ROTOR/AUXILIARY BEARING CONTACT ISSUES IN MAGNETIC BEARING SYSTEMS

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ABSTRACT

This paper assesses the characteristics of displacements in a rotor/magnetic bearing system when contact with an auxiliary bearing occurs. The dynamic rotor response is examined in fixed and rotating frames of reference with initial conditions to induce contact. In the rotating frame, it is demonstrated that the phase lag in the synchronous response vector relative to the unbalance vector may change by more than 90 deg due to contact. This is of particular relevance to controllers that utilise the synchronous harmonic components of measured signals.

The analytical study in the paper considers contact forces derived from Hertzian theory. This allows the frictional heat source to be specified over the contact zone. A simple method of estimating surface temperature around the contact zone is also presented.

INTRODUCTION

The use of auxiliary bearings in parallel with magnetic bearings prevents damaging interaction between rotor and stator lamination stacks. The problem is then transferred to the interaction of the rotor with a replaceable auxiliary bearing. Typically, the life of an auxiliary bearing is quoted in terms of a limited number of touchdowns. Although the reliability of magnetic bearing components has improved to the point when failure is considered to be remote, rotor/auxiliary bearing interactions are still possible when a magnetic bearing is fully functional. Any external shock or base motion with a sufficiently high acceleration level will cause contact. Transport applications will inevitably include a range of base motion inputs, which are likely to exceed the magnetic bearing load carrying capacity. The inputs may be of short duration, though persistent contacts could follow. To minimize auxiliary bearing damage it is important to restore position control of the rotor at the earliest opportunity. This will require an understanding of how displacement feedback signals are affected by contact.

There have been many studies directed to the dynamic interaction of a rotor with a stator component. The primary case to avoid is that of backward induced whirl [1, 2], which results in large and damaging contact forces that will exceed the load capacity of any practically designed magnetic bearing. This means that the restoration of position control is not feasible. However, resilient mounting of the auxiliary bearing and reduction of contact friction are design options that will reduce the occurrence of backward whirl.

The remaining contact modes involve a range of dynamic characteristics, which depend on system parameters. These include synchronous rub [3, 4], sub-synchronous rub [5-7], and chaotic response [8, 9]. The work reported in [10] considers the regions of rub and bounce type response. Other notable experimental procedures involve contact forces and vibration [11, 12], and realistic drop tests [13-15]. The development of non-linear dynamic analysis tools has yielded further in-depth studies of the dynamic contact problem [16-19]. With regard to magnetic bearing systems most studies of dynamic behavior involving contact do not consider the options for restoration of position control. The work reported in [20-22] focuses on the problem when control force is available.

The purpose of this paper is to bring attention to other issues that are associated with the rotor/auxiliary bearing contact problem. In addition to the changes in dynamic feedback signals caused by contact, thermal heating arising from the contact dynamics is discussed. The latter point has received little attention in the open literature. It is conceivable that the limited life of auxiliary bearings may be due to thermoelastic distortion of races and rolling elements. A first step towards assessing the level of this effect is the evaluation of contact temperatures.

NOTATION

a	contact zone half-length
c_b	resilient mounting viscous damping rate
c_r	rotor/auxiliary bearing radial clearance
c_m	rotor/magnetic bearing radial clearance
e	unbalance eccentricity
E	Young's modulus
f_c, $f_{mx,y}$	normal contact, magnetic bearing forces
k_b	resilient mounting stiffness
$k_{f,m}$	magnetic bearing force coefficients
K	thermal conductivity
l_b	auxiliary bearing length
m	mass
p	contact zone pressure
Pe	Peclet number
q	contact zone heat flux
R	radius
s	tangential contact zone coordinate
T	temperature
u,v	deflections in a rotating reference frame
V	initial velocity
$V_{0,c}$	magnetic bearing reference, control voltages
x,y	deflections in a fixed reference frame
α	heat partition coefficient
β	initial angle of incidence
δr	rotor to auxiliary bearing radial deflection
ϕ	unbalance phase
κ	thermal diffusivity
μ	friction coefficient
ν	Poisson's ratio
θ	polar coordinate
ω_c	angular slip speed in contact zone
Ω	rotor angular speed
Subscripts	
r	refers to rotor section
b	refers to auxiliary bearing

