GEOMETRICAL COUPLING BETWEEN AXIAL AND RADIAL DYNAMICS OF AMB

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ABSTRACT

The coupling between angular and radial motion in rotors supported by AMB has been studied in a previous paper by the Authors. The present paper generalized these results to conical AMB actuators taking into account also radial-axial coupling. The aim is to assess how much these effects influence the dynamics in the small of the rotor but does not restrict to the traditional linearization. The linearized model is studied together with the complete nonlinear one, to deal also with second-order effects.

INTRODUCTION

Active magnetic bearings are usually modelled as linear system, but they actually are intrinsically nonlinear. The reasons for nonlinearities are many, namely the nonlinear dependence of the magnetic forces from the displacements and currents in open loop and the nonlinearities of all components of the control loop [1, 2]. The aim of the present paper is building a mathematical model of a general conical heteropolar bearing, taking into account the first two causes of nonlinearity. The control loop can be closed by accounting for other sources of nonlinearity, such as saturation of the power amplifiers and of the sensors. The model can be linearized, yielding results close to those obtained using the conventional formulae; however some axial-radial coupling, usually neglected, is present also in the linearized solution.

Although devised for heteropolar configurations, the model can be modified to deal with homopolar bearings. An example related to a satellite reaction wheel on two six-poles conical bearings shows how the present model can be used for a specific application.

MAGNETIC CIRCUIT

Consider one of the electromagnets of a conical magnetic bearing (Fig. 1a). The geometrical configuration is general for heteropolar bearings (if $\delta = 0$ a cylindrical bearing is obtained), while for homopolar bearings some modifications to the model are needed.

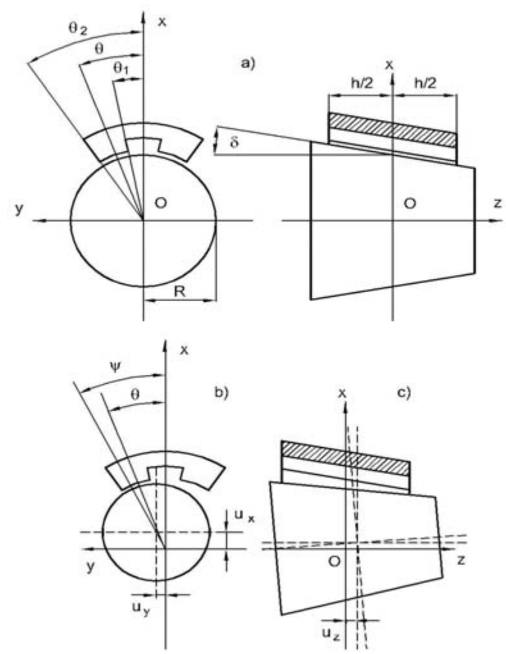

Figure 1: (a) Geometric definitions. (b) Displacement of the journal in the midplane of the bearing. (c): Axial displacement and rotations.

Point O is the center of the bearing. The components of the displacement of the journal are u_x, u_y and u_z and its rotations about axes x and y are ϕ_x and ϕ_y. Under the small rotations assumptions, the order in which the two rotations are considered needs not to be stated [2].

The magnetic circuit can be studied by neglecting the reluctance of the iron parts of the circuit. The total reluctance then reduces to that of the two airgaps, which are equal only if the shaft is in a symmetrical position with respect to xz plane (u_y and ϕ_x vanishingly small). In general

$$\mathcal{R} = \mathcal{R}_1 + \mathcal{R}_2 \ , \tag{1}$$

subscript 1 referring to the airgap with $y > 0$.

