ROBUSTNESS OF SELF SENSING MAGNETIC BEARINGS USING AMPLIFIER SWITCHING RIPPLE

Eric Maslen, Dominick Montie, and Tetsuya Iwasaki
University of Virginia, Department of Mechanical and Aerospace Engineering
Charlottesville, VA 22904, USA
ehm7s@virginia.edu

ABSTRACT

Self–sensing magnetic bearings replace explicit position sensors with estimates obtained from measurements of coil voltage and current. The benefits include potential reductions in cost, weight, and hardware complexity. Recent results exploring the robustness limits of such systems have revealed (apparently) that they are fundamentally and substantially inferior to systems with explicit sensors. The reported limits of performance are so poor as to preclude commercial realization. The present work explores the premise that switching ripple can mitigate these limitations. If the ripple results in a periodic perturbation of bias flux, the system becomes linear periodic and can exhibit dramatically enhanced performance, making commercial realization viable. This development lays the basis for a renewed interest in self–sensing AMBs and real optimism for their future promise.

INTRODUCTION

Figure 1 illustrates a very simple active magnetic bearing (AMB) [2] in which two opposing electromagnets are used to suspend a magnetically permeable rotor. The position x of the rotor must be known in order to coordinate the actions of the magnets and is typically measured either optically or magnetically. This sensor introduces problems of cost, reliability, and some loss of dynamic performance when it cannot be colocated with the magnetic actuator. Consequently, there is an interest in eliminating this position sensor. Active magnetic bearings which determine rotor position by measuring only coil voltage and current are referred to as “self–sensing” [23].

Despite extensive study, no commercially useful self–sensing AMB system has yet been reported. Systems reported in the literature tend to be overly susceptible to noise or very sensitive to system parameters and tuning. Both problems are symptomatic of excessive *sensitivity*. Perhaps the most important relevant publications are [22, 23] which laid the central groundwork for the problem, [6] which first examined the sensitivity of linear time invariant (LTI) self–sensing realizations, and

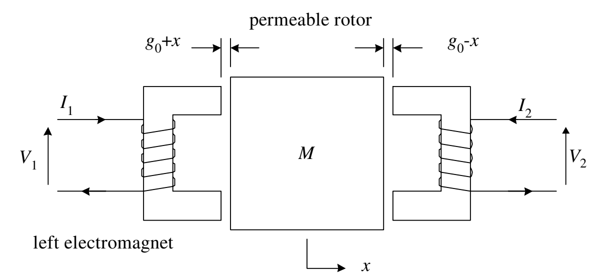

Figure 1: Active magnetic bearing

[15, 20] which established the fundamental sensitivity bounds for this LTI realization.

This literature can be readily classed according to two distinct lines of inquiry. One focuses on methods based in rigorous theory: state space LTI realizations with explicit synthesis methods [1,12,22,23]. This work is challenged by [6, 15] which establish unacceptably poor robustness limitations. The 1998 publication of the very discouraging observations by Morse *et al.* [15] seemed to spell an end to useful development of self-sensing magnetic bearings. Up to that point, most workers remaining in the field operated on the assumption that the poor robustness of experimental systems would ultimately yield to steadily improving signal processing techniques. However, [15] established that the LTI model for self–sensing bearings (first described in [23] and maintained by the rest of the field since) presents fundamentally poor potential robustness independent of clever signal processing or control schemes.

The other class consists of *ad–hoc* estimation techniques that make explicit use of the high frequency switching ripple [10, 11, 14, 16, 17, 19] produced by switching amplifiers. These results seem to suggest that the limitations in [15] are too severe. However these approaches lack a rigorous theoretical basis to permit a clear comparison to the limitations of the LTI problem. Compelling contradictory data is not available and would be suspect even if it were: the limitations of [15] fundamentally characterize the mapping from input (voltage control) signals to output (current sensor) signals.

Such limitations can *only* be relaxed by changing the input/output or the plant model in some very fundamental manner. Thus, the present work explores the key difference between plants with switching ripple and those without: the model in [15] ignores switching ripple.