CONTACT FORCES/FRICTIONAL SOURCE

Figure 1 shows a rotor section in contact with a resiliently mounted auxiliary bearing. The absolute displacement of the auxiliary bearing in a fixed reference frame is (x_b,y_b) while that of the rotor is (x_r,y_r). The rotor will be in contact with the auxiliary bearing if $\delta r = \sqrt{(x_r - x_b)^2 + (y_r - y_b)^2} \geq c_r$, where c_r is the radial clearance. According to Hertzian theory, the length of the contact zone (2a) is related to the normal contact force on the rotor, f_c, by

$$a = \sqrt{4R^* f_c / \pi E^* l_b} \tag{1}$$

where

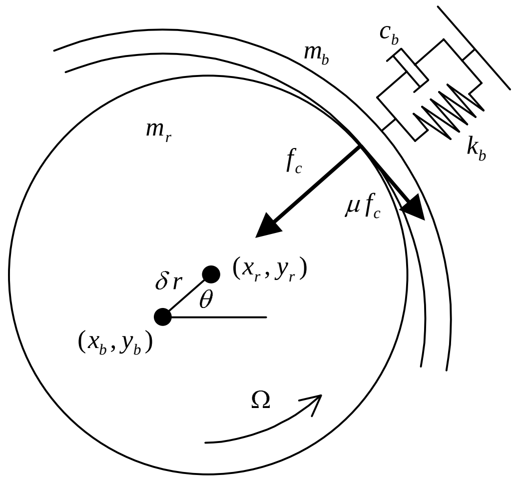

FIGURE 1: Rotor forces due to contact with auxiliary bearing

$$\left.\begin{aligned} R^* &= R_b(R_b - c_r)/c_r \\ 1/E^* &= (1-\nu_r^2)/E_r + (1-\nu_b^2)/E_b \end{aligned}\right\} \tag{2}$$

The non-linear contact force/deflection relation is then given by

$$\delta r - c_r = \frac{f_c}{\pi E^* l_b}\left(\frac{2}{3} + \ln\frac{4R_r R_b}{a^2}\right) \tag{3}$$

A variable s tangential to the contact is used to define the contact zone $-a \leq s \leq a$ over which the pressure is

$$p = \frac{E^*}{2R^*}\sqrt{a^2 - s^2} \tag{4}$$

Assuming that a simple Coulomb friction law applies, the total heat flux developed in the contact zone is

$$q = \mu p R_r \omega_c = \frac{\mu E^* R_r \omega_c}{2R^*}\sqrt{a^2 - s^2} \tag{5}$$

where ω_c is the angular slip speed between the rotor and auxiliary bearing.

MAGNETIC BEARING FORCES

A standard model for an eight-pole radial magnetic bearing in a differential driving mode is used. The magnetic bearing forces, including a simplified saturation limit, are written as

$$f_{mx,y} = -\frac{k_m}{k_f}\tanh k_f\left(\frac{(V_0+V_c)^2}{(c_m+z)^2} - \frac{(V_0-V_c)^2}{(c_m-z)^2}\right) \tag{6}$$

where $z = x_r$ or y_r, and it is taken that $c_r < c_m$. Control of the rotor in each axis is achieved using a PID feedback law on the rotor displacements (x_r,y_r), which are assumed to correspond with measurement signals.

DYNAMIC EQUATIONS OF MOTION

The equations of motion for the auxiliary bearing and rotor are

$$\left.\begin{aligned} m_b\ddot{x}_b + c_b\dot{x}_b + k_b x_b &= f_c\cos\theta - \mu\, f_c\sin\theta \\ m_b\ddot{y}_b + c_b\dot{y}_b + k_b y_b &= f_c\sin\theta + \mu\, f_c\cos\theta \end{aligned}\right\}$$

$$\left.\begin{aligned} m_r\ddot{x}_r &= f_{mx} - f_c\cos\theta + \mu\, f_c\sin\theta + m_r e\Omega^2\cos(\Omega t+\phi) \\ m_r\ddot{y}_r &= f_{my} - f_c\sin\theta - \mu\, f_c\cos\theta + m_r e\Omega^2\sin(\Omega t+\phi) \end{aligned}\right\} \tag{7}$$

which includes unbalance. Static forces on the rotor are considered to be compensated by integral control action and are thus omitted. The initial conditions are

$$\left.\begin{aligned} & x_b(0) = y_b(0) = \dot{x}_b(0) = \dot{y}_b(0) = 0 \\ & x_r(0) = x_{r0},\ y_r(0) = y_{r0} \\ & \dot{x}_r(0) = V\cos\beta,\ \dot{y}_r(0) = V\sin\beta \end{aligned}\right\} \tag{8}$$

where the velocity V is applied at an angle of incidence β relative to the x_r axis. These conditions could be considered as arising from external base acceleration.

When viewed in the fixed reference frame, a synchronous rotor response will typically correspond with a circular orbit (with or without contact). Synchronous control strategies process the time data in (x_r,y_r) to extract synchronous harmonic components. These are subsequently used to apply harmonic control forces using an open loop gain matrix. An alternative view of synchronous harmonic components is in a synchronously rotating reference frame. In this frame the rotor orbit (u_r,v_r) is given by

$$\begin{bmatrix} u_r \\ v_r \end{bmatrix} = \begin{bmatrix} \cos\Omega t & \sin\Omega t \\ -\sin\Omega t & \cos\Omega t \end{bmatrix} \begin{bmatrix} x_r \\ y_r \end{bmatrix} \tag{9}$$

Thus synchronous responses will appear as stationary vectors in the rotating frame.

CONTACT TEMPERATURES

The full prediction of the transient thermal response of the rotor/auxiliary bearing system is beyond the scope of this paper. Instead, the local flash temperature concept of Blok [23] is used to estimate the surface temperatures. The total frictional heat flux of equation (5) must be partitioned into $q_b = \alpha q$ and $q_r = (1 - \alpha)q$ entering the bearing and rotor respectively. The partitioning coefficient α is dependent on the slip speed [24] and is taken as

$$\alpha = 1\Big/\left(1 + \frac{K_b}{K_r}\sqrt{\frac{1+Pe_b}{1+Pe_r}}\right) \tag{10}$$

