

## A NONLINEAR AMB MODEL & IDENTIFICATION METHODS

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### ABSTRACT.

This paper presents a single axis nonlinear model of the Active Magnetic Bearing (AMB). In the AMB technology properly designed control algorithm is the most important element. The basic knowledge in the controller design process is the model of the system. It is well known that the AMB cannot operate without control due to structural instability and strong nonlinearities. Thus the modeling studies and controller performance are important stages in the AMB development.

The laboratory test rig containing the AMB is presented. The system is controlled from a standard PC computer, where the integrated real-time control and rapid prototyping environment is used. The dedicated I/O board with custom logic applied in the FPGA chip is used for measurements and control signals formulation. Rotor position is measured by proximity sensors. To obtain the full information about the actuator unit operating mode the coil current is measured too. A number of identification procedures is used to obtain parameters of the AMB model. The open and closed loop identifications are used. The physical fundamentals of the behavior of the electromagnet are used in the modeling process. They are extended by nonlinear functions due to results obtained from the identification.

### INTRODUCTION

In recent years a number of rotary machines using Active Magnetic Bearings were designed in order to eliminate the lubricant medium, vibration, noise and to achieve high velocities and loads. A wide range of control algorithm is being tested to obtain desired performance.

A number of publications that discuss the AMB construction, modeling and controller synthesis [1], [2], [3] have appeared all over the world.

The magnetic bearing system is characterized by nonlinearities coming from electromagnetic forces, state and control constraints and electrical characteristic of the power-supply actuator unit. These strong nonlinearities make the linear model obtained due to local linearization work well in a very small operating area around the operating point. If the operating point

moves from its original place, the system can become unstable due to neglected nonlinearities (Example: tracking problem in the spindle application [4]).

Generally, for precise control of AMB systems nonlinear controllers are better but more complicated than linear ones. Therefore, a wide range of linear models and controllers were tested and implemented for AMB systems [5]. Some of them approximate the process quite well around the operating point. With development of today's PC/DSP based technology nonlinear methods are more often used in AMB control algorithm application [6].

### TEST RIG DESCRIPTION

The AMB laboratory model (Fig. 1) consists of the rotor suspended in two magnetic bearings [7]. Each bearing is controlled in two perpendicular axes. The rotor position is measured by proximity sensors located at each axis. Two proximity probes at 90° apart are installed. In this way the actual displacement of the rotor could be seen.

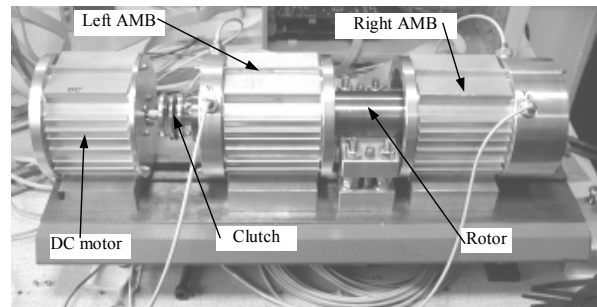


Fig. 1. AMB - Laboratory model

The electromagnet coils are controlled from a power driver using PWM technique. Generally the system consists of 12 measurements (rotor positions and coil currents) and 8 control signals (for both AMB). The AMB system is connected to the computer with the RT-DAC3 multi I/O board [8] equipped with a FPGA programmable chip. A user defined logic (Fig. 2) dedicated for the AMB system was applied in this case, drastically increasing the control loop throughput and

making the real-time characteristics of the controller compatible with sampling requirements of the process.