The objective of this paper is to prove that the addition of switching ripple can produce a self–sensing AMB with better robustness than the LTI prediction of [15]. Our approach is based on the simplifying hypothesis that periodicity of the bias flux is the essential feature produced by switching ripple permitting better robustness. To prove this, we construct a linear periodic (LP) model of the AMB, define a robustness measure in terms of the $\mathcal{L}_2$ induced norm of the sensitivity operators, and examine how the periodicity affects achievable robustness.

Our results show that the LP model analysis predicts much better robustness than the LTI model [15]. Moreover, the LP prediction gets worse as the periodic aspect of the model is diminished and converges to the LTI prediction in the limit, confirming the consistency of our and previous analyses.

PROBLEM FORMULATION

The model

The simplest useful nondimensional model for the active magnetic bearing system depicted in Figure 1 is [13]:

$$\begin{aligned}
\dot{x} &= v && \text{(1a)}\\
\dot{v} &= \phi_b \phi && \text{(1b)}\\
\dot{\phi} &= -\eta\phi + \eta\phi_b x + u && \text{(1c)}\\
\dot{\phi}_b &= -\eta\phi_b + \eta\phi x + u_b && \text{(1d)}\\
y &= \phi - \phi_b x && \text{(1e)}\\
I_b &= \phi_b - \phi x && \text{(1f)}
\end{aligned}$$

whose nondimensional states and signals are defined by

$$x \equiv \frac{X}{g_0}, \qquad v \equiv \frac{dX}{dT}\frac{\tau}{g_0}, \qquad \tau \equiv \sqrt{\frac{\mu_0 g_0 M}{A_g B_{sat}^2}},$$
$$\phi \equiv \frac{\Phi_1 - \Phi_2}{A_g B_{sat}}, \qquad \phi_b \equiv \frac{\Phi_1 + \Phi_2}{A_g B_{sat}},$$
$$y \equiv \frac{I_1 - I_2}{I_{sat}}, \qquad I_b \equiv \frac{I_1 + I_2}{I_{sat}}, \qquad I_{sat} \equiv \frac{2 B_{sat} g_0}{\mu_0 N},$$
$$u \equiv \frac{V_1 - V_2}{V_0}, \qquad u_b \equiv \frac{V_1 + V_2}{V_0}, \qquad V_0 \equiv \frac{N B_{sat} A_g}{\tau}$$

The states X and dX/dT are the position and velocity of the mass, while T is the dimensional time variable. Φ_1 and Φ_2 are the magnetic fluxes in the left and right magnets; I_1 and I_2 are the currents in the corresponding coils; V_1 and V_2 are the driving voltages. Time is nondimensionalized by τ; $\dot{x} \equiv dx/dt$, etc. where $t \equiv T/\tau$ is the nondimensionalized time variable.

The parameter η in (1) is defined in terms of the physical properties of the system:

$$\eta \equiv \frac{2 g_0 R}{\mu_0 N^2 A_g}\ \tau \qquad (2)$$

Table 1: Reference Physical Parameters, from [15]

symbol	definition	value	units
μ_0	permeability of air	1.26×10^{-6}	H/m
N	coil turns	220	-
A_g	pole area	4.84×10^{-5}	m^2
M	rotor mass	0.1315	kg
g_0	nominal air gap	0.4	mm
R	coil resistance	2.2	ohms
I_0	bias current	0.5	A
L	coil inductance	3.68	mH
The remaining parameters are derived from [15] based on an assumed magnetic saturation density of 1.2 Tesla.			
B_{sat}	magnetic sat. density	1.2	Tesla
Φ_0	nondim bias flux	0.288	-
η	ratio of time scales	0.582	-
σ	Φ_0/η	0.496	-

whose various parameters are defined in Table 1. The parameter η represents the ratio between mechanical and electrodynamic time scales.

The standard approach [15, 23] to approximately linearizing this system is to assume that the state variable ϕ_b can be forced to track some trajectory Φ(t) by appropriate choice of u_b. The result (neglecting the dynamics of the slaved state ϕ_b) is the linear (time varying) system

$$\frac{d}{dt}\begin{bmatrix} x \\ v \\ \phi \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & \Phi(t) \\ \eta\Phi(t) & 0 & -\eta \end{bmatrix}\begin{bmatrix} x \\ v \\ \phi \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \qquad \text{(3a)}$$

$$y = \begin{bmatrix} -\Phi(t) & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ v \\ \phi \end{bmatrix} \qquad \text{(3b)}$$

In the ensuing discussion, the input/output mapping from u to y will be denoted by P.