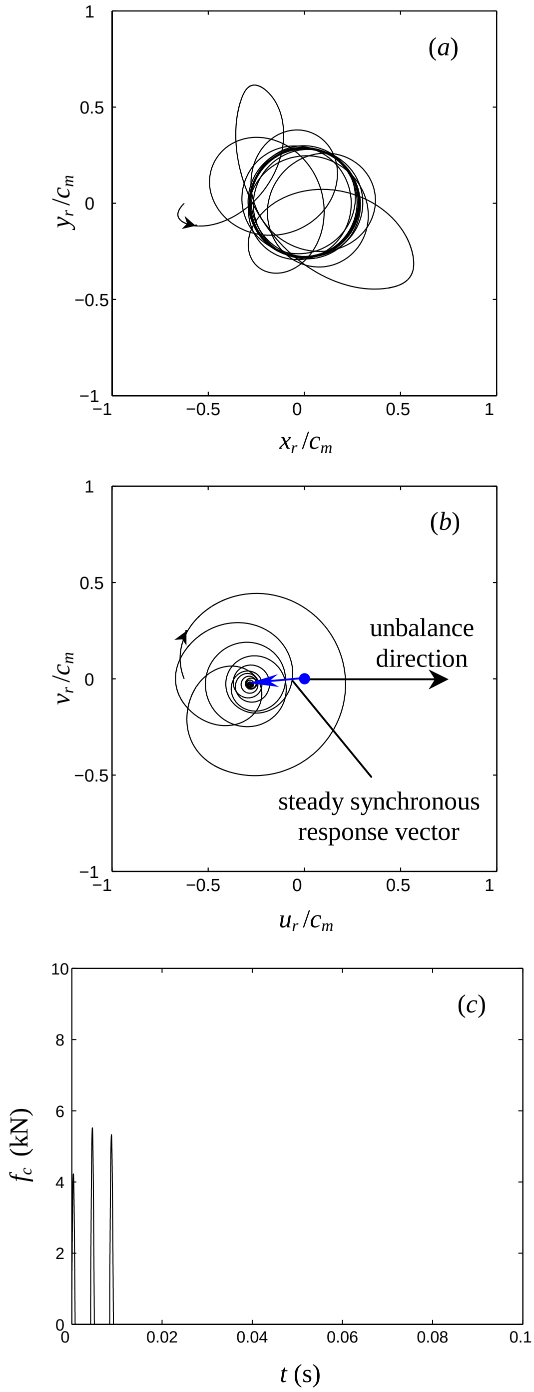

FIGURE 2: Rotor motion in fixed frame (a), rotating frame (b). Contact force response (c). Initially ($x_r(0)$, $y_r(0)$) = ($-c_r$, 0), V = 0.2 m/s, $\beta = -135$ deg

where $Pe_b = R_r\omega_c a/2\kappa_b$ and $Pe_r = R_r\omega_c a/2\kappa_r$ are Peclet numbers. A method for evaluating the surface temperature is given in [25] (page 269). Using the defined heat flux q_b, the surface temperature of the auxiliary bearing in a region local to the contact zone is estimated from

$$T_b = \alpha \frac{\mu \kappa_b^2 E^*}{\pi K_b R_r \omega_c R^*} \times \int_{S-Pe_b}^{S+Pe_b} e^u K_0(|u|) \sqrt{Pe_b^2 - S^2 + 2uS - u^2}\, du \tag{11}$$

where $S = R_r \omega_c s / 2\kappa_b$.

A CASE STUDY

The following rotor/auxiliary/magnetic bearing data were chosen for a case study:

m_r = 20 kg, R_r = 44.5 mm, e = 0.2 mm, $\phi = 0$,
m_b = 0.1 kg, R_b = 45.0 mm, l_b = 10.0 mm,
$c_b = 10^4$ Ns/m, $k_b = 2\times10^8$ N/m,
$E_r = E_b = 2.1\times10^{11}$ N/m^2, $\nu_r = \nu_b = 0.3$,
$K_r = K_b$ = 35 W/m/K, $\kappa_r = \kappa_b = 8.9\times10^{-6}$ m^2/s,
c_r = 0.5 mm, c_m = 0.8 mm,
$\mu = 0.1$, Ω = 1000 rad/s.

The magnetic bearing PID parameters were chosen to give a linearized radial stiffness of around 1MN/m and linearized radial damping of around 1 kNs/m. Thus the rotor angular speed of 1000 rad/s was well above the natural frequency (225 rad/s) of the levitated rotor. With these parameters, the expected steady synchronous response of the rotor without auxiliary bearing contact is a near circular forward whirl orbit of radius close to e = 0.2 mm. Also, radial oscillation of the auxiliary bearing mass alone on the mounting spring/damper is over-critically damped.

To induce rotor/auxiliary bearing contact, the initial velocity was selected at V = 0.2 m/s with initial rotor position $(x_r(0),y_r(0)) = (-c_r,0)$, just touching the auxiliary bearing. Figure 2 shows the system response for an angle of incidence of $\beta = -135$ deg. The rotor maintains contact for three bounces and then loses contact. In the fixed reference frame (figure 2(a)), the rotor is seen to settle in the expected forward whirl orbit. In the rotating frame (figure 2(b)), the steady synchronous response vector position has a phase lag relative to the unbalance vector of nearly 180 deg.