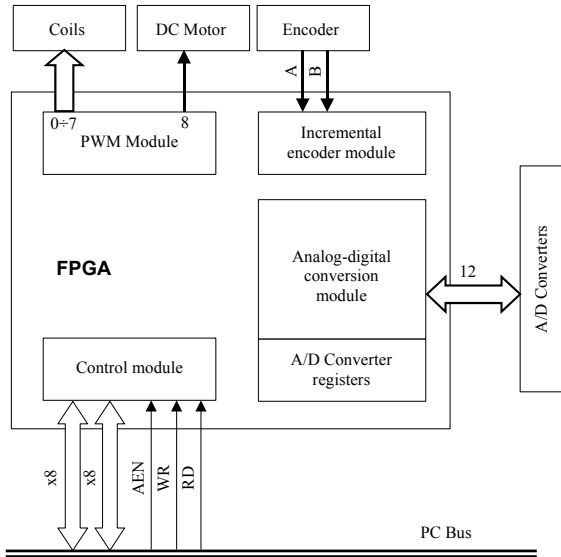


Fig. 2. FPGA Logic

The set of drivers is required to establish a connection between software and hardware layer. System drivers are available in the operating system. The I/O board drivers are dedicated to the current I/O board logic. To provide the real-time control in the Windows system the Real Time Windows Target [9] is used. This MATLAB/Simulink extension allows to set the sampling frequency up to 20kHz. The maximum rate depends on hardware architecture. The control loop is introduced in the MATLAB/Simulink environment.

### MODEL DESIGN

Models of real systems are of fundamental importance and can be useful for system analysis. Models are required for the design of new processes and analysis of existing ones. Advanced techniques for the design of controllers, fault detection, diagnosis (etc.) are also based on models [10].

A single magnetic bearing consists of 4 electromagnets controlled separately. Electromagnets are located at the top and bottom of each axis. Figure 3 presents the configuration of the magnetic bearing and forces acting on the rotor.

Assuming that the axial configuration is identical in both bearings a single axis control problem can be analyzed and then applied to both. Another possibility is to identify each of the electromagnets separately to obtain individual actuator parameters. The electromagnetic force ( $F_X$  or  $F_Y$ ) acting on the suspended mass is calculated as a sum of electromagnetic forces produced by the lower and upper electromagnet in the considered axis.

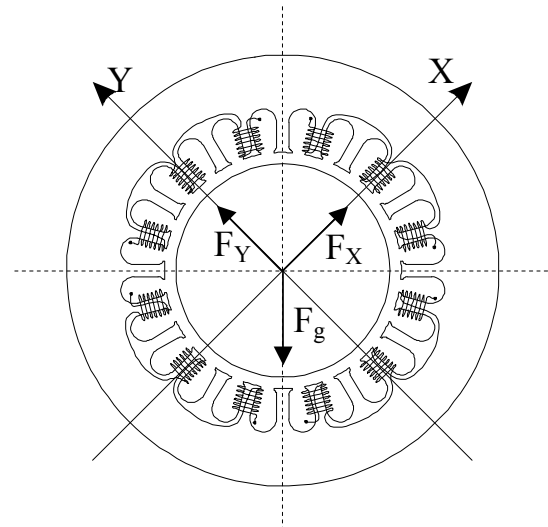


Fig. 3. AMB – Forces acting on the rotor

The electromagnetic force model is based on the dependence of coil inductance  $L(x_1)$  on the distance  $x_1$  of the rotor from the surface of the electromagnet.

$$L'(x_1) = \frac{dL(x_1)}{dx_1} = -\frac{L_0}{b} \exp\left(\frac{-x_1}{b}\right) + 2cx_1 \quad (1)$$

where  $L_0$ ,  $b$ ,  $c$  are constants, and  $x_1$  is the axial rotor position. The electromechanical model of a single axis is given in (1). The control signal is applied in the differential mode.

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{1}{2m} [L^{(1)}(x_1) \cdot x_3^2 - L^{(1)}(d - x_1) \cdot x_4^2] + \frac{F_{gx}}{m} \\ \dot{x}_3 &= \frac{a_3(u_{c3} - u) + b_3 - P1_3 x_3}{P2_3} \\ \dot{x}_4 &= \frac{a_4(u_{c4} + u) + b_4 - P1_4 x_4}{P2_4} \end{aligned} \quad (2)$$

where:

- $x_1$  axial mass position [m],
- $x_2$  axial mass velocity [m/s],
- $x_3$  current in upper coil of the considered control axis [A],
- $x_4$  current in lower coil of the considered control axis [A],
- $u$  control signal – PWM duty,
- $u_{c3}$  constant control value,
- $u_{c4}$  constant control value,
- $m$  mass suspended in bearing 3.48 [kg],

- $d$  maximum air gap  $8.0 \cdot 10^{-4}$  [m],
- $F_g$  gravitation force acting in control axis  
 $F_{gX} = F_{gY}$  [N],
- $L(\cdot)$  coil inductance [H],
- $a_i, b_i, P1_i, P2_i$  actuator parameters  $i = \{3,4\}$ .

States and control signals are bounded as follows:

$$x_1 \in [0, d], x_2 \in \mathfrak{R}, x_3 \in [0, 5], x_4 \in [0, 5],$$

$$(u_{c3} - u) \in [0, 1], (u_{c4} + u) \in [0, 1].$$

Keeping the right side of equations (2) to zero and keep count about boundaries the steady state points can be obtained:

$$\begin{aligned} x_{10} &\in [0, d], x_{20} = 0 \\ x_{40} &\in [0, 5] \\ x_{30} &= \sqrt{\frac{L^{(1)}(d - x_1)x_{40}^2 - 2F_{gX}}{L^{(1)}(x_1)}} \\ u_0 &= 0 \\ u_{c3} &= \frac{P1_3 x_{30} - b_3}{a_3} \\ u_{c4} &= \frac{P1_4 x_{40} - b_4}{a_4} \end{aligned} \quad (3)$$

At the operating point the axial velocity is equal to zero. For  $x_{40} = 0$  the upper electromagnet produces electromagnetic force that compensates gravity. The magnetic bearing stiffness can be adjusted by choosing an appropriate value of  $x_{40}$ . The constant control values  $u_{c3}$  and  $u_{c4}$  depend on the values of current chosen for the selected operating point.

### VERIFICATION OF POSITION SENSORS

The rotor position is measured by the displacement proximity sensors [11] located at each axis. The sensors accuracy is given by the manufacturer. The main idea of the designed automatic verification algorithm is to move the rotor around the interior of the bearing. The rotor movement is possible because of the AMB construction – the presence of electromagnets. The consecutive coils are powered to produce the electromagnetic force and to attract the rotor to the electromagnet surface. The coil control sequence is illustrated in Figure 4 and the result of the reeling process for the left AMB is presented in Figure 5. The control sequence is applied to both AMB to ensure the symmetrical movement of the rotor across the bearing plane.

Using this experiment - in the case of AMBs mounted separately it is possible to certify that the

bearings are set axially – then the noncircular shape will be obtained from rotor position measurements.

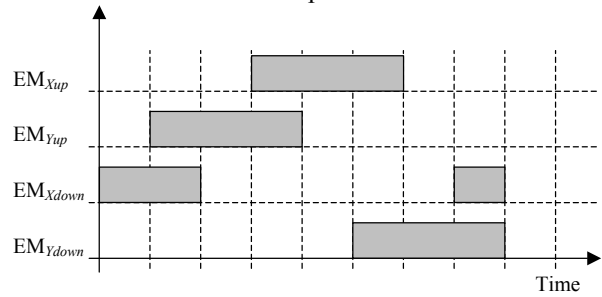


Fig. 4. AMB – Electromagnets control sequence

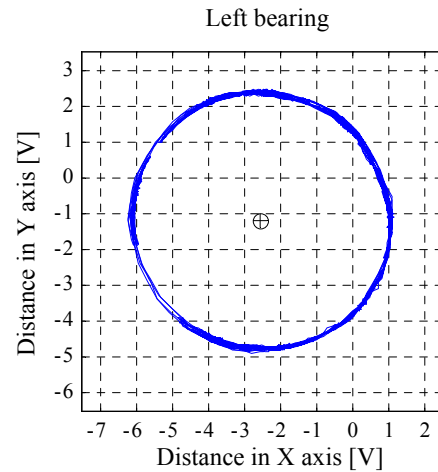


Fig. 5. AMB – Electromagnets control sequence

### ACTUATOR PARAMETERS

Third and fourth equation in a set (2) represents the actuator (electromagnets and power amplifiers) parameters. Because the power amplifiers are controlled by the PWM technique it is required to select PWM signal frequency. The automatic identification procedure based on setting of PWM duty, PWM frequency change and current measurement is used to obtain a set of characteristics for a different rotor locations.