Robustness measure

For the purposes of the present discussion, robustness will be quantified in terms of the sensitivity function, as illustrated in Figure 2. The primary reason for this is to permit direct comparison to the results in [15], but also because of its connection to the Nyquist criterion and relative ease of measurement.

Figure 2 defines two possible sensitivity functions: the output sensitivity, S_o, and the input sensitivity, S_i, by way of the two potential uncertainty operators, Δ_i and Δ_o:

$$S_o \ :\ y = S_o v = (I - PC)^{-1} v \qquad (4)$$
$$S_i \ :\ u = S_i w = (I - CP)^{-1} w \qquad (5)$$

Products of operators PC imply successive operation.

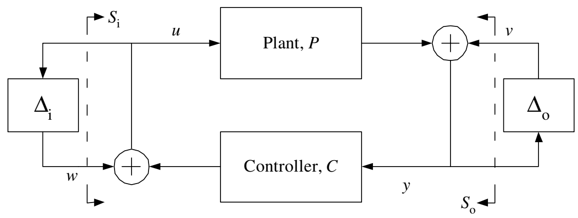

Figure 2: Definitions of sensitivity functions.

In order to directly interpret the sensitivity functions in terms of the amount of uncertainty (either Δ_o or Δ_i) the plant can tolerate, the $\mathcal{L}_2$ induced norm of the sensitivity function is examined, which is well defined for linear, time varying (LTV) plants such as (3). This (input or output) sensitivity norm is defined as

$$\|S\| := \sup_{w \in \mathcal{L}_2 \setminus \{0\}} \frac{\|Sw\|_2}{\|w\|_2} \tag{6}$$

in which S stands for S_i or S_o, $\|\cdot\|_2$ is the $\mathcal{L}_2$ norm of a signal, and $\mathcal{L}_2$ is the set of square integrable signals. The significance of the norm, implied by the small gain theorem, is that a plant will tolerate (i.e., maintain stability in the presence of) any general LTV uncertainty operator whose $\mathcal{L}_2$ induced norm is smaller than the reciprocal of the $\mathcal{L}_2$ induced norm of the corresponding sensitivity function. Hence, a small $\mathcal{L}_2$ induced norm for the sensitivity function implies tolerance of a large uncertainty operator: good robustness.

Problem statement

This paper examines the robustness limitation of (3) as measured by the induced $\mathcal{L}_2$ norm of the sensitivity operators. Denoting the dependence of the sensitivity S on the plant P and the controller C as $S(P, C)$, the best achievable robustness for P is formally defined as

$$\varphi_*(P) := \inf_{C \in \mathbf{C}} \|S_*(P, C)\| \tag{7}$$

$\mathbf{C}$ is a class of stabilizing controllers, and $*$ is either i (input) or o (output).

Recall that the plant P in (3) is, in general, an LTV system that depends on the choice of the bias flux trajectory $\Phi(t)$. This paper focuses on the two cases where $\Phi(t)$ is either constant or periodic. The conjecture is proffered that periodic biasing can substantially enhance the best achievable robustness $\varphi_*(P)$ when compared with constant biasing. The validity of this conjecture is assessed through numerical analysis based on the carefully constructed AMB model (3).

EXISTING RESULTS

The conjecture that an LP model for self–sensing AMBs might provide a more optimistic assessment of their potential performance arose from attempting to reconcile some nominally conflicting existing results. The ensuing section reviews these results, laying the background for the LP conjecture.

Theoretical prediction of robustness for the LTI case

Most typically (as in [15,21,23]), it is assumed that $\Phi(t)$ in (3) is some constant, Φ_0, to produce an LTI realization. In this case, the transfer function from u to y is given by

$$\frac{y(s)}{u(s)} = \frac{s^2 - \Phi_0^2}{s^3 + \eta s^2 - \eta \Phi_0^2} \tag{8}$$

Some simplification arises by introducing the definitions $\lambda \equiv \frac{s}{\Phi_0}$ and $\sigma \equiv \frac{\Phi_0}{\eta}$ so that

$$\frac{y(\lambda)}{u(\lambda)} = \frac{1}{\eta} \frac{\lambda^2 - 1}{\sigma \lambda^3 + \lambda^2 - 1} \tag{9}$$

This transfer function has a right half plane zero at $\lambda = 1$ and a right half plane pole lying between 0 and 1, depending on the value of σ. For physically reasonable designs, σ ranges between about 0.3 and 3: the pole ranges from about 0.6 to about 0.9.