The reluctance R_j can be computed as

$$\frac{1}{\mathcal{R}_j} = \int_A \frac{\mu_0}{d} dA = \mu_0 \int_{-h/2}^{h/2} \int_{\theta_1}^{\theta_2} \frac{r(z)}{d(z,\theta)\cos(\delta)} dz\ d\theta \ . \tag{2}$$

where A is the area of the j-th pole piece and $d(z,\theta)$ is the local air gap thickness. The latter can be evaluated with good approximation as

$$d = [c - u_z \tan(\delta) - \xi \cos(\theta) - \eta \sin(\theta)] \cos(\delta) \ , \tag{3}$$

where

$$\begin{cases} \xi = u_x + \phi_y \left[\frac{z}{\cos^2(\delta)} - R\tan(\delta)\right] & -c < \xi < c \\ \eta = u_y - \phi_x \left[\frac{z}{\cos^2(\delta)} + R\tan(\delta)\right] & -c < \eta < c \end{cases}$$

The integration along angle θ can be performed in an approximated way, obtaining

$$\frac{1}{\mathcal{R}_j} \approx \mu_0 \int_{-h/2}^{h/2} \frac{R + z\tan(\delta)}{S_j + T_j z}\ dz \ , \tag{4}$$

where

$$\begin{cases} S_j = \left\{\alpha_j [c - u_z \tan(\delta)] + [u_y + R\phi_x \tan(\delta)]\beta_j + \right. \\ \qquad \left. + [u_x - R\phi_y \tan(\delta)]\gamma_j\right\} \cos^2(\delta) \\ T_j = -\phi_x \beta_j + \phi_y \gamma_j \ . \end{cases} \tag{5}$$

and α_j, β_j and γ_j are three constants which depend only on the angular width and position of the pole piece (see Appendix 1). θ_1 and θ_2 are here considered as independent from z, which yields a pole piece circumferentially thicker at the larger diameter. α_j, β_j and γ_j can be considered as functions of z, but this would result in much more complex formulae, without substantial increase of the accuracy of the model.

If the electromagnet is located symmetrically with respect to x-axis, it is possible to define the values of the parameters for the the whole electromagnet

$$\alpha = \alpha_1 = \alpha_2 \ , \quad \beta = \beta_1 = -\beta_2, \quad \gamma = \gamma_1 = \gamma_2 \ . \tag{6}$$

By performing the integration, equation (4) becomes

$$\frac{1}{\mathcal{R}_j} = \mu_0 \left[\frac{h\tan(\delta)}{T_j} + \frac{RT_j - S_j\tan(\delta)}{T_j^2} \ln\left(\frac{2S_j + T_j h}{2S_j - T_j h}\right)\right] \ . \tag{7}$$

If the axis of the journal remains parallel to that of the bearing, T_j vanishes and equation (7) reduces to

$$\frac{1}{\mathcal{R}_j} = \mu_0 \int_{-h/2}^{h/2} \frac{R + z\tan(\delta)}{S_j}\ dz = \frac{\mu_0 R h}{S_j} \ . \tag{8}$$

The magnetic flux density for the j-th pole is linked to the number of turns of the coil N and the total current i by the relationship [3]

$$B_j(z,\theta) = \frac{\mathcal{R}_j}{\mathcal{R}_1 + \mathcal{R}_2} \frac{\mu_0 N I}{d(z,\theta)} \ . \tag{9}$$

The magnetic energy is then

$$\mathcal{E} = \frac{1}{2\mu_0} \sum_{j=1,2} \left[\int_{V_1} B^2\ dV\right] \ . \tag{10}$$

Since the magnetic flux density is constant along the lines perpendicular to the pole pieces (with the above mentioned approximations), the volume integral can be transformed into an area integral

$$\mathcal{E} = \frac{\mu_0 N^2 i^2}{2(\mathcal{R}_1 + \mathcal{R}_2)^2} \sum_{j=1,2} \left[\mathcal{R}_j^2 \int_{-h/2}^{h/2} \int_{\theta_1}^{\theta_2} \frac{r}{d\cos(\delta)} dz\ d\theta\right] \ . \tag{11}$$

