, the residual matrix of the r^{th} mode $R_r^{(n)}$ whose rank is usually n is truncated as optimal rank one approximation $R_r^{(1)}$ through SVD as shown in (3).

$$R_r^{(n)} = U_r \cdot \Sigma_r \cdot V_r^T \approx R_r^{(1)} = u_{r,1} \cdot \sigma_{r,1} \cdot v_{r,1} \tag{3}$$

Here, U_r and V_r are orthogonal matrices with columns $u_{r,i}$, $v_{r,i}$. Σ_r is diagonal and contains the singular values $\sigma_{r,i}$ with descending magnitudes.

Necessity of a FPFG

Role of a FPFG is explained using a simple SISO AMB system that consists of one rigid and one flexible modes. The transfer function of the SISO AMB system $G(s)$ excluding the additional dynamics can be expressed by the pole and residual of each mode as shown in (4).

$$G(s) = \frac{R_1}{s^2 + 2ds - p^2} + \frac{R_2}{s^2 + 2\delta\omega s + \omega^2} \tag{4}$$

The FPFG is a linear transformation that turns real poles of the rigid modes into complex conjugate poles like flexible modes, which boosts visibility of the rigid modes in the FRF[1]. The characteristic equation of the closed-loop system G_{cl} produced by the FPFG, K, is given by

$$(s^2+2ds-p^2+R_1K)(s^2+2\delta\omega s+\omega^2+R_2K)-R_1R_2K^2 = 0 \tag{5}$$

The pole movement can be described using the sign of the characteristic equation at zero frequency. The characteristic equation at zero frequency can be simplified as

$$K(\frac{R_1}{p^2} - \frac{R_2}{\omega^2}) - 1 = 0 \tag{6}$$

If the equation (6) is less than zero, the closed-loop system still has real poles of the rigid mode. As the FPFG increases, the real poles of closed-loop system approach to the imaginary axis. Then, if the equation (6) is an equality, the poles of the rigid mode are located at the origin (assuming that $d = 0$), which is the transitional point from real to complex conjugate poles. In this condition, the characteristic equation is singular and a pole-zero cancellation doesn't happen. Therefore, the numerical condition becomes poor and the calculation result is not accurate any more. When the left hand side of the equation (6) becomes less than zero due to increase of the FPFG, the real poles of the rigid mode are converted into complex conjugate poles. Although the visibility of the poles of the rigid modes poles is improved as the FPFG increases, the complex conjugate poles of the flexible mode are also affected by the FPFG.

Over-parametrization error

Residuals of the curve-fitted SIMO model are approximated through the maximum singular value. The SVD procedure reveals identification error of the curve-fitting procedure due to over-parametrization of the MIMO system via a SIMO system. This over-parametrization error is the main identification error considering the accuracy of the curve-fitting procedure.

Additional dynamics

The measured ETFE of an AMB rotor system includes not only rigid and flexible modes but also the additional dynamics including sensor, actuators, and discretization. These additional dynamics have different characteristics from rigid and flexible modes, that is, they affect all transfer function matrix equally. Therefore, big error could occur by under-parameterizing these additional dynamics during SVD procedure.

NEW IDENTIFICATION SCHEME

Synthesis of optimal FPFG

Minimization of over-parametrization error

The FPFG transforms the real poles of rigid modes into complex conjugate poles in order to improve the visibility of the rigid modes. However, the identification error of still remains and finally results in the over-parametrization error during the SVD procedure. In particular, the responses of rigid modes at low frequency are very big and their identification errors are considerable.

The idea is as follows. First, responses at zero frequency are used to represent the rigid modes without the effect of the both additional dynamics and flexible modes. Second, the FPFG is determined to minimize the over-parameterization error of rigid modes due to SIMO modeling. The over-parameterization errorcan be minimized through reducing the rank one approximation error of the virtual closed-loop response at zero frequency. However, the relative rank one approximation error can be represented by $\sigma_2(G_{cl})/\sigma_1(G_{cl})$, which is the inverse of the condition number if $\mathrm{rank}(G_{cl})$ is 2. Therefore, as the relative rank one approximation error decreases, the condition number increases and the identification error increases significantly due to ill-conditioned denominator matrix.