When C and P are SISO LTI operators, they commute so the input S_i and output S_o sensitivity functions are the same. Further, for LTI operators, the $\mathcal{L}_2$ induced norm is the same as the achievable $\mathcal{H}_\infty$ norm and, as developed in [8], a hard lower bound on the achievable $\mathcal{H}_\infty$ norm of S_o or S_i can be computed directly in the case that P has a pole p_o and zero z_o on the positive real axis:

$$\varphi(P) \geq \left| \frac{p_0 + z_0}{p_0 - z_0} \right| \tag{10}$$

Applying (10) to (9) produces a limit that ranges from about 17 for $\sigma = 0.3$ down to about 4 for $\sigma = 3$ as illustrated in Figure 3. The bound for the nominal case explored in [15] is 11.5. Commercial AMB systems are normally expected to have a peak sensitivity gain of less than 3: even the most extreme choice of σ will not yield a commercially acceptable solution.

Experimental results: switching ripple systems

An example self–sensing AMB based on a switching ripple demodulation scheme is presented in [14] and later reinterpreted in [13]. For that system, the *real* gain margin was explicitly measured by simply varying the controller gain to find lower and upper limits of stability. The resulting margin reported there suggests a complementary sensitivity ($T_i = 1 - S_i$) gain of 2.2 while the theoretical lower bound on T_i for the parameters of that system is 7.4. This theoretical result is provided in [13] and was derived using (10).

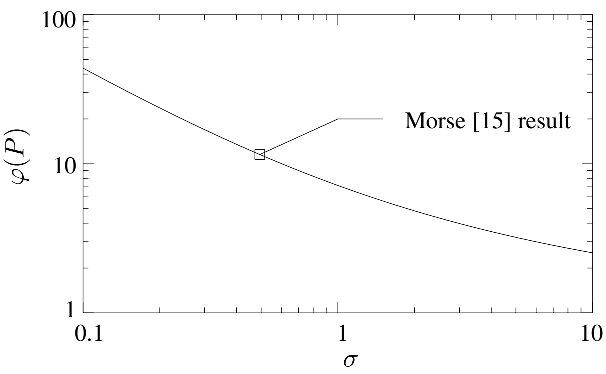

Figure 3: Effect of σ on sensitivity norm bound.

Another self–sensing scheme is explored in [18] where the controller is explicitly LP. In that work, the sensitivity function is measured directly using swept sine tests and compared to the theoretical bound for an equivalent LTI system. In this case, the measured peak S_i is 3.5 while the theoretical bound for this plant is 4.94.

Both of these results *appear* to directly violate the hard bounds predicted by [15] and strongly suggest practical feasibility of self-sensing AMB in spite of the discouraging result of [15]: and it is tempting to conclude that the actual robustness of the self-sensing AMB might be better than the prediction of [15]. If this is the case, then the discrepancy must be attributed to some shortcoming of the model used in the theoretical prediction. The model in [15] is LTI while the actual systems in [13, 18] are time–varying due to the switching ripple. This suggests that enhanced robustness of the self–sensing AMB with switching ripple may be theoretically predicted if a suitable time–varying model is used.

LP ANALYSIS FRAMEWORK

Central idea: periodic biasing

The exact influence of amplifier switching on this plant is difficult to assess because it requires including a detailed model of the (nonlinear) switching rules governing the amplifiers. However, consider the conjecture that the simple presence of switching ripple may account for the apparently higher robustness. To test this conjecture, assume that the effect of amplifier switching ripple can be approximated as a sinusoidal dither in the state ϕ_b. Instead of ϕ_b tracking a constant value Φ_0, it instead tracks some function of time:

$$\phi_b \to \Phi(t) = \Phi_0(1 + \gamma \sin \omega t) \qquad (11)$$

The basis for this assumption lies in recognizing that switching ripple can be modelled as noise added to the input signals u and u_b. However, since u is a free control variable, the presence of noise in this signal cannot influence the potential *robustness* of the system. So switching ripple appearing on u may be ignored. In contrast, switching ripple appearing on u_b will produce ripple in $\Phi(t)$ if the bias control is not of sufficient bandwidth to prevent this. Practical considerations impose this bandwidth limitation *a priori* (the control bandwidth is axiomatically less than half of the amplifier switching rate – the Nyquist frequency for what is essentially a sampled data problem) so it is reasonable to assume that $\Phi(t)$ is periodic as described by (11).