By comparing equation (11) with equation (2), it follows

$$\mathcal{E} = \frac{N^2 i^2}{2(\mathcal{R}_1 + \mathcal{R}_2)} \ . \tag{12}$$

MAGNETIC FORCES

The forces and moments exerted on the journal are

$$Q_i = \frac{\partial \mathcal{E}}{\partial q_i} = -\frac{N^2 i^2}{2(\mathcal{R}_1 + \mathcal{R}_2)^2} \frac{\partial (\mathcal{R}_1 + \mathcal{R}_2)}{\partial q_i} \ . \tag{13}$$

By introducing the permeance $C = 1/R$, the expression of the magnetic energy becomes

$$\mathcal{E} = \frac{N^2 i^2 \mathcal{C}_1 \mathcal{C}_2}{2(\mathcal{C}_1 + \mathcal{C}_2)} \ . \tag{14}$$

The generalized forces are then

$$\begin{aligned} Q_i = \tfrac{N^2 i^2}{2(\mathcal{C}_1 + \mathcal{C}_2)^2} &\left[\mathcal{C}_2^2 \left(\tfrac{\partial \mathcal{C}_1}{\partial T_1}\tfrac{\partial T_1}{\partial q_i} + \tfrac{\partial \mathcal{C}_1}{\partial S_1}\tfrac{\partial S_1}{\partial q_i}\right) + \right. \\ &\left. + \mathcal{C}_1^2 \left(\tfrac{\partial \mathcal{C}_2}{\partial T_2}\tfrac{\partial T_2}{\partial q_i} + \tfrac{\partial \mathcal{C}_2}{\partial S_2}\tfrac{\partial S_2}{\partial q_i}\right)\right] \ . \end{aligned} \tag{15}$$

BEARING MODEL

A bearing is made of a number n of electromagnets. The forces and moments exerted by the i-th electromagnet whose x-axis is rotated by angle Γ_i with respect to the X-axis of the bearing can be written in the reference frame XYz of the latter as

$$\mathbf{Q}_{i_{XYz}} = \mathbf{R}_i \mathbf{Q}_{i_{xyz}} \ , \tag{16}$$

where $Q = [F_x,\ F_y,\ F_z,\ M_x,\ M_y]^T$ and

$$\mathbf{R}_i = \begin{bmatrix} \cos(\Gamma_i) & -\sin(\Gamma_i) & 0 & 0 & 0 \\ \sin(\Gamma_i) & \cos(\Gamma_i) & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \cos(\Gamma_i) & -\sin(\Gamma_i) \\ 0 & 0 & 0 & \sin(\Gamma_i) & \cos(\Gamma_i) \end{bmatrix}_{i_{xyz}} \tag{17}$$

The forces can be linearized about any position as

$$\mathbf{Q}_i = \mathbf{Q}_{0_i} + \mathbf{J}_{0_i}\mathbf{q} \ , \tag{18}$$

where $\mathbf{q}$ is a column matrix in which the generalized displacements are listed, $\mathbf{Q}_{0_i}$ is the value of $\mathbf{Q}_i$ computed in the reference position and $\mathbf{J}_{0_i}$ is the Jacobian matrix $\partial Q_{ik}/\partial q_j$ computed in the same position. Owing to the intricacy of the general expressions of the forces and of their derivatives, both $\mathbf{Q}_{0_i}$ and $\mathbf{J}_{0_i}$ are better computed numerically in any particular case. If the behaviour of the bearing is uncoupled, $\mathbf{J}_0$ reduces to a diagonal matrix.

The linearized expression of the total force is

$$\mathbf{Q}_{XYz} = \sum_{i=1}^{n} \mathbf{R}_i \mathbf{Q}_{0_i} + \left(\sum_{i=1}^{n} \mathbf{R}_i \mathbf{J}_{0_i} \mathbf{R}_i^T \right) \mathbf{q}_{XYz} \ , \tag{19}$$

where also the displacements $\mathbf{q}_i$ must be expressed in the reference frame of the bearing