The angle of incidence was then changed to β = 135 deg. Figure 3 indicates increased interaction between the rotor and auxiliary bearing. In the fixed frame (figure 3(a)), the rotor becomes steady in a forward circular rub mode. In the rotating frame (figure 3(b)), the circular rub is seen to be synchronous with the running speed and the phase lag of the steady synchronous response vector is now around 40 deg. Compared with that in figure 2(b), this phase lag is consistent with the relatively high mounting stiffness influencing the rotor response. The contact force behavior of figure 3(c) shows that multiple bounces occur with a peak levels over 40 kN before continuous

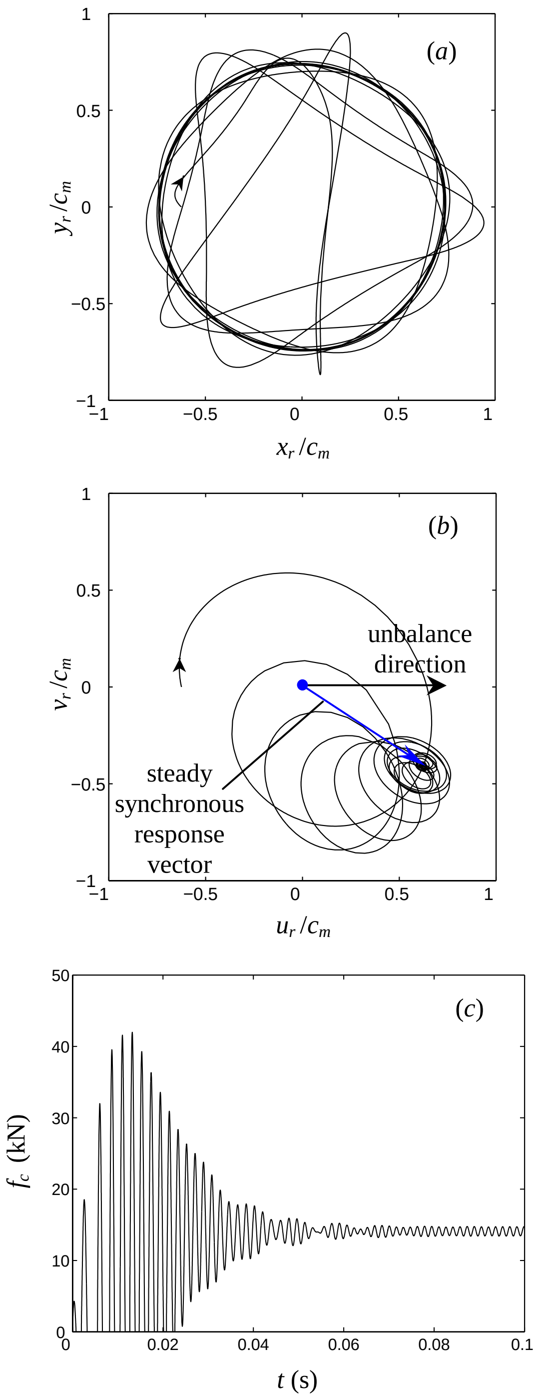

FIGURE 3: Rotor motion in fixed frame (a), rotating frame (b). Contact force response (c). Initially $(x_r(0), y_r(0)) = (-c_r, 0)$, V = 0.2 m/s, β = 135 deg

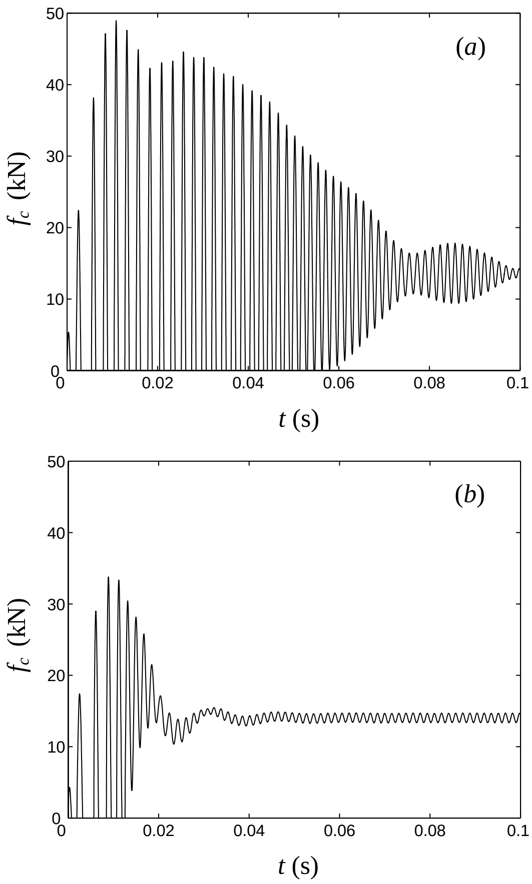

FIGURE 4: Rotor contact force responses. Data changes from FIGURE 3: (a) $k_b = 3\times10^8$ N/m; (b) $c_b = 2\times10^4$ Ns/m

rub occurs with an eventual steady state of around 15 kN. The oscillation seen in the contact force prior to the steady state is dominated by the natural frequency of the rotor mass m_r on the mounting stiffness k_b.

Figure 3(a) shows that the rotor deflections due to the bounce motion may be close to the rotor/magnetic bearing clearance circle. However, for large auxiliary bearing deflection, the mounting stiffness characteristic will be non-linear and increasing. The value of the mounting stiffness was therefore increased by 50% to $k_b = 3\times10^8$ N/m and the contact forces are shown in figure 4(a). Although the rotor defection is reduced (not shown), the rotor bounce motion is more severe and persistent with peak levels almost at 50 kN. Figure 4(b) shows the effect of increasing the mount damping. With all other data as for figure 3, the damper rate $c_b = 2\times10^4$ Ns/m resulted in fewer contacts and peak forces around 35 kN.

The dynamic simulation results indicate that a range of contact force levels is likely depending on the initial conditions. A preliminary assessment of contact

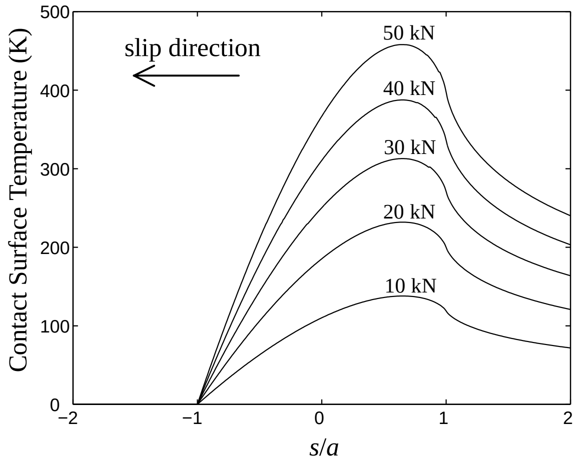

FIGURE 5: Surface temperature rise around contact zone $-1 \le s/a \le 1$, with $\mu = 0.1$and various f_c shown

temperatures may be made through evaluation of equation (11), considering f_c as a steady component. It is also assumed that the angular slip speed $\omega_c = \Omega$. Figure 5 shows the predictions of surface temperature rises local to the contact zone for a range of f_c. These show that significant temperature rises occur with maxima towards the trailing edge of the contact zone.

DISCUSSION AND CONCLUSIONS

It has been demonstrated that the phase lag of the synchronous response vector of the rotor orbit relative to the unbalance vector may change by more than 90 deg due to contact. This has significant consequences for synchronous controllers that are designed to minimize rotor vibration through gains that are appropriate to the system without contact. Any attempt to use the same gains to restore rotor position control from a synchronous rub mode is likely to result in even harder contact.

There are several issues concerned with the contact temperature predictions. Firstly, they are based on steady contact forces, though the dynamic predictions are highly transient. However, the temperatures are confined to a skin depth, which will have a sufficiently small thermal mass to accommodate the source fluctuations. A second issue relates to the assumption that the angular slip speed is equal to the rotor angular speed. This is the case for a static bush, but the slip speed will reduce in a rolling element bearing as the inner race is accelerated by friction. However, this may be offset against the chosen friction coefficient of $\mu = 0.1$. Under dry contact conditions, a value of $\mu = 0.3$ is more likely, which will increase predicted temperature rises. Finally, skewed contact towards the edge of an auxiliary bearing could result in a thinner contact zone and hence higher temperatures.

The results bring attention to the fact that significant contact temperatures may occur in rotor/auxiliary bearing interactions. The need for a proper transient analysis of the thermal response is recommended.

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