Control of Magnetic Bearing Levitated Flexible Rotor System: A Convex Optimization Approach

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ABSTRACT

This paper reports on an investigation of the control of active magnetic bearings (AMB) in suspended rotor systems. The AMB model is nonlinear inherently, and parameter uncertainty has to be taken into consideration for effective control design. For a lightly damped flexible structure rotor, the closed-loop system stability and performance are very sensitive to errors in the natural frequencies, while the high speed spinning rotor displays strong elastic characteristics and gyroscopic effects, which entail a linear parameter varying (LPV) system model. The effective control system design is a challenge. To meet robust control objectives and to design the controller in an LPV framework, we propose a robust LPV control design based on convex optimization. The overall LPV system model with uncertainty characterizations is formulated as a convex hull. A new H_∞ analysis condition which utilizes an extra variable on LMI conditions is formulated. A parameter dependent Lyapunov function for the closed loop convex hull can be constructed to greatly reduce controller design conservatism and take advantage of the acceleration/deceleration information of the rotor.

Keywords: Magnetic bearings, LPV, LMI, control performances

1 INTRODUCTION

This work is intended to develop a systematic design approach to the control of active magnetic bearings (AMB) for levitated high speed rotor systems. Because of the obvious advantages of AMB systems, such as high reliability, high precision of the rotor position control, low wear and tear, low maintenance, magnetic levitated high speed rotor systems have been studied and utilized in a wide range of industrial applications, such as energy storage devices, machine tools, hybrid vehicles ([1]). We carry out our design based on an AMB controls test rig set up at the University of Virginia. It is a prototype of a flywheel energy storage system, which consists of a flexible rotor equipped with a gyroscopic disk, a set of AMBs to suspend the spinning rotor, a set of sensors, amplifiers and a digital control platform. It is built to investigate the control design and properties of high speed rotor systems supported by AMBs. Fig. 1 shows the schematic of the rotor system.

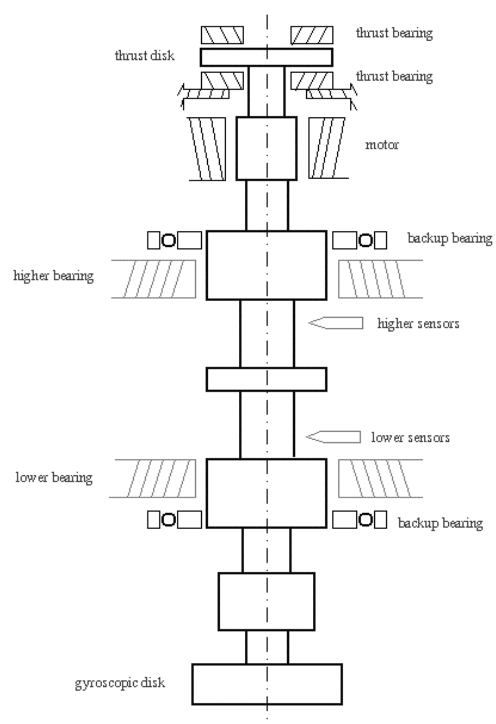

Figure 1: Schematic of the rotor

Experimental results show that the resonance of the lightly damped flexible rotor and its supporting frame can vary significantly in different operating conditions, and the closed-loop system stability and performance are very sensitive to the errors in the natural frequencies, deviation of the system parameters, etc. Moreover, the high speed spinning rotor displays strong elastic characteristics and gyroscopic effects, which requires a linear parameter varying (LPV) system model. Effective control system design is a challenge. Traditional PID or H_∞ controller may have significant difficulties in the presence of system uncertainties and may not be able to stabilize the rotor. A controller that can robustly stabilize the system is essential.

Based on the structured uncertainty specifications, μ synthesis is an effective tool to design a robust controller. For our system, a μ controller has been designed based on the linear time invariant (LTI) system model and tested successfully for a small range of speed variations. The rotor speed is considered as an uncertainty of the system. Theoretically, the LTI μ controller can not guarantee the system performance in face of the time varying rotor speed. Technically, the μ controller works only when the rotor speed changes slowly ([2]).