With this latter assumption, the general linear time varying model (3) becomes linear periodic:

$$\frac{d}{dt}\begin{bmatrix} x \\ v \\ \phi \end{bmatrix} = A(t;\gamma,\omega)\begin{bmatrix} x \\ v \\ \phi \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \qquad (12a)$$

$$y = C(t;\gamma,\omega)\begin{bmatrix} x \\ v \\ \phi \end{bmatrix} \qquad (12b)$$

$$A(t;\gamma,\omega) \equiv \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & \Phi_0 \\ \eta\Phi_0 & 0 & -\eta \end{bmatrix} + \gamma \sin \omega t \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \Phi_0 \\ \eta\Phi_0 & 0 & 0 \end{bmatrix} \qquad (12c)$$

$$C(t;\gamma,\omega) \equiv \begin{bmatrix} -\Phi_0 \\ 0 \\ 1 \end{bmatrix}^\top + \gamma \sin \omega t \begin{bmatrix} -\Phi_0 \\ 0 \\ 0 \end{bmatrix}^\top \qquad (12d)$$

Sensitivity Analysis

The recent introduction of convenient tools for LP systems [4] permits synthesis of optimal controllers that minimize the $\mathcal{L}_2$ induced norm in the discrete–time setting and computation of the resulting $\mathcal{L}_2$ induced norm performance measures. These tools sidestep the approximate nature of previous approaches such as [3, 5, 7] and permit direct assessment of the impact of the parameters characterizing the periodicity (both amplitude and finite frequency.)

To see the effect of the parameters γ (dither amplitude) and ω (dither frequency), LP controllers for the system (12) were computed using cost functions selected to minimize the $\mathcal{L}_2$ induced norm of either the input or output sensitivity function using the method described in [4]. The sensitivity function norms of the resulting closed loop systems were then evaluated.

Input Sensitivity

Figure 4 illustrates the computed input sensitivity function norm bound, $\varphi_i(P)$, as a function of γ and ω for

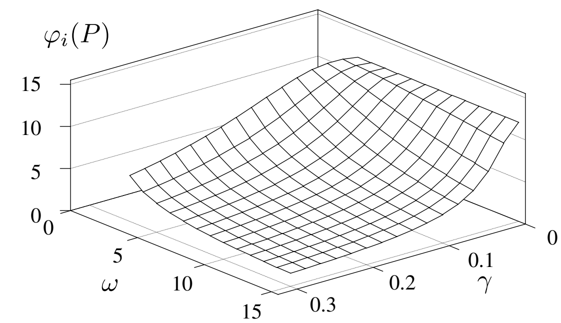

Figure 4: Effect of γ and ω on the input sensitivity function norm.

$\Phi_0 = 0.288$ and $\eta = 0.582$ as in [15]. From this figure, it is clear that driving γ toward zero maximizes the sensitivity norm: the upper limit of 11.7 agrees well with that obtained for the nominal LTI problem in Section . Driving both γ and ω large produces a lower limit: about 1.5 for the range of γ and ω explored in Figure 4. This lower limit is much better than that for the LTI system and is low enough for commercial acceptance: $\gamma = 0.12$ and $\omega = 15$ achieves $\varphi_i(P) = 2.9$ which is at the practical margin of acceptance.

Output Sensitivity

Although space doesn't permit a full development here, the reader is referred to [9] for a discussion of the output sensitivity problem. In that case, as long as the uncertainty operator itself is LTI, similar improvements to robustness are seen with increasing amplitude and frequency of ripple.

Physical interpretation

To understand why the input sensitivity of the system can be improved so dramatically with the LP realization, recognize that the LP output matrix in (12b) applies periodic modulation to x but not to ϕ. Consequently, the output signal can be *synchronously demodulated* by multiplying y by $\sin\omega t$ to yield a nearly LTI plant whose outputs are essentially both x and ϕ.