$$\mathbf{q}_{i_{XYz}} = \mathbf{R}_i \mathbf{q}_{i_{xyz}} \ . \tag{20}$$

Equation (19) is linear in $\mathbf{q}$ but not in the currents i_i. It can be linearized by introducing a bias current i_b, a compensation current i_0, to balance static forces, and a control current i_c[1, 2]

$$i_i = i_{b_i} + i_{0_i} + i_{c_i} \ . \tag{21}$$

Since the control current is assumed to be small if compared to the other two components, the force can then be expressed as

$$\mathbf{Q}_{XYz} = \mathbf{Q}_{b_0} - \mathbf{K}_{ol}\mathbf{q}_{XYz} + \mathbf{G}_c \mathbf{i}_c \ , \tag{22}$$

where

$$\mathbf{Q}_{b_0} = \sum_{i=1}^{n} \mathbf{R}_i \mathbf{Q}'_{0_i} \left(i_{b_i} + i_{0_i}\right)^2 \tag{23}$$

is the force exerted by the bearing at zero displacement and control current,

$$\mathbf{K}_{ol} = \sum_{i=1}^{n} \left(i_{b_i} + i_{0_i}\right)^2 \mathbf{R}_i \mathbf{J}'_{0_i} \mathbf{R}_i^T \tag{24}$$

is the open-loop stiffness matrix (negative defined) and $\mathbf{G}_c$ is the current gain matrix having 5 rows and as many columns as there are coils in the bearing

$$\mathbf{G}_c = 2 \left[\ \ldots \quad \left\{ \mathbf{R}_i \mathbf{Q}'_{0_i} \left(i_{b_i} + i_{0_i}\right) \right\} \quad \ldots \ \right] \tag{25}$$

and $\mathbf{Q}'_{0_i} = \mathbf{Q}_{0_i}/i_i^2$ and $\mathbf{J}'_{0_i} = \mathbf{J}_{0_i}/i_i^2$.

EXAMPLE: RIGID-BODY DYNAMICS OF A REACTION WHEEL ON CONICAL AMBs

The system is described in detail in [4]. The main inertial rigid-body characteristics of the system are: mass $m = 0.821$ kg, polar moment of inertia $J_p = 0.0374$ kg m^2, transversal moment of inertia $J_t = 0.0194$ kg m^2. The distance between the midplanes of the bearings is $l = 27$ mm, with the centre of mass located at midspan ($a = b = l/2$). The relevant data of the six-poles conical AMBs are: $N = 96$, bias current $i_b = 0.2$ A, average radius $R = 29$ mm, length $h = 7$ mm, radial clearance $c = 0.3$ mm, $\phi = 15°$, $\theta_1 = 9°$, $\theta_2 = 29°$. The values of the parameters which approximate the reluctance of the bearing are

$$\alpha = 2.8648 \ , \quad \beta = -0.9354 \ , \quad \gamma = -2.7030 \ . \tag{26}$$

Two cases will be considered: microgravity (no static load) and Earth gravity, with the static load supported by a single pole (x-axis directed upwards)

In the first case, no compensation current is present. Since the center of mass of the rotor is at midspan ($a = b$), in the other case the two bearings are equally loaded and the only nonzero compensation current is $i_{0_1} = 1.137$ A.

The displacement vector q_i at the $i-$th bearing can be obtained from the displacement of the center of mass as

$$\mathbf{q}_i = \mathbf{T}_i \mathbf{q}_G = \begin{bmatrix} 1 & 0 & 0 & 0 & z_i \\ 0 & 1 & 0 & -z_i & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \left\{ \begin{array}{c} u_x \\ u_y \\ u_z \\ \phi_x \\ \phi_y \end{array} \right\}_G . \tag{27}$$

In the same way, the forces due to a bearing referred to a frame located in the center of mass are obtained from those referred to the bearing itself as

$$\mathbf{f}_G = \mathbf{T}_i^T \mathbf{f}_i \ . \tag{28}$$

The open loop stiffness matrix (referred to the center of mass) of the whole suspension is obtained from those of the bearings as

$$K_{ol} = \sum_{\forall i} T_i^T K_{ol_i} T_i \ . \tag{29}$$

The mass and the gyroscopic matrices are

$$M = diag \left[\ m \quad m \quad m \quad J_t \quad J_t \ \right] \ , \tag{30}$$

$$\mathbf{G} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -J_p \\ 0 & 0 & 0 & J_p & 0 \end{bmatrix} . \tag{31}$$

The state space equation is then

$$\dot{\mathbf{z}} = \mathbf{A}\mathbf{z} + \mathbf{B}_c \mathbf{u}_c + \mathbf{B}_e \mathbf{u}_e \ . \tag{32}$$

where the dynamic matrix

$$\mathbf{A} = \begin{bmatrix} \Omega \mathbf{M}^{-1}\mathbf{G} & \mathbf{M}^{-1}\mathbf{K}_{ol} \\ \mathbf{I} & \mathbf{0} \end{bmatrix} \tag{33}$$

depends on the spin speed.

The control input is a column with six terms, namely the control currents in the six coils and the control input gain matrix is

$$\mathbf{B} = \begin{bmatrix} \mathbf{M}^{-1}\mathbf{T}_1^T\mathbf{G}_{c_1} & \mathbf{M}^{-1}\mathbf{T}_2^T\mathbf{G}_{c_2} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \tag{34}$$

The inputs to the control system are the airgap widths measured by optical sensors. Consider a decentralized control strategy, where in each bearing there are three optical sensors measuring the airgap width at the centre of the electromagnet. The output of the sensors can be computed from equation (3)

$$y_i = k_s d_{z=0,\theta=0} \ , \tag{35}$$

where k_s is the gain of the sensor.

This approximation can be quite rough, since the optical sensors read the light flux through the airgap, which is proportional not to the airgap at the center of the bearing, but in the narrowest point. This however cannot be accounted for precisely in a linearized model, so the model obtained above will be used.

An ideal PD control is assumed in for the linearized analysis. Also the derivatives of the signals obtained from the displacement sensors are thus included in the outputs of the open loop system and the output gain matrix $\mathbf{C}$ has 12 rows and 10 columns.

Closing the loop, the control inputs $\mathbf{u}_c$ can be written in the form

$$\mathbf{u}_c = -\mathbf{K}_c\mathbf{C}\mathbf{z} = -\begin{bmatrix} k_d\mathbf{I} & k_p\mathbf{I} \end{bmatrix} \mathbf{C}\mathbf{z} \ , \tag{36}$$

where the derivative and proportional gains k_d and k_p are overall gains referred to the controller and the power amplifier and the identity matrices are of order 6.

The closed loop linearized dynamics is then studied through the equation

$$\dot{\mathbf{z}} = (\mathbf{A} - \mathbf{B}_c\mathbf{K}_c\mathbf{C})\,\mathbf{z} + \mathbf{B}_e\mathbf{u}_e \ . \tag{37}$$

If instead of using the state space approach the equations were written in the configuration space, it would have been clear that the closed loop stiffness and damping matrices are slightly non symmetrical, giving way to a gyroscopic matrix (of little importance, since a far larger true gyroscopic matrix is already present) and to a circulatory matrix. They can easily be justified by the observation that the system is slightly non co-located, since the sensors read the displacements not exactly in the same place where the forces act. However the circulatory matrix is too small to jeopardize stability.

The Campbell diagram can thus be computed from the homogeneous equation associated to equation (37). Assuming

$$k_s = 4 \text{ kV/m} \ , \ \ k_p = 250 \text{ A/V} \ , \ \ k_d = 0.025 \text{ A/Vs} \tag{38}$$

the results computed for the 2 cases are shown in Fig. 2, full lines. The unbalance grade assumed for the computation of the unbalance response is G2.5, a fairly large value for an application of this type.

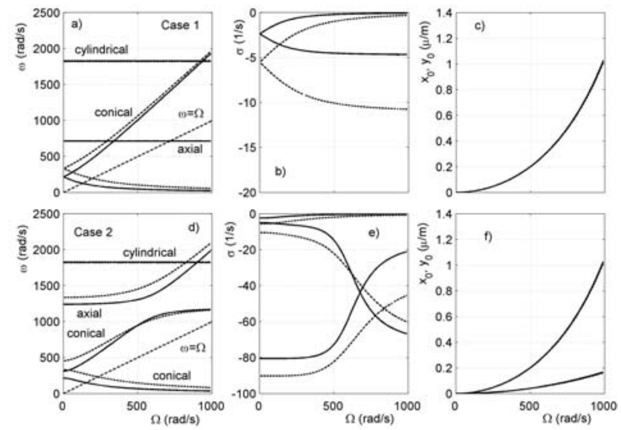

Figure 2: Campbell diagram, decay rate plot and unbalance response (in terms of semi-axes of the elliptical orbit). Case 1: microgravity; case 2: 1g, with 1 electromagnet carrying the load. Full lines: considering couplig, present linearized model; dashed lines: no coupling, usual model.

The presence of coupling affects the Campbell diagram and the decay rate plot in a non-negligible way , while the linearized unbalance response is computed by the two model with almost identical results. The system is found to be stable, at least in the small.

The linearized results are however not much satisfactory in the present case since the bearings are not working in class A, i.e. the currents supplied by the power amplifier change their signs in at least one coil. This is linked to the large value of the unbalance and the low value of the bias current assumed. When this occurs (class B regime) the linearized models lose their validity for the computation of the unbalance response (the Campbell diagram and the decay rate plot still hold for the motion in the small). A nonlinear analysis was then performed with the aim of assessing whether the system is stable in the large and whether the bearings are able to compensate for the unbalance forces.

The orbits of the center of mass obtained for case 1 are shown in Fig. 3. The four figures refer to the linearized model, computed using the closed loop dynamic matrix, a model in which the linearization in the currents and the displacements are performed in the computation of the forces of the single electromagnets, a model with linearization in the displacements but not in the currents and a fully nonlinear model. The initial conditions on displacements are $x_0 = y_0 = z_0 = 0.5$ μm, $\phi_{x0} = \phi_{y0} = 5 \times 10^{-5}$ rad. The initial velocities are assumed all equal to 0.

From the figure it is apparent that the nonlinear solution is fairly different from the linearized one, in that it contains higher order harmonics superimposed on the 1x component and that what really matters is the nonlinear effect on the currents, while

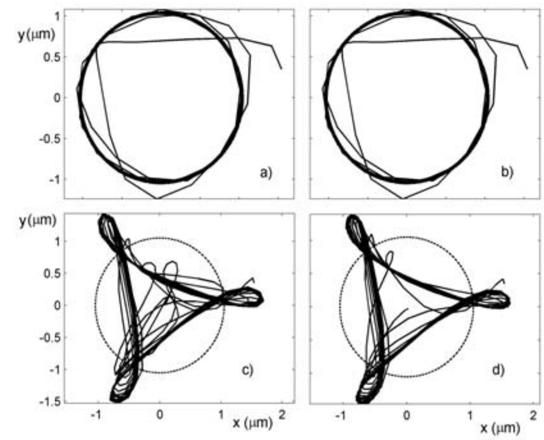

Figure 3: Case 1; orbit of the center of mass of the reaction wheel. a) and b): linearized analysis performed using the state space equation and computing the forces; c) numerical integration performed by linearizing the functions of the displacements; d): fully nonlinear analysis. The dashed lines represent the orbit computed as a particular solution of the complete linearized equation.

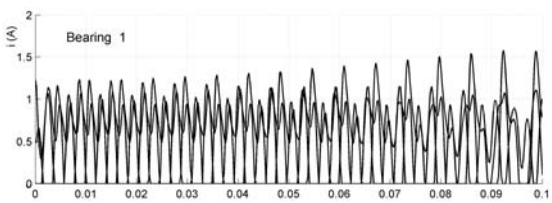

Figure 4: Total currents in the coils of one of the bearings. The system works as a Class B bearing, since all coils are periodically switched off.

the linearization on the displacements is acceptable. Moreover the system is stable also in the large, since the initial perturbation, larger than the steady state displacement, does not affect the final solution.

This result is however not general and refers only to the case studied. Actually in a way they were easily predictable: the displacements are small, even if the unbalance is fairly large, being about 1/30 of the airgap. The system works well within the range in which linearization is acceptable. On the contrary, the forces exerted by the coils and hence the control currents, are fairly large. This causes the bearing to work in Class B, as clearly shown in Fig. 4.

The simulation was repeated with a smaller value of the unbalance, namely unbalance grade G 0.25. The nonlinear solution for the unbalance response is shown in Fig. 5.

As it is clear from Fig. 6, the bearings work in Class A. That notwithstanding, the orbit contains higher order harmonics and is close to a triangle.

Even if the bearings work in Class A, the nonlinear effects due to the currents are strong, while, also in this case, those due to displacements are negligible. Axial-radial coupling is not negligible, and from the ϕ_x, ϕ_y plot it is clear that a low frequency backward whirling starts. It takes a long time to be significantly damped.

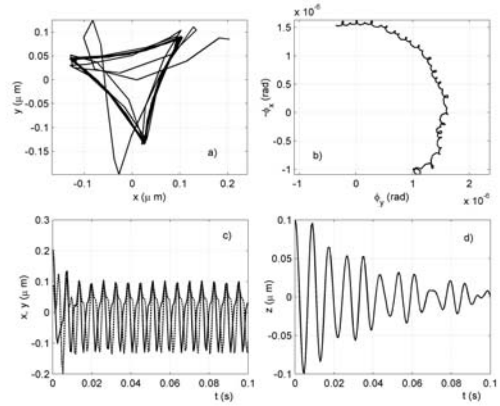

Figure 5: Orbit of the center of mass, angles ϕ_x and ϕ_y and time histories of the displacements in x, y and z directions. Case 1: microgravity (G=0.25).

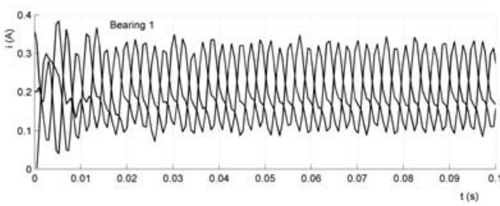

Figure 6: Total currents in the coils of one of the bearings. The system works as a Class A bearing.

If the static load due to weight is considered the bearings behave in an asymmetrical way, with the highest stiffness is in the direction of the load (x-direction). The results obtained in 1g conditions with the load supported by one electromagnet, with the same unbalance (G 0.25) and the same initial conditions as in Fig. 5 are reported in Fig. 7 Only the nonlinear solution is shown. Again, the bearings work in Class A (Fig. 8).

This time the nonlinear effects due to the currents are almost as negligible than those due to displacements, since the static components of the currents are much larger than the control currents. The large value of the compensation current makes the bearings much more stiff than in the microgravity case, yielding a lower displacements and a more stable working of the system.

CONCLUSIONS

A model for heteropolar conical active magnetic bearings which takes into account, although in a

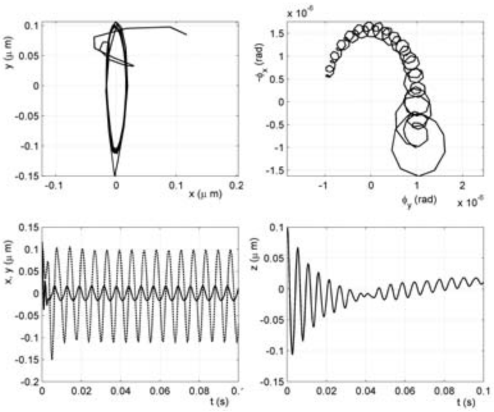

Figure 7: Orbit of the center of mass, angles ϕ_x and ϕ_y and time histories of the displacements in x, y and z directions. Case 2.