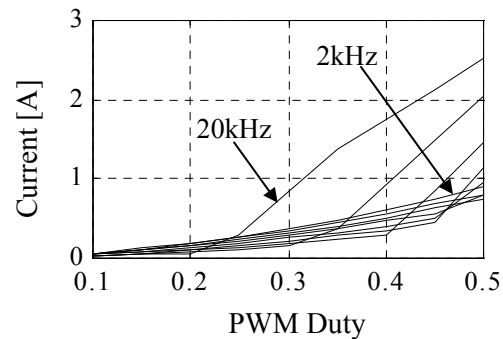


Fig. 6. AMB – Current characteristics – rotor removed from the AMB

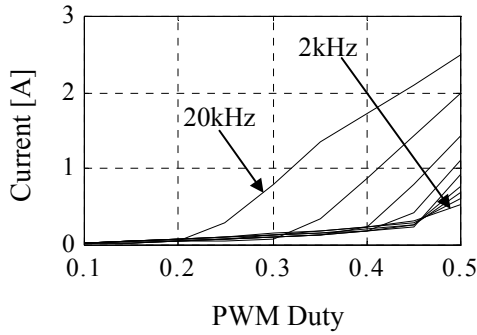


Fig. 7. AMB – Current characteristics – rotor located at the AMB center

Experimental results presented in Fig. 6 and 7 shows that the highest frequency can be chosen as operational. In this case there is no sensitivity of rotor position and current oscillations are minimized. The parameters  $a_i$  and  $b_i$  are calculated from obtained characteristics.

Parameters  $P1_i$  and  $P2_i$  are calculated using step responses of the actuator by the least square optimization method. Actuator step characteristics can be approximated by the first order equation for small PWM duty changes (see Fig. 8).

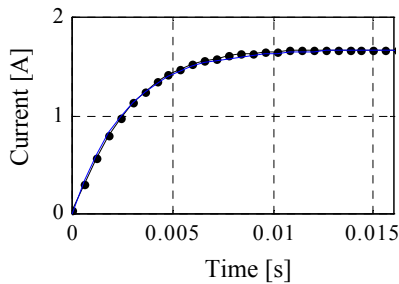


Fig. 8. Step response of the actuator unit

**ELECTROMAGNETIC FORCE**

The electromagnetic force based on the inductance change (eq. 1) is calculated using closed loop identification experiment. Any control algorithm with integration can be applied. Parameters of the electromagnetic force can be calculated theoretically using the standard formula [1], [2]. Then controller parameters can be synthesized and applied for different operating points across the bearing air gap. The rotor is stabilized in several rotor positions. The system can be rotated 45 degrees to produce the electromagnetic force in a single AMB electromagnet for gravity compensation. Then the nonlinear electromagnetic force approximation, included in second equation of (3) can be optimized. Thus the standard formula is modified by the proposed one. For the operating point with the rotor located at the bearing center the

electromagnetic force characteristics vs. rotor position and coil current have been calculated. As shown in Figures 9 and 10, the force sensitivity to deviations from the selected operating point is high.