Since the system model is in the LPV form with the varying parameter (rotor speed) measured on line, it is desirable to construct an LPV controller that can adjust itself according to the online rotor speed measurement. There have been various approaches to addressing the LPV control.

One approach is the piecewise μ-synthesis controller ([10]). The idea is to design several μ controllers for different rotor speed zones and switch between the LTI controllers according to the current rotor speed. Bumpless transfer techniques are used during the switch. This heuristic approach is effective technically, but theoretically it cannot guarantee the global performance and stability.

Another approach is the traditional gain-scheduling LPV control ([6]), which does not take into consideration the structured uncertainty specifications, and is only applicable to system design with measurable uncertainties.

A modified and improved approach is the robust LPV control with a single Lyapunov function specified for the convex hull of the LPV plant ([11]). By convex characterization, the robust control of the LPV system has been reformulated as a convex optimization problem. However, because a single Lyapunov function has to be satisfied for all the vertices of the convex hull, the synthesis method is conservative, especially when the number of the vertices is large, or when the range of the varying parameter or parametric uncertainty is wide, or when the varying parameter changes slowly.

To deal with these issues, several methods have been proposed ([4, 5]). Generally, a parameter dependent Lyapunov function is used instead of a single constant one. However, the stability condition

$$P(p)A(p) + A(p)^{\mathrm{T}}P(p) + \dot{P}(p) < 0, \quad \forall p \in \Delta,$$

then implies an infinite dimensional LMI set. And because of the coupled term $P(p)A(p) + A(p)^{\mathrm{T}}P(p)$ in the stability condition, we cannot simply substitute $P(p)$ with vertices P_i and $A(p)$ with A_i. A compromise and practical method is to grid p, so that the problem is tractable ([6, 8]). But this method cannot guarantee a global solution to the problem, and the computational load will dramatically increase with the increase of the gridding density.

We propose a new analysis and synthesis method by introducing an extra variable on the LMI conditions, which can circumvent the coupled terms between the Lyapunov function matrix $P(p)$ and the time varying system matrix $A(p)$. Different Lyapunov functions can be easily synthesized for all the vertices of the convex hull. The conservatism is significantly reduced but the system robust stability and performance are retained. A parameter dependent Lyapunov function for the closed loop convex hull can be constructed to take advantage of the acceleration/deceleration information of the rotor. Moreover, it can be extended to multi-objective control and obtain good performance with less conservatism.

2 SYSTEM MODEL

Our AMB test rig plant model is mainly composed of the following four components: AMBs, rotor, sensors and amplifiers.

The AMB model describes the magnetic bearing force acting upon the rotor as a function of the currents applied to the magnetic bearing coils and the air gap width between the bearing and the rotor. Assuming negligible bearing magnetic flux leakage, we can estimate the force acting upon the rotor due to one magnetic bearing pole as:

$$f = K\frac{I^2}{g^2},$$

where I is the coil current, g is the air gap width (distance between the rotor and the bearing pole), and K is a constant factor determined by the cross-sectional area of the pole, number of coil turns and permeability of free space ([7]). To accommodate the LPV form of the system model and facilitate our convex optimization approach, we first linearize the magnetic bearing model as follows.

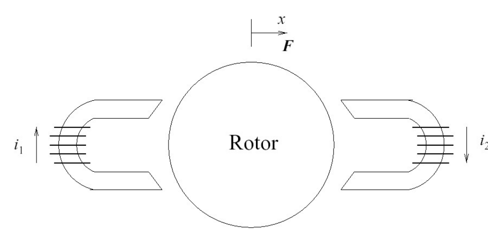

Figure 2: One axis of a magnetic bearing

Consider two opposed magnetic bearing poles working together to suspend the rotor along one axis of the bearing as in Fig. 2.

Let

$$\begin{cases} i_1 = i_b + i \\ i_2 = i_b - i \end{cases} \quad \text{and} \quad \begin{cases} g_1 = g_0 - x \\ g_2 = g_0 + x. \end{cases}$$

Assuming small i and x, we can linearize the force about the nominal bias current i_b and centered position (centered air gap width is g_0 for both poles) as

$$F = k_i i + k_x x, \tag{1}$$

where k_i is the current stiffness and k_x is the displacement stiffness of the AMB. The linear model is too simple. It is not sufficient for effective control design. To address the nonlinear properties of the magnetic bearing model, we use the LPV model to describe and compensate the nonlinearity and uncertainty. The nonlinearity can be modeled by an uncertain parameter set as

$$k_i = \delta_{k_i} \in \left[\underline{k}_i, \bar{k}_i\right],$$

$$k_x = \delta_{k_x} \in \left[\underline{k}_x, \bar{k}_x\right],$$

where $\underline{k}_i$, $\bar{k}_i$, $\underline{k}_x$ and $\bar{k}_x$ are the extreme values of the parameter set, i.e.,

$$F = \delta_{k_i} i + \delta_{k_x} x, \tag{2}$$

which is in an LPV form.

The sensor and amplifier models are also inherently nonlinear. Similarly, we linearize them under normal operational conditions and use the LPV model to describe the difference between the linear terms and the nonlinear terms while taking into consideration the model uncertainties.

The rotor dynamic model, which is constructed by FEM and tuned by modal testing and model reconciliation, describes the displacement of the rotor (air gap width variation) in response to the external force (magnetic bearing force). For the sake of effective control design and to cover the frequency of the rotor spin speed, we model up to two flexible modes of the rotor and six modes for the flexible supporting substructure, resulting in a 36th order linear model. Due to the gyroscopic effects of the spinning rotor, the rotor dynamic properties are linearly dependent upon the rotor speed, which also entails an LPV model.

We stack up the four channel magnetic bearing set model and integrate with the rotor dynamic, sensor and amplifier models to obtain an overall LPV state space model in the following form:

$$\begin{cases} \dot{x} = (A_\delta(\delta) + pA_p)x + B_1 w + B_2 u, \\ z = C_1 x + D_{11} w + D_{12} u, \\ y = C_2 x + D_{21} w, \end{cases} \tag{3}$$

where $p \in [p_1, p_2] = [p_{\min}, p_{\max}]$ is the rotor speed, which can be measured online, in the form

$$A_\delta(\delta) = A_0 + \delta_1 A_1 + \cdots + \delta_r A_r$$

represents the nonlinearity and uncertainty of the system. For our test rig system model, we formulate the diagonalized nonlinearity and uncertainty structure with $r = 5$, representing the natural frequency uncertainty (δ_{n}), AMB displacement stiffness and current stiffness nonlinearity ($\delta_{\mathrm{k_x}}$ and $\delta_{\mathrm{k_i}}$), sensor and amplifier gain nonlinearity (δ_{sag}) and substructural models uncertainty (δ_{sub}).

3 H_∞ ANALYSIS

To illustrate our LPV controller synthesis approach, we begin with our H_∞ analysis. Consider a general LPV closed loop system

$$\begin{cases} \dot{x} = A(p)x + B(p)w, \\ z = C(p)x + D(p)w, \end{cases} \tag{4}$$

where $A(p), B(p), C(p), D(p)$ are affine matrix functions of the external varying parameter vector

$$p := (p_1(t), p_2(t), \cdots, p_r(t)),$$

which is a function of time. And

$$p_i \in [p_{i\min}, p_{i\max}],$$

where $p_{i\min}$ and $p_{i\max}$ are the limits of the trajectory of the varying parameter $p_i(t)$. The LPV plant constitutes a convex hull due to the affine characteristics of the matrix functions. We can thus formulate the system as