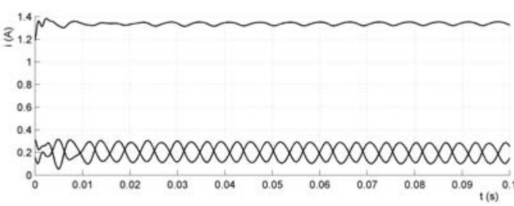

Figure 8: Total currents in the coils of one of the bearings. The system works as a Class A bearing.

simplified way, geometrical nonlinearities has been built. Such a model can be associated with nonlinear models of all components of the control loop (sensors, controller, amplifiers) to perform numerical simulations of the whole system. The main simplifications still present are linked to neglecting stray flux and the reluctance of the magnetic circuit when compared with that of the airgap. Both assumptions are reasonable, and the latter becomes unacceptable only when a pole piece is close to contacting the rotor, an occurrence which must be avoided and in most cases is prevented by the presence of emergency bearings. When, as usual, the minimum airgap cannot become less than half the average airgap c, the way in which the forces are computed and also the simplifications introduced in the computation of the reluctance are well acceptable.

The example related to a reaction wheel shows that radial-axial coupling is not negligible even in a case in which the center of mass of the rotor is at midspan between two identical bearings, yielding a further uncoupling between conical and cylindrical modes. In this condition the axial modes couple only with the conical ones, while cylindrical modes remain uncoupled.

The reaction wheel is stable both in microgravity conditions and in 1g operations, but in the former case the low stiffness causes large amplitudes to be present. Owing to the low bias currents (suggested by the need of reducing power consumption), the bearings work in class B and the system behaves in a strongly nonlinear way. No linearized analysis is possible. Only with unrealistic low values of unbalance a linearized analysis can retain some meaning.

In 1g operation the system is much better behaved and, since the bearings work in Class A, the linearized model predicts the behaviour of the system with precision.

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APPENDIX 1: APPROXIMATION OF THE INTEGRAL OF EQUATION (2)

Consider the following integral

$$I = \int_{\theta_1}^{\theta_2} \frac{d\theta}{1 - a\sin(\theta) - b\cos(\theta)} \ , \tag{39}$$

where

$$-1 < a < 1 \ , \quad -1 < b < 1 \ , \tag{40}$$

and assume that its value can be approximated as

$$I = \frac{1}{\alpha + \beta a + \gamma b} \ , \tag{41}$$

The values of constants α, β and γ are computed in the following conditions:

$$a = b = 0 \ ; \ a = \pm a_1 \ , \ b = 0 \ ; \ a = 0 \ , \ b = \pm b_1 \ , \tag{42}$$

where a_1 and b_1 can be as large as 0.9.

When $a = b = 0$, α is readily obtained

$$\alpha = \frac{1}{\theta_2 - \theta_1} \tag{43}$$

When $a = \pm a_1$, $b = 0$ the values I_{11} and I_{11} of the integral is easily computed in closed form. In a similar way the values I_{21} and I_{22} of the integral are computed stating $a = 0$, $b = \pm b_1$. The values of β and γ can be obtained using a least square approach, yielding

$$\begin{cases} \beta = \frac{1}{2a_1}\left(\frac{1}{I_{11}} + \frac{1}{I_{12}} - \frac{2}{I_0}\right) \\ \gamma = \frac{1}{2b_1}\left(\frac{1}{I_{21}} - \frac{1}{I_{22}} - \frac{2}{I_0}\right) \ . \end{cases} \tag{44}$$

In the case of a 8-pole bearing with $\theta_1 = 11.25°$ and $\theta_2 = 33.75°$ and stating $a_1 = b_1 = 0.9$, it follows:

$$\alpha = 2.5465 \ , \quad \beta = -0.9780 \ , \quad \gamma = -2.3484 \ .$$