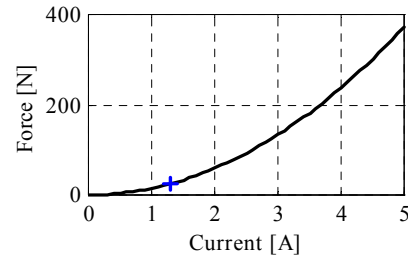


Fig. 9. Force vs. current for fixed rotor position

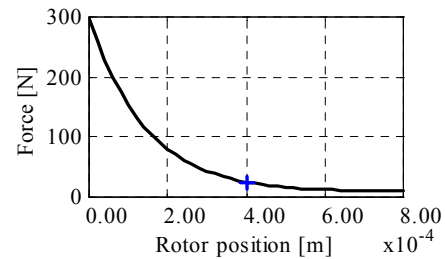


Fig. 10. Force vs. rotor position for fixed current value

**CONTROLLER TESTING**

Thus the linear model will operate properly in a very small area around the steady-state point (see Fig. 11). Better performance can be achieved while the nonlinear controller is used (e.g. Takagi-Sugeno Fuzzy Logic Controller [12]) (see Fig. 12.).

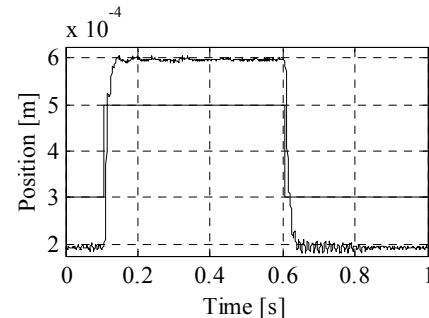


Fig. 11. LQ controller – desired and rotor position

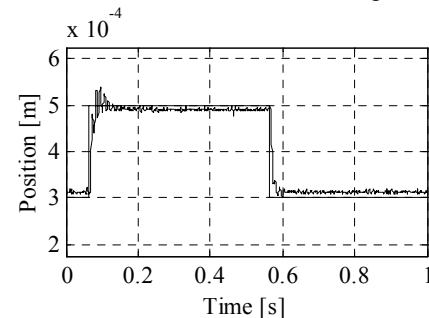


Fig. 12. Fuzzy logic controller – desired and rotor position

During time-optimal controller synthesis the following problem occurs: the bang-bang control signal obtained by the synthesis procedure applied in the test rig was resulted in another rotor trajectory. A number of test were performed applying the control signal in the open loop. The conclusion is that the model of actuator unit does not match the real actuator for large control changes of the bang-bang type. To solve this problem the power supply unit can be rebuilt or the model can be changed.

### MODEL MODIFICATION

The analysis of current behavior shows that the characteristic differs while the direction and control values are changing rapidly. In this case the third and fourth equations in (2) can be replaced by the neural-network trained with real system responses. Two layers were used: the first containing 5 neurons with *tansig* transfer function, the second containing 1 neuron with *purelin* transfer function. Neural network was trained using Levenberg-Marquardt backpropagation method. The medium square error between model response and real signals is equal to  $1.11485 \cdot 10^{-5}$ .

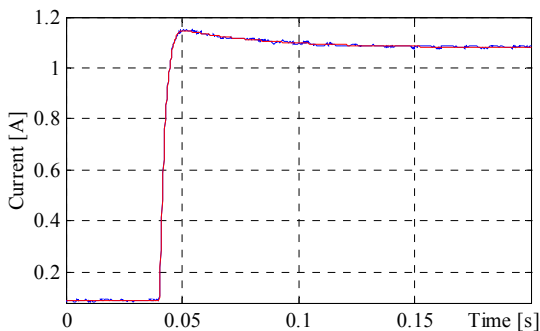


Fig. 13. Current response for step control change

### CONCLUSIONS

A number of open and closed loop identification procedures were presented. The simulation model was verified using real signals. The desired controllers show that the control algorithms work properly while rotor position is not changing rapidly. This situation appears for standard bearing operation – levitation at the bearing center.

The modeling of the AMB is a complex process and a wide range of techniques can be used. To obtain a good model for the control purposes it is necessary to model each part of the system precisely. Due to strong nonlinearities the linear controller operates properly only in a small region around the steady state point. More advanced controllers must be used to protect the AMB in critical situations (high disturbances or loads). Thus the global nonlinear model is required to test the controller at the simulation stage.

Further research will be focused on signal analysis and analytical formulation of actuator behavior. It seems to be necessary to equip the test rig with some additional sensors.

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