To see this, assume that the signals x and ϕ have low bandwidth spectra relative to ω: $y = -\Phi_0(1 + \gamma\sin\omega t)x + \phi$. Adding a signal multiplied by $\sin\omega t$ produces

$$\begin{aligned} y_1 &= \begin{bmatrix} \sin\omega t \\ 1 \end{bmatrix} y \\ &= \begin{bmatrix} -\Phi_0(\sin\omega t + \gamma\sin^2\omega t) & \sin\omega t \\ -\Phi_0(1+\gamma\sin\omega t) & 1 \end{bmatrix} \begin{bmatrix} x \\ \phi \end{bmatrix} \end{aligned}$$

Now, with the assumption that x and ϕ consist of signals with much lower frequency than ω, they may be considered essentially constant over one cycle: $t - T$ to t where $T = 2\pi/\omega$. Thus, apply a moving boxcar averaging filter and some simple scaling to y_1 to obtain

$$y_2(t) = \begin{bmatrix} -\frac{1}{\gamma\Phi_0} & 0 \\ -\frac{2}{\Phi_0} & 1 \end{bmatrix} \int_{t-T}^{t} y_1(t)\, dt = \begin{bmatrix} x \\ \phi \end{bmatrix}$$

This separates x from ϕ which permits (in the sense of the approximations of [3]) arbitrary reassignment of the zeros of the transfer function from u to y_2.

In the limit as $\omega \to \infty$, the low–pass nature of the plant eliminates the effect of periodicity in (12a) while the synchronous demodulation shifts the ϕ–dependent component of the signal to a very high frequency where it is easily removed with a low–pass filter. By eliminating the mixing of x and ϕ in the plant output, the zero in the right half plane immediately adjacent to the unstable pole is eliminated, thereby mitigating the associated robustness limits. Thus, it would be expected that, for large values of ω, the sensitivity might approach the best gain of 1.0 achieved with measurement of both x and y (or x and ϕ) as discussed in Section .

DISCUSSION

The main observation to be drawn from this study is that robust self–sensing AMB may be achievable through exploiting switching ripple. A simple way to view this effect is the periodic bias model developed here. Fortuitously, this periodic biasing (or something similar to it) may often be the most natural mode of operation so that the advantage is obtained without significant direct cost except in the resulting signal processing complexity.

An important extension of this observation is that the present work develops a quantitative assessment of the impact of ripple amplitude and frequency. The critical conclusion to draw from this is that it is not possible to accomplish robust sensitivity with arbitrarily small ripple. Indeed, the level of ripple needed to attain acceptable robustness is substantial and probably means that such systems will inherently need to use two–state switching amplifiers rather than three–state as is the current trend in industrial practice. This means that self–sensing AMBs can be expected to exhibit higher amplifier losses and somewhat higher acoustic emissions: self–sensing doesn't come for free.

REFERENCES

[1] H. Bleuler and D. Vischer. Magnetic bearing systems with minimal hardware requirement. In *ROMAG '91: Magnetic Bearings and Dry Gas Seals International Conference and Exhibition*, March 1991.

[2] Hannes Bleuler. Survey of magnetic levitation and magnetic bearing types. *JSME International Journal, Series 3: Vibration, Control Engineering, Engineering for Industry*, 35(3), September 1992.

[3] N. N. Bogoliubov and Y. A. Mitropolsky. *Asymptotic Methods in the Theory of Non-Linear Oscillations*. Hindustan Publishing Corporation (India), 1961.

[4] Geir E. Dullerud and Sanjay Lall. A new approach for analysis and synthesis of time-varying systems. *IEEE Transactions on Automatic Control*, 44(8):1486–1497, August 1999.

[5] P T. Kabamba, S M. Meerkov, and E K. Poh. Closed loop vibrational control: State and output feedback stabilizability. volume 3, pages 2375–2376, Piscataway, NJ, USA.

[6] L. Kucera. Robustness of self-sensing magnetic bearing. In *Proceedings of MAG'97 Industrial Conference and Exhibition on Magnetic Bearings*, pages 261–270, Alexandria, Virginia, August 1997.

[7] S. Lee, S. M. Meerkov, and T. Runolfsson. Linear periodic feedback: Zeros placement capabