EXPERIMENTAL RESEARCH ON THE NUTATIONAL STABILITY OF MAGNETICALLY SUSPENDED MOMENTUM WHEEL IN CONTROL MOMENT GYROSCOPE (CMG)

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ABSTRACT

Aimed at the stability problem of magnetically suspended momentum wheel (MSMW) caused by gyroscopic effect, a criterion based on the frequency characteristic of control circuits is used to analysis and predict whirl modes stability of the flywheel system, and a proportional cross feedback control algorithm based on decentralized control is proposed to stabilize both precession and nutation. Particular simulation and experimental research on nutation stability shows that the phase lag of controller, especially the nonlinearity of amplifier caused by limited current rate is the main factor that influences the stability of nutation at high speed. A series of measurements and experiments are executed on a MSMW prototype under development at Beijing University of Aeronautics and Astronautics (BUAA). Results show good agreement between the analysis and the experiment. The maximum design speed 24,000 rpm is achieved through parameters adjusting.

INTRODUCTION

Control Moment Gyroscope (CMG) is the key actuator of attitude control for large-scale, long-life spacecraft such as space station. For years, ball bearing supported momentum wheel is used as part of CMG. Compared with traditional ball bearing, no contact and lubrication are the remarkable features of the magnetic bearings, which solve the long-life and high-speed rotation problems of momentum wheel. MSMW offers the most significant high-speed, long life, small volume and weight, and high reliability advantages. Therefore, MSMW is considered as an ideal actuator for future spacecraft attitude control [1].

In order to increase angular momentum, the rotor of moment wheel is designed to be a flat disk with great ratio of moments of inertia (**Jp**/**Je**>1, **Jp**, polar moment of inertia, **Je**, equator moment of inertia). Owing to the strong gyroscopic coupling effect of the rotor dynamics characteristic, the two whirl modes of the rotor, forward whirl (nutation) and backward whirl (precession) may become unstable at high speed due to the decrease of mode damping, while decentralized PID control is used. The integrator part can cause instability for the precession mode and the low-pass filter will tend to destabilize the nutation mode [2].

To solve the instability problem induced by gyroscopic effect, lots of methods have been introduced [3], [4]. Traditionally, cross feedback control, as a simple effective way, is used to compensate the gyroscopic effect. A properly adjusted cross feedback controller can stabilize precession fairly well, but for nutation with high frequency, the design of cross feedback controller becomes complex, and the regulation of control parameters are difficult [5].

For MSMW with remarkable gyroscopic effect, the paper analyses theoretically and researches experimentally on the stability of nutation mode, which largely determines the maximum speed of the flywheel system. A stability analysis method of whirling motion based on the frequency characteristic of control circuit and a proportional cross feedback control method based on decentralized PID control are proposed. A practical model of amplifier is established and used to predict critical speed. The main ideas of controller design and parameters regulation are given and verified through experiments.

STABILITY ANALYSIS OF HIGH SPEED MAGNETICALLY SUSPENDED GYROSCOPIC ROTOR

When momentum wheel is placed in level, the coupling of flywheel rotor between axial motion and radial motion is small. The influence that the axial motion brings to the radial motion will not be considered, and further the influence of gravity is neglected.

The rigid rotor structure of MSMW is shown in FIGURE 1. The radial magnetic bearings lie at $\mathbf{a}$, $\mathbf{b}$ ($\mathbf{a}>0$, $\mathbf{b}<0$), and the displacement sensors at $\mathbf{a}_s$, $\mathbf{b}_s$ ($\mathbf{a}_s>0$, $\mathbf{b}_s<0$) respectively, $\mathbf{\Omega}$ is rotating speed of the rotor, $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ the slopes of rotor shift around coordinate axes **x**, **y**. While the rotor is bilateral symmetry ($\mathbf{a}=-\mathbf{b}$, $\mathbf{a}_s=-\mathbf{b}_s$), and the parameters of the two radial magnetic bearings are the same (they have the same force-displacement factor $\mathbf{k}_x>0$, the same force-current factor $\mathbf{k}_i$). For decentralized control (refer to FIGURE 2), the equations of motion of the rotor in mass centric coordinates are

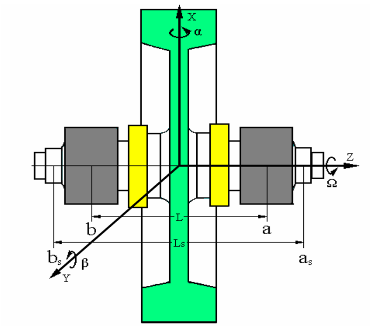

FIGURE 1 Rotor structure of flywheel

$$\begin{cases} m\ddot{x}-2k_x x+2k_i i_d(x)=0 \\ m\ddot{y}-2k_x y+2k_i i_d(y)=0 \end{cases} \tag{1}$$

$$\begin{cases} J_e\ddot{\alpha}+J_p\Omega\dot{\beta}-2a^2k_x\alpha+2aa_sk_ii_d(\alpha)=0 \\ J_e\ddot{\beta}-J_p\Omega\dot{\alpha}-2a^2k_x\beta+2aa_sk_ii_d(\beta)=0 \end{cases} \tag{2}$$

Where **$i_d(\bullet)$** is algorithm of decentralized control circuit. The translation motion and the whirl motion of rotor system are decoupled. Equation (1) describes the translation motion of the rotor, and equation (2) the rotation motions of the rotor around coordinate axes **x**, **y**, viz. the whirl motions of the rotor, while the rotating speed is zero, the two rotation motions are decoupled.

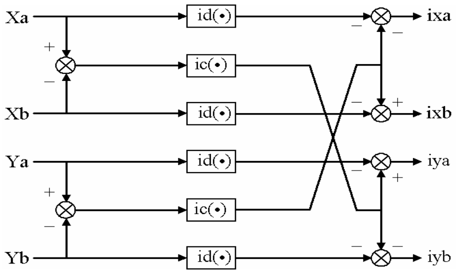

FIGURE 2 Decentralized controller with Cross feedback

While cross feedback is introduced (FIGURE 2), Equation (1) maintains the same, which indicates that the introduction of cross feedback does not influence the translation motion of symmetry rotor. Equation (2) transforms to

$$\begin{cases} J_e\ddot{\alpha}+J_p\Omega\dot{\beta}-2a^2k_x\alpha+2aa_sk_ii_d(\alpha)+4aa_sk_ii_c(\beta)=0 \\ J_e\ddot{\beta}-J_p\Omega\dot{\alpha}-2a^2k_x\beta+2aa_sk_ii_d(\beta)-4aa_sk_ii_c(\alpha)=0 \end{cases} \tag{3}$$

Where **$i_c(\bullet)$** is algorithm of cross feedback circuit.

It is supposed that the transfer function of decentralized control circuit and the cross feedback circuit can be written as ratios of two polynomials respectively

$$i_d(\cdot)=n_d(\cdot)/d_d(\cdot) \tag{4}$$

$$i_c(\cdot)=n_c(\cdot)/d_c(\cdot) \tag{5}$$

Appling Laplace transformation to equation (3), the characteristic equation of the whirl motions is

$$\begin{aligned} &[(J_es^2-2a^2k_x)d_d(s)d_c(s)+2aa_sk_in_d(s)d_c(s)]^2 \\ &+[J_p\Omega sd_d(s)d_c(s)+4aa_sk_id_d(s)n_c(s)]^2=0 \end{aligned} \tag{6}$$

According the definition of imaginary unit, **j^2**=-1, equation (6) is transformed to

$$\begin{aligned} &[(J_es^2-2a^2k_x)d_d(s)d_c(s)+2aa_sk_in_d(s)d_c(s)] \\ &=\pm j[J_p\Omega sd_d(s)d_c(s)-4aa_sk_id_d(s)n_c(s)] \end{aligned} \tag{7}$$

Rearrange equation (7), gives

$$\begin{aligned} i_d(s)\pm 2ji_c(s)&=\frac{n_d(s)}{d_d(s)}\pm 2j\frac{n_c(s)}{d_c(s)} \\ &=\frac{-J_es^2\pm jJ_p\Omega s+2a^2k_x}{2aa_sk_i} \end{aligned} \tag{8}$$

When the rotor system is marginal stable, let **$s=j\omega$** and incorporate it into equation(8), gives

$$i_d(j\omega)\pm 2ji_c(j\omega)=\frac{J_e\omega^2\mp J_p\Omega\omega+2a^2k_x}{2aa_sk_i} \tag{9}$$

Equation (9) is the marginal stability condition for the whirl modes of rotor system with cross feedback control. For decentralized control, the marginal stability condition is

$$i_d(j\omega)=\frac{J_e\omega^2\mp J_p\Omega\omega+2a^2k_x}{2aa_sk_i} \tag{10}$$

In equation (9), the left of the equal mark is a plural coefficient sum of frequency characteristics between the decentralized control circuit and the cross feedback control circuit, called frequency characteristic of whirl control circuit. The right of the equal mark is a real number. The marginal stability of the whirl modes means that the frequency characteristic curve of the whirl control circuit crosses over the real axis. The frequency at the cross-over point is the critical whirl frequency **ω_ω**, labels the gain at the cross-over point as **i_ω**, then

$$i_\omega=\left[i(j\omega)\pm 2ji_c(j\omega)\right]\Big|_{\omega=\omega_\omega} \tag{11}$$

The critical speed of rotor system at critical whirl frequency can be calculated by equation (9), written as following

$$\Omega=\frac{J_e}{J_p}\omega_\omega+\frac{2a^2k_x-2aa_sk_ii_\omega}{J_p\omega_\omega} \tag{12}$$

In equation (12), for critical precession frequency **ω_p**, **$\omega_p=\omega_\omega$**<0, means that it has an inverse direction in contrast to the rotating direction of rotor shift. For critical nutation frequency **ω_n**, **$\omega_n=\omega_\omega$**>0.

With whirl mode critical stability condition defined in equation (9) and critical speed of whirl mode defined in equation (12), the stability of actual control system can be determined easily by calculating and measuring the frequency characteristic of whirl control circuit.

As for undamped decentralized control system, the control circuit frequency characteristic $\mathbf{i_d(j\omega)}$ is a constant. Let it be $\mathbf{i_k}$. Undamped oscillation frequencies of precession and nutation can be deduced from equation (10). The corresponding equations are

$$\omega_n = \frac{1}{2}\left[\frac{J_p}{J_e}\Omega + \sqrt{\left(\frac{J_p}{J_e}\Omega\right)^2 + 8a(a_s k_i i_k - a k_x)}\right] \qquad (13)$$

$$\omega_p = \frac{1}{2}\left[\frac{J_p}{J_e}\Omega - \sqrt{\left(\frac{J_p}{J_e}\Omega\right)^2 + 8a(a_s k_i i_k - a k_x)}\right] \qquad (14)$$

Equations (13) and (14) indicate, when rotational speed is zero, whirl frequency is unique. The initial frequency is determined by control circuit gain. Nutation mode whirls with the same direction as that of rotor shift rotating, while precession mode whirls inversely. With the rising of speed, nutation frequency increases and approaches **Jp/Je•Ω**, while precession frequency reduces and approaches zero.

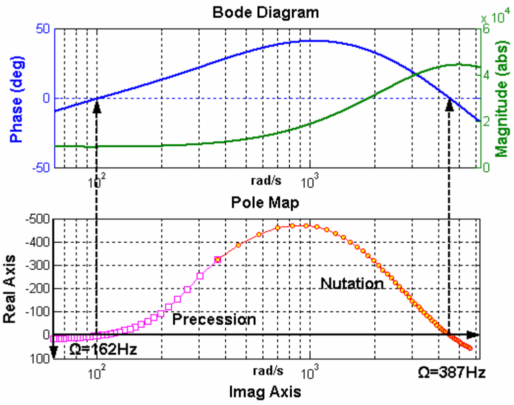

FIGURE 3 Relationship between bode diagram and pole map with decentralized PID control

For decentralized PID control system includes phase delay parts, such as low pass filter, the relationship between critical whirl frequencies and frequency characteristic curve of decentralized control circuit is shown in FIGURE 3. Critical whirl frequencies of precession and nutation are just the undamped oscillation frequencies where the phase shift is zero in the phase-frequency characteristic curve of control circuit. For symmetry flywheel rotor system with decentralized controller, the stability of whirl modes is decided mainly by phase–frequency characteristic of control circuit. Whirl mode is stable at the frequency where the control circuit has phase lead, whirl mode is unstable at the frequency where the control circuit has phase lag. Phase shift at low frequency affects stability of precession mode mainly and phase shift at high frequency affects stability of nutation mode basically.

For actual magnetically suspended flywheel system, the introduction of integrator leads to the phase delay in low frequency segment, which affects the stability of precession. Besides low pass filter used to suppress noise and notch filter used to restrain vibration of rotor at its natural frequency in controller, power amplifier with low pass characteristic and nonlinear characteristic (discussed in another section) create phase delay in high frequency segment also, which affected stability of nutation. For digital control system, the input-output delay and control delay caused by limited sampling frequency and computing speed are responsible for the instability of rotor system at high speed.

PROPORTIONAL CROSS FEEDBACK CONTROL STABILIZING BOTH NUTATION AND PRECESSION MODES

The instability of whirl modes restrict that flywheel rotor reach its rated speed, it is required that the control system provide phase compensation for whirl mode at high speed. For precession mode, phase lag can be improved by reducing integral coefficient and increasing differential coefficient. However, precession mode still limited stability at high speed. For nutation mode, phase lag can be improved by increasing differential coefficient and system bandwidth, adding phase lead circuit into the controller. As for practical control system, some of those methods can introduce noises, and large gain at high frequency can increase nutation mode frequency. In the case of that other high frequency modes are considered, those methods show their limitations.

Through experiment and simulation, a proportional cross feedback control algorithm based on decentralized control is used in the MSMW under research. The algorithm of proportional cross feedback control circuit, $\mathbf{i_c(\bullet)}$ is shows in FIGURE 4.

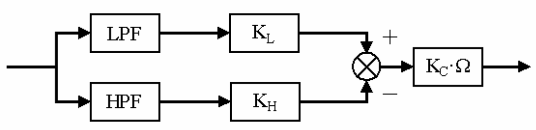

FIGURE 4 Algorithm of proportional cross feedback control circuit

The main idea of this algorithm is introduced here. First, based on decentralized PID controller, the whirl motions are separated from the displacement signals. Second, according to their difference in frequency, precession signal and nutation signal are separated by using a low-pass filter (**LPF**) and a high-pass filter (**HPF**). Finally, cross phase compensation is applied to precession mode and nutation mode depending on their whirling direction and the phase lead-lag relationship between radial motion signals. In practical control, the cross feedback control signal for precession and nutation are contrary due to their inverse movement direction. In addition, considering the direct proportion relationship between gyro coupling moment and rotational speed, and the character that precession frequency rises with rotational speed while nutation frequency falls down, a cross proportion coefficient **k_c•Ω** which has direct ratio relationship with rotational speed is introduced to guarantee the stability of rotor at different speed.

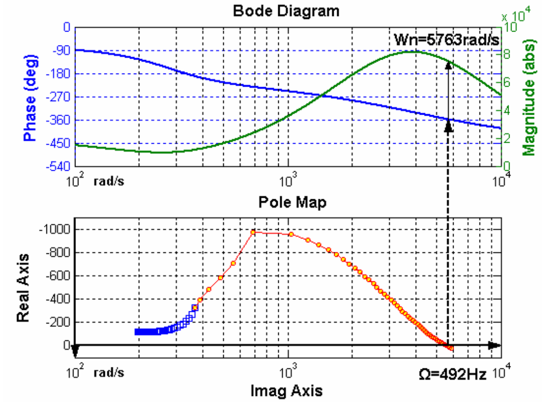

FIGURE 5 Relationship between bode diagram and pole map with proportional cross feedback control

As an example, nutation mode stability is analyzed using the marginal stability condition for whirl modes defined by equation (9). As for proportional cross feedback control circuit show in FIGURE 4, root locus diagram of whirl mode and frequency characteristic curves of nutation control circuit are shown in FIGURE 5, the critical speed rise clearly (refer to FIGURE 3).

NONLINEARITY AND EQUIVALENT PHASE FREQUENCY CHARACTERISTIC OF POWER AMPLIFIER

In the course of testing the adopted PWM power amplifier, it is found that performance of power amplifier changes with different input signals. Especially, the phase-frequency characteristic of power amplifier is not only related to frequency of input signal, but also influenced by input signal's amplitude, which displays remarkable nonlinearity. The main reason is that the output current of power amplifier has the maximum rising rate and falling rate, which caused by the inductance coil load in magnetic bearings.

As the rate of input signal of power amplifier exceeds a certain of number, the output current of amplifier is no longer linear vs. input voltage, but changes at its maximum current rate. The output becomes sawtooth wave, which will bring harmonic wave to the magnetically suspended flywheel system. Measured input and output signals of power amplifier are show in FIGURE 6.

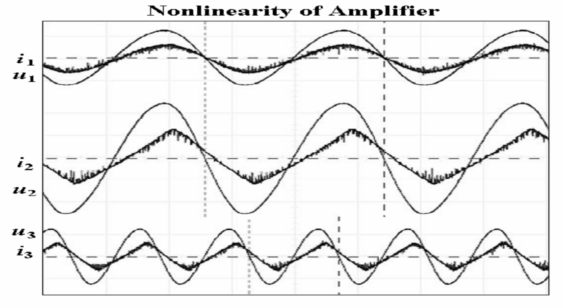

FIGURE 6 Output current of power amplifier at different input voltage

According to the measured maximum current rising rate and falling rate of the power amplifier, a nonlinear mathematical simulation model is established. FFT is used to get the equivalent frequency characteristic curve by comparing of basic frequency signals in magnitude and phase. The calculated equivalent frequency characteristic curve with different input amplitude is Show in FIGURE 7.