Cost function

A trade-off between conflicted two requirements is inevitable in choosing an apropos FPFG: ***FPFG should be close to the transitional point*** in order both to minimize the relative optimal rank one approximation error and to reduce the changes of the poles of the flexible modes. ***Condition number of the denominator at zero frequency*** should not be too large in order to maintain good numerical condition. Therefore, a cost function is proposed as follows considering the points above. An optimal gain can be determined through a simple search method near the transitional point.

$$J = \arg\min_k \left[\frac{k}{k_t} + \left(\frac{\mathrm{cond}(D(0, k_t \cdot I))}{\mathrm{cond}(D(0, k \cdot I))} \right)^m \right] \; (|k| > |k_t|) \tag{7}$$

Here, k_t is the largest real scalar value to satisfy $\det(D(0, k \cdot I)) = \det(I + G^R(0) \cdot k_t \cdot I) = 0$ and m is a weighting power for the numerical condition.

The k_t can be computed through solving a simple second order equation, $\det(\hat{G}^R(0)) \cdot k_t^2 + \mathrm{tr}(\hat{G}^R(0)) \cdot k_t + 1 = 0$ if G^R=real(G). In addition, the condition number is not infinite even at the transitional point since the k_t is computed ignoring the imaginary part.

Parametrization of the additional dynamics

Since the additional dynamics affects all transfer function matrix equally, the additional dynamics are separated and parameterized by

$$G_{total}(s) = G_{add}(s) \cdot G(s) \tag{8}$$

Here, $G_{add}(s)$ is a transfer function of additional dynamics, and $G(s)$ denotes a MIMO AMB rotor system of rigid and flexible modes.

The order of the G_{add} is automatically determined since the order is generally chosen as the lowest order without significant degradation of residual error during the curve-fitting procedure. After curve-fitting of a SIMO model, poles and zeroes of the additional dynamics can be easily distinguished from those of rigid and flexible modes, since their poles and zeros are far from the imaginary axis.

Robust estimation of zero frequency response

The zero frequency response should be estimated accurately because it directly affects identification quality. Zero frequency response is estimated using non-zero frequency responses near zero frequency since zero frequency response cannot be measured directly. During the estimation of the zero frequency response, noise effect can be reduced through a proper weighting function like (9).

$$\hat{G}(\omega_0) = \frac{1}{\sum_k w_k} \sum_k w_k G(\omega_k) \tag{9}$$

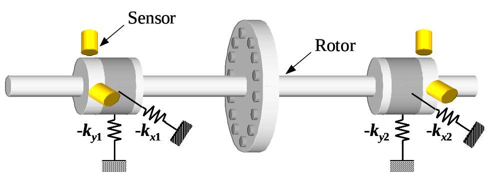


Figure 2: Schematic of the flexible rotor system

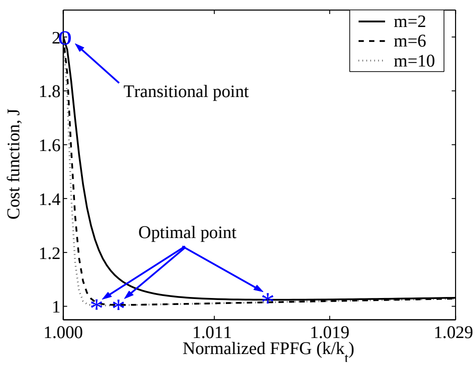
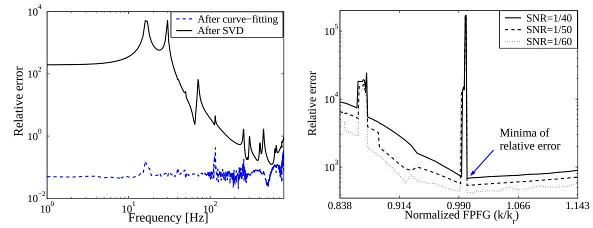


Figure 3: The Proposed cost function values



(a) Error due to SVD (b) Noise effect

Figure 4: Simulation results

Here, $G(\omega_0)$ is the response in zero frequency, $w_k = W_\gamma(\omega_k - \omega_0)$ that is a modified frequency for window function W_γ.

SIMULATION

The performance of the proposed identification scheme is tested through simulations of a theoretical flexible AMB rotor system. The flexible AMB rotor system consists of two AMBs, two sensors and a rotor as shown in Fig. 2. The frequency responses including white noise are produced to mimic the real situation closely. Measured frequency response of a test rig is used for actuator while a theoretical model is used for sensor. In addition, the discretization effect is ignored.

Cost function values are calculated with increasing the FPFG and the weighting power m, as shown in Fig. 3. The optimal point can be adjusted through changing the weighting power considering the numerical condition.

Identification of the flexible AMB rotor system is performed using the proposed scheme and optimal FPFG. Then, relative identification error at each step are shown in Fig.4 (a). The over-parametrization error after the SVD procedure is still much bigger than the error after the curve-fitting procedure although the optimal gain minimizes the over-parametrization error. Relative identification errors are calculated with various FPFG and SNR. The optimal FPFG doesn't change regardless of the SNR values as shown in Fig. 4 (b). There are two peaks of identification errors, which indicates two transitional points where two real poles of rigid modes are moved into the origin, respectively.

COMPARISON WITH PREVIOUS METHODS USING EXPERIMENTAL DATA

There are two previous identification methods for AMB rotor systems: Gahler's and Ahn's methods. The proposed identification scheme is compared with two previous methods using measured frequency responses. Frequency responses of AMB rigid and flexible rotor systems are used to represent two kinds of typical AMB rotor systems. In addition, both cases of including and excluding the additional dynamics are identified to investigate the effect of the additional dynamics. The identification performances are evaluated using two indies: absolute and relative errors as follows.

$$E_{ABS} = ||G - \hat{G}||_F, \; E_{REL} = ||G - \hat{G}||_F / ||G||_F \quad (10)$$

Three identification methods are summarized in Fig. 5. As shown in Fig. 5, the proposed identification scheme minimizes the SVD error through both FPFG synthesis and separate parametrization of the additional dynamics, and also simplifies the identification procedure by identifying the system poles and zeros at a time. The order of the identified model is different depending on identification methods and the additional dynamics. In case of Gahler's method, the FPFG is selected through trial and error since the identification error depends on not only identification of poles, but also identification of zeroes.

Flexible AMB rotor system

Since Ahn's method cannot be applied to a flexible AMB rotor system, the proposed scheme is compared only with Gahler's method. Figure 6 shows

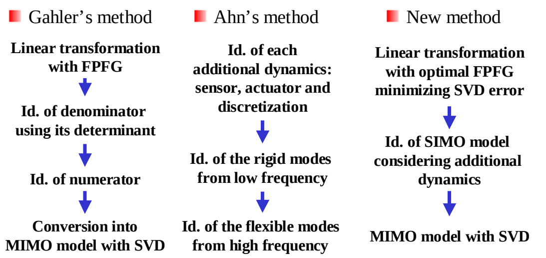


Figure 5: Summary of identification methods

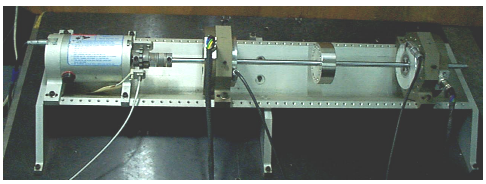
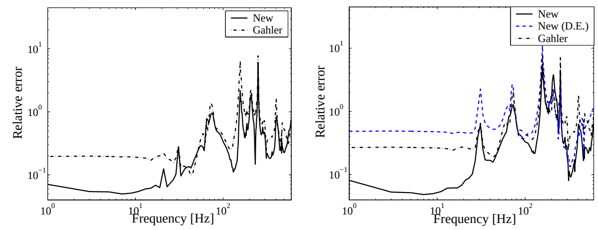


Figure 6: Flexible rotor system



(a) W/O add. dynamics (b) With add. dynamics

Figure 7: Identification errors of the flexible rotor

Table 1: Identification errors of the flexible rotor

Error	Add. dyn.	New (D.E.)	Gahler
Rel.	X	25.37	39.97
	O	26.46(47.67)	49.30
Abs.	X	9.28	10.64
	O	11.40(26.63)	16.92

the AMB flexible rotor system. The test rig consists of two AMBs, two cylindrical capacitive sensors and an flexible rotor, and its specifications are the same as the simulation model. Frequency responses are estimated from closed-loop responses after the system is stabilized with a PID control (10kHz sampling) [2]. Single sinusoidal excitation is performed from 1 Hz to 599 Hz by 2 Hz step. Gyroscopic effect is ignored since the frequency responses are measured without rotation.

Excluding the additional dynamics

The additional dynamics are identified and separated from the measured frequency responses. Then, identifications are performed using the proposed scheme and Gahler's method. The order of the identified model is 10 for both methods. Relative identification errors are shown in Fig. 7 (a) and sum of the identification errors are summarized in Table 1. As shown in Fig. 7 (a) and Table 1, the proposed scheme shows better results than Gahler's method. In case of Gahler's method, system poles are identified using determinant of frequency responses and then system zeroes are identified using identified system poles. Therefore, it is very hard to minimize the identification error through an optimization.

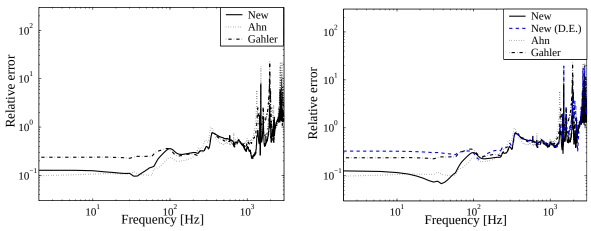
Including the additional dynamics

First, the system with a small additional dynamics is identified after separating the discretization effect. Then, the system that includes discretization effect is identified in order to compare identification performance in case of a large additional dynamics. In case of excluding the discretization effect, identified models with the proposed scheme and Gahler's method are 16^{th} and 10^{th} order, respectively. The identified model order with Gahler's method is low since the SVD procedure is applied to the additional dynamics and the additional dynamics is under-parameterized. In case of including the discretization effect, identified model with the proposed scheme and Gahler's method are 20^{th} and 12^{th} order, respectively.

Relative identification errors are shown in Fig. 7 (b) and sum of the errors are shown in Table 1 (D.E. denotes including discretization effect). The proposed scheme shows much smaller identification error than Gahler's method. The identification errors in case of including additional dynamics are larger than those in case of excluding additional dynamics. In particular, identification errors increase much in case of Gahler's method due to under-parameterization error of the additional dynamics.

AMB rigid rotor system

In case of AMB rigid rotor system, the rigid modes can be separated from flexible modes compared with the flexible AMB rotor system and Ahn's method can be applied. Therefore, the proposed scheme is compared with Gahler's and Ahn's method using measured frequency responses of the AMB rigid rotor system. The AMB rigid rotor system consists of two AMB and three cylindrical capacitive sensors. The frequency responses of the paper [2] are used for the comparison of identification methods.



(a) W/O add. dynamics (b) With add. dynamics

Figure 8: Identification errors of the rigid rotor

Table 2: Identification errors of the rigid rotor

Error	Add. dyn.	New (D.E.)	Ahn	Gahler
Rel.	X	135.82	221.31	191.19
	O	88.46(129.75)	221.31	584.83
Abs.	X	20.90	23.78	25.5
	O	17.19(24.04)	20.48	73.62

Excluding the additional dynamics

The additional dynamics are identified and separated from the measured frequency responses. Then, identifications are performed with three identification methods. The order of the identified model is 8 for three methods. Relative identification errors are shown in Fig. 8 (a) and sum of the identification errors are summarized in Table 2. The proposed scheme shows better results than Gahler's and Ahn's methods. Although Ahn's method focuses on the rigid modes and shows good match in the low frequency, frequency responses at high frequency don't match very well.

Including the additional dynamics

In case of excluding the discretization effect, identified models with the proposed scheme, Gahler's method and Ahn's method are 20^{th}, 12^{th} and 22^{nd} order, respectively. Relative identification errors are shown in Fig. 8 (b) and the sums of the errors are shown in Table 2. The proposed scheme shows much smaller identification error than other two methods. The absolute identification errors including additional dynamics decrease compared with those excluding additional dynamics since responses in high frequency, where most identification errors appear, decrease due to the additional dynamics. Also, in case of including the discretization effect, the identification errors increase since the uncertainty in estimating the additional dynamics becomes large as the effect of the additional dynamics becomes large.

CONCLUSION

This paper presents an improved identification scheme of AMB rotor systems considering sensor and actuator dynamics. First, this paper discussed the necessity and a selection criterion of the FPFG. The identification error due to over-parametrization of the MIMO system via a SIMO modeling was minimized using an optimal FPFG. Second, identification procedure became simple and efficient through estimating all system poles and zeros simultaneously. Third, more improvement was achieved through separate parametrization of the additional dynamics. The performance of the proposed identification scheme was verified through severla simulations. Finally, the proposed scheme is compared with previous identification methods using experimental data. A great improvement in model quality and large amount of time saving can be achieved with the proposed method.

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