$$\begin{cases} \dot{x} = \sum_{i=1}^n \alpha_i A_{vi} x + \sum_{i=1}^n \alpha_i B_{vi} w, \\ z = \sum_{i=1}^n \alpha_i C_{vi} x + \sum_{i=1}^n \alpha_i D_{vi} w, \end{cases} \tag{5}$$

where $\alpha_i = \alpha_i(p(t))$ is the varying parameter coefficient and $\alpha_i \in [0,1]$, $\sum_{i=1}^n \alpha_i = 1$, where $n = 2^r$ is the number of the vertices of the convex hull. $A_{vi}, B_{vi}, C_{vi}, D_{vi}$ are the vertices of the convex hull of the closed-loop system. We denote the corresponding transfer function of the vertices as

$$G_i(s) := \begin{pmatrix} A_{vi} & B_{vi} \\ C_{vi} & D_{vi} \end{pmatrix}. \tag{6}$$

The control objectives for the LPV system can be cast into the H_∞ problem for the vertices of the convex hull. We have the following H_∞ analysis theorem:

Theorem 1 *Consider the closed-loop LPV system in the form of* (5)*. Let* $\gamma > 0$ *be given. Then the following statements are equivalent:*

(1) *The closed-loop LPV system is asymptotically stable and the* H_∞ *norm of the closed-loop transfer function from* w *to* z *is less than* γ *for any* $\alpha_i \in [0,1]$.

(2) *There exist real matrices E and $P = P^{\mathrm{T}} > 0$ such that for $i = 1, 2, \cdots, n$,*

$$\begin{bmatrix} -(E+E^{\mathrm{T}}) & * & * & * & * \\ A_{vi}{}^{\mathrm{T}}E+P & -P & * & * & * \\ B_{vi}{}^{\mathrm{T}}E & 0 & -\gamma I & * & * \\ E & 0 & 0 & -P & * \\ 0 & C_{vi} & D_{vi} & 0 & -\gamma I \end{bmatrix} < 0. \tag{7}$$

Sketch of the proof. For convenience, rewrite (5) as

$$\begin{cases} \dot{x} = A_\Sigma x + B_\Sigma w, \\ z = C_\Sigma x + D_\Sigma w. \end{cases} \tag{8}$$

By the principle of convexity, condition (2) is equivalent to

$$\begin{bmatrix} -(E+E^{\mathrm{T}}) & * & * & * & * \\ A_\Sigma{}^{\mathrm{T}}E+P & -P & * & * & * \\ B_\Sigma{}^{\mathrm{T}}E & 0 & -\gamma I & * & * \\ E & 0 & 0 & -P & * \\ 0 & C_\Sigma & D_\Sigma & 0 & -\gamma I \end{bmatrix} < 0. \tag{9}$$

After some matrix manipulations and applying the Projection Lemma ([3]), (9) is equivalent to

$$\begin{bmatrix} A_\Sigma{}^{\mathrm{T}}P+PA_\Sigma-P+\frac{1}{\gamma}C_\Sigma{}^{\mathrm{T}}C_\Sigma & * & * \\ B_\Sigma{}^{\mathrm{T}}P+\frac{1}{\gamma}D_\Sigma{}^{\mathrm{T}}C_\Sigma & -\gamma I+\frac{1}{\gamma}D_\Sigma{}^{\mathrm{T}}D_\Sigma & * \\ P & 0 & -P \end{bmatrix} < 0. \tag{10}$$

Considering the Lyapunov function

$$V(x) := x^{\mathrm{T}}Px > 0,$$

and using the Schur complement on (10), we can show that

$$\|z\|_2^2 - \gamma^2\|w\|_2^2 + \dot{V} < 0.$$

This is exactly the H_∞ condition for the closed loop transfer function of the given LPV system, i.e.,

$$\|G_\Sigma\|_\infty = \sup_{\omega \neq 0} \frac{\|z(t)\|_2}{\|w(t)\|_2} < \gamma. \tag{11}$$

□

By introducing an extra matrix E in the H_∞ analysis LMI set (7), we decouple the term $PA_\Sigma + A_\Sigma{}^{\mathrm{T}}P$ in the LMI condition for system stability. This has the obvious potential and advantage in analyzing LPV system using a parameter dependent Lyapunov function to reduce conservatism and also in multi-objective feedback control synthesis.