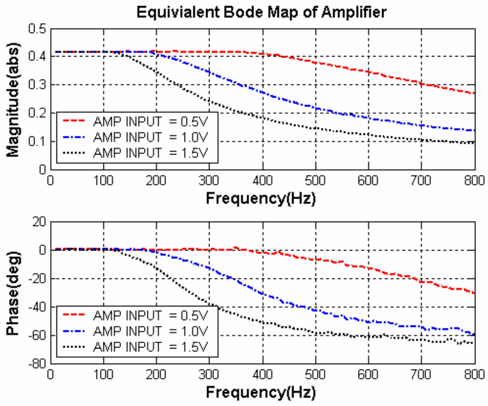

FIGURE 7 Equivalent frequency characteristic curves of power amplifier

When the input frequency is low, the output current of power amplifier changes with the input signals without distortion and phase shift. When input frequency rise, the output amplitude fade down, and the phase lag increase. Moreover, when the input amplitude increase, the output amplitude decay and the phase lag is more severe, however, the maximum phase lag will not exceed 90 degrees.

The phase-frequency characteristic of amplifier is influenced not only by frequency, but also by amplitude of the input signal. As the amplitude of the input signal is increasing, the equivalent bandwidth narrows. Therefore, the stability of high frequency mode is influenced directly by its vibration amplitude. When the vibration amplitude of the nutation mode increases for poor damping, the phase lag increases, and the critical nutation frequency decreases, which results in out of control of the rotor system ahead of schedule. On the contrary, when the rotor is out of control after critical speed, rotor vibration amplitude is quite large. Phase lag of power amplifier will be more severe than that at critical speed, Even though the speed of flywheel decreases violently, the stability of the system will not resume quickly.

In addition, the stability of high frequency mode is also influenced by lower frequency vibration with high amplitude. Owing to the limit of maximum current rate, The output current of power amplifier will not response to the low amplitude, high frequency signal when it following the high amplitude, median or low frequency signals.

SYSTEM TESTING AND EXPERIMENT RESULT

FIGURE 8 MSMW system for CMG

FIGURE 8 shows the MSMW system for CMG, which is under research at BUAA. Parameters are listed in TABLE 1.

TABLE 1 Parameter table of MSMW

Rated speed	20000	rpm
Maximum design speed	24000	rpm
Angular momentum	200	Nms
Rotor mass	12.6	kg
Polar moment of inertia	0.096	kg/m^2
Equatorial moment of inertia	0.056	kg/m^2
Displace stiffness coefficient	1×10^6	N/m
Current stiffness coefficient	200	N/A

Cross feedback control can restrain the gyroscopic effect, improve whirl modes stability of rotor. However, frequency characteristic of control circuit is required, especially, the phases lag at high frequency, which is critical to the control of nutation.

The controller of magnetic bearing is mainly made up of linear circuits, whose frequency characteristic can be calculated or measured easily. However, for power amplifier with nonlinear characteristics, the system model error will be large when linear simulation model is used. A method of combining experiment testing with nonlinear simulation is adopted to get the equivalent frequency characteristic curves for nutation mode. First, the critical nutation frequency is measured in experiment. Second, according to the stability condition of nutation mode, the phase lag-angle of power amplifier is calculated. Then, the nonlinear simulation model is used to determine the equivalent input voltage amplitude of power amplifier (corresponding to nutation vibration amplitude at critical speed). Finally, the equivalent frequency characteristic curve is calculated.

FIGURE 9 shows trajectory of instantaneous rotor shift deviation at critical speed. The frequency spectrum at bottom of the figure shows that the critical nutation frequency is 153 Hz, and the critical speed is 70 Hz.

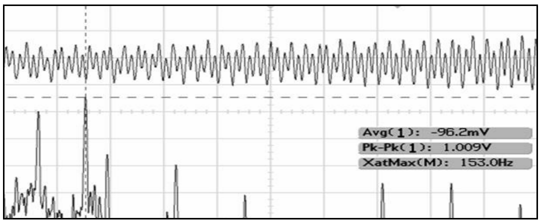

FIGURE 9 trajectory of instantaneous rotor shift deviation at critical nutation stability

With the foregoing method, the equivalent frequency characteristic curve of power amplifier for nutation mode is tested and calculated, shown in FIGURE 10. Four times test results are shown in the figure also, which demonstrates the power amplifier phase lag-angle tested. It coincides very well with the equivalent frequency characteristic curve.

The power amplifier's phase lag is about 50 degrees at 800 Hz, which contribute the chief phase lag to control system.

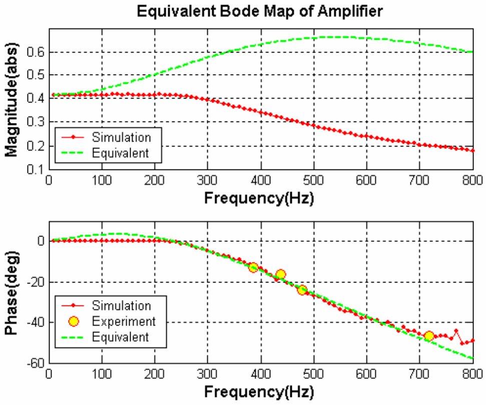

FIGURE 10 Equivalent frequency characteristic curves of power amplifier for nutation mode

According to the marginal stability condition for whirl modes of rotor system with cross feedback control, the influences of system bandwidth, cross mode and cross proportion coefficient on the stability of nutation are simulated and tested. The results (TABLE 2) show good agreement between the analysis and the experiment.

TABLE 2 Parallel table of simulation and experiment results on critical stability of nutation

System band width		Hz	1k	1.5k	3k	3k
Cross mode (N, nutation cross) (P, precession cross)			P	P	P	P, N
Cross proportion		Hz	494	606	704	2416
critical speed	experiment	Hz	189	225	248	348
	simulation	Hz	185	212.5	247.5	358
	relative error	％	-2.10	-5.60	-0.20	2.90
critical nutation frequency	experiment	Hz	386	438	479	718
	simulation	Hz	385	430	486	720
	relative error	％	-0.30	-1.80	1.50	0.30

Based on the system modal established and the method introduced in the paper, parameter optimization of the proportional cross feedback controller is performed. The MSMW system runs stably form 0 rpm to its maximum design speed, 24,000 rpm with regulated cross feedback controller. FIGURE 11 shows waterfall plot of the rotor shift deviation. It can be seen that the basic frequency corresponding to rotational speed, various harmonic frequencies (odd harmonic frequency is major), abundant low frequency below 50 Hz and direct current component. Test result shows that the stability margin of nutation mode is improved obviously. At the same time, it is proved that the method used to analyses stability of nutation mode is effective and practical.

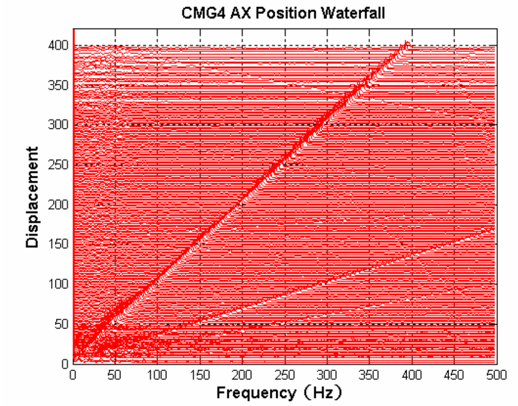

FIGURE 11 Waterfall plot of rotor displacement while rotor speed falls from 24,000 rpm to 0 rpm

CONCLUSION

For symmetry flywheel rotor system, stability of whirl mode largely depends on the phase-frequency characteristic of whirl mode control circuit. Control system phase lag, in particular, the nonlinear characteristic of power amplifier caused by limited current rate is the main factor that influences the stability of nutation.

Proportional cross feedback control can improve the stability of whirl modes effectively. The natural of it is to compensate the phase lag of control system by utilizing phase relationship of whirling motion on geometric position. Considering inverse whirling direction, different vibrating frequency of precession and nutation, filters are required to separate these two vibrations, and thus precession and nutation are controlled with different cross feedback subcircuit.

A stability analysis method of whirling motion based on the frequency characteristic of control circuit is presented and tested in the paper. The criterion used to determine critical whirl frequencies and the equation used to calculate critical speed are given. The method can reflect the contribution of each control circuit to the whole control system and their effect on system stability. Owing to the detectability of frequency characteristic curves, it is convenient for control system analysis and cross control parameter setting.

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