For our LPV controller design, Theorem 1 entails a straightforward synthesis method. The finite number of vertices of the convex hull capture the characteristics of an infinite number of possible trajectories and rates of the varying parameters in the closed loop system. So a set of LMI constraints (7) at the vertices of the system convex hull impose an LPV controller meeting the stability and H_∞ performance requirements for the closed loop LPV system. From the proof, we can see that the Lyapunov function is only a function of the state variable x, and is independent of the external varying parameter p. Thus the closed-loop system performance is guaranteed for any trajectory and rate of the varying parameters in the given range. However, this method is inevitably conservative if we choose the same P and E for the H_∞ synthesis at all the vertices. To take advantage of the extra matrix variable E in the H_∞ condition, less conservative LPV control synthesis methods are given in the following section.

4 H_∞ SYNTHESIS

Considering our system model (3), we define the uncertainty parameter set as

$$\Delta := \{\delta = (\delta_1, \delta_2, \cdots, \delta_r) : \delta_i \in [\underline{\delta}_i, \bar{\delta}_i]\},$$

where each parameter δ_i could be any value in a predefined range $[\underline{\delta}_i, \bar{\delta}_i]$.

The matrix $A_\delta(\delta)$ can then be considered as an affine function mapping from Δ to the system matrix set Ω_A with the parameter vector variable $\delta \in \Delta$, i.e.,

$$A_\delta : \Delta \to \Omega_A, \text{ such that } A_\delta(\delta) \in \Omega_A \text{ for all } \delta \in \Delta.$$

The coefficient matrices $A_0, A_1, \cdots, A_r$ are known constant matrices determined by the system dynamics, nonlinearity and uncertainty structure.

Rewrite the LPV plant model (3) in a convex set representation:

$$\begin{cases} \dot{x} = \sum_{i=1}^{2}\sum_{j=1}^{2^r} \alpha_{p_i}\alpha_{\delta_j} A_{ij}x + B_1 w + B_2 u, \\ z = C_1 x + D_{11} w + D_{12} u, \\ y = C_2 x + D_{21} w, \end{cases} \tag{12}$$

where $\alpha_{p_i} \in [0,1]$ and $\alpha_{\delta_j} \in [0,1]$ represent the coefficients for varying parameter p, nonlinearity and parameter uncertainty δ in the convex set. Note that

$$\alpha_{p_1} = \frac{p_2 - p}{p_2 - p_1} \in [0,1], \quad \alpha_{p_2} = \frac{p - p_1}{p_2 - p_1} \in [0,1]$$

satisfy

$$\sum_{i=1}^{2} \alpha_{p_i} = 1, \quad \sum_{j=1}^{2^r} \alpha_{\delta_j} = 1, \quad \sum_{i=1}^{2}\sum_{j=1}^{2^r} \alpha_{p_i}\alpha_{\delta_j} = 1.$$

The coefficient matrix A_{ij} is the corresponding vertex of the system matrix in the convex hull. To reduce conservatism and to take advantage of the acceleration information, we suppose the bound of the acceleration of the varying parameter is known, i.e.,

$$\dot{p} \in [p_{d1}, p_{d2}],$$

where p_{d1} and p_{d2} are the minimum and maximum acceleration of the varying parameter, which in our case, is the rotor speed.

Assuming full state is available for feedback, our desired LPV H_∞ controller is of the form

$$u = \sum_{i=1}^{2} \alpha_{p_i} K_i x, \tag{13}$$

then the closed-loop state feedback system can be represented by

$$Gs: \begin{cases} \dot{x} = \sum_{i=1}^{2} \sum_{j=1}^{2^r} \alpha_{p_i} \alpha_{\delta_j} (A_{ij} + B_2 K_i) x + B_1 w, \\ z = \sum_{i=1}^{2} \alpha_{p_i} (C_1 + D_{12} K_i) x + D_{11} w. \end{cases} \tag{14}$$

For convenience, we denote

$$Gs: \begin{cases} \dot{x} = Asx + Bsw, \\ z = Csx + Dsw. \end{cases} \tag{15}$$

The vertices of the convex set (14) is denoted as

$$Gs_{ij} := \begin{pmatrix} As_{ij} & Bs_{ij} \\ Cs_{ij} & Ds_{ij} \end{pmatrix} := \begin{pmatrix} A_{ij} + B_2 K_i & B_1 \\ C_1 + D_{12} K_i & D_{11} \end{pmatrix}, \tag{16}$$

for $i = 1, 2, \quad j = 1, 2, \cdots, 2^r$.

We then have the following procedures to construct our H_∞ state feedback controller.

STEP 1. For the LPV plant model (12) and desired closed-loop H_∞ norm γ, solve the following set of LMIs (17),

$$\begin{bmatrix} -(E+E^{\mathrm{T}}) & * & * & * & * \\ A_{ij}E + B_2 M_i + P_{ij} & -P_{ij} + Q_{ij} & * & * & * \\ C_1 E + D_{12} M_i & 0 & -\gamma I & * & * \\ E & 0 & 0 & -P_{ij} & * \\ 0 & {B_1}^{\mathrm{T}} & {D_{11}}^{\mathrm{T}} & 0 & -\gamma I \end{bmatrix} < 0, \tag{17}$$

where E, $P_{ij} = {P_{ij}}^{\mathrm{T}} > 0$, M_i are unknown, and Q_{ij} is defined as

$$Q_{ij} = p_{di} \frac{P_{2j} - P_{1j}}{p_2 - p_1},$$

for $i = 1, 2, \quad j = 1, 2, \cdots, 2^r$.

STEP 2. The desired state feedback LPV controller is of the form (13), where

$$K_i = M_i E^{-1}. \tag{18}$$

Theorem 2 *The closed-loop system consisting of the LPV plant* (12) *and the LPV state feedback law* (18) *has the following property: It is asymptotically stable and the* H_∞ *norm of its transfer function from* w *to* z *is less than* γ *for any time varying* $p \in [p_{\min}, p_{\max}]$, $\dot{p} \in [p_{d1}, p_{d2}]$ *and* $\delta \in \Delta$.

This theorem can be proved by constructing a parameter dependent Lyapunov function

$$V(p,\delta) := x^{\mathrm{T}} P(p,\delta) x = \sum_{i=1}^{2} \sum_{j=1}^{2^r} \alpha_{p_i} \alpha_{\delta_j} x^{\mathrm{T}} P_{ij} x,$$

and applying Theorem 1 for the closed loop LPV system analysis. We omit the proof here (See [9]). The LMI condition in Theorem 2 specifies a parameter dependent Lyapunov function which is less conservative than the constant Lyapunov function condition following Theorem 1. This can be seen by setting $P_{ij} = P_0$ for $i = 1, 2, j = 1, 2, \cdots, 2^r$, and consequently $Q_{ij} = 0$, then we get the constant Lyapunov function condition, which guards against an arbitrarily fast changing parameter without taking into consideration the acceleration information.

For the output feedback design, we construct a state observer and use the same techniques to obtain the observer gain. The observer design is just the dual form of the state feedback design.

5 SIMULATION/IMPLEMENTATION

To verify the robustness and performance of our LPV controller, we compare our design with a μ synthesis controller. Fig. 3 gives the comparison of the H_∞ norm of different design approaches. As we can see, the performance of our robust LPV design is better than the μ design in the simulation for the rotor speed from 0 to $1000\ rad/s$.

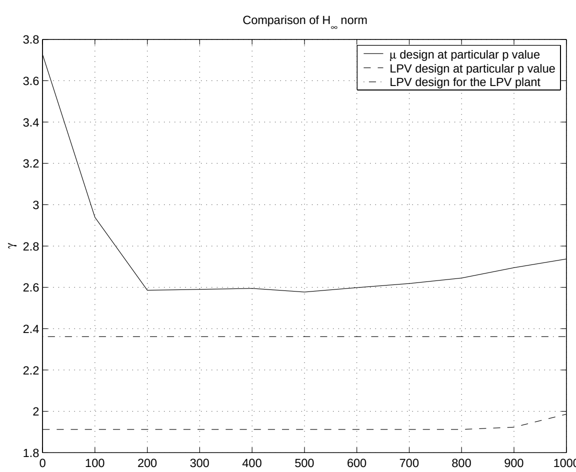

Figure 3: Comparison of H_∞ norm

We demonstrated our controller in the AMB test rig. The rotor was successfully levitated and spun up to $12,000\ rpm$. The vibration level for the flexible modes of the rotor is significantly suppressed.

For an AMB system, the nominal bias current is an important design parameter. Generally a low bias current entails low power consumption, but may induce in-sufficient actuator force and possible instability of the control system. Based on our robust LPV design, the controller should be able to maintain stability and performance in the presence of the system nonlinearity and uncertainty, including the potential variation or decrease of the bias currents in the coils of the magnetic bearings. For test purposes, we decrease the nominal bias current and test the control system stability and performance. The bias current can be reduced by around 75% without losing system stability. Fig. 4 and 5 are the performance comparison of the two cases. The experimental results demonstrate the effectiveness and performance of our robust LPV controller in face of system nonlinearities and uncertainties.

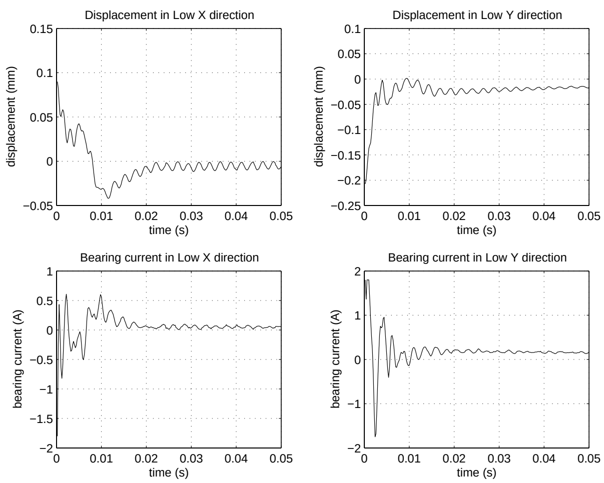

Figure 4: Nominal bearing coil bias current

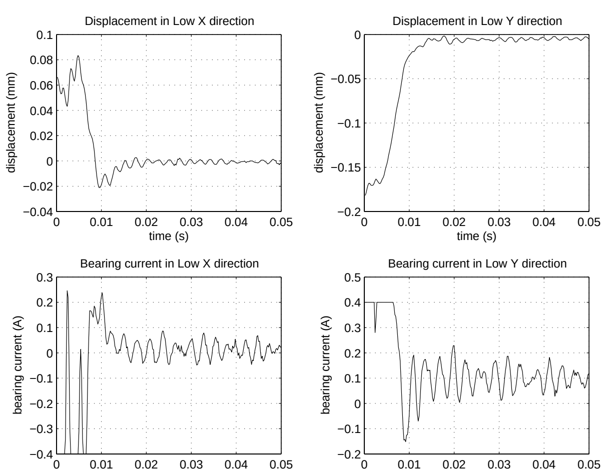

Figure 5: Decreased bearing coil bias current

6 CONCLUSIONS

We developed a new approach to H_∞ analysis and synthesis of LPV system with both measured varying parameter and unmeasured uncertainties and nonlinearities. A parameter dependent Lyapunov function was constructed to reduce conservatism and take advantage of the acceleration/deceleration information. Simulation and experimental results confirmed the effectiveness of our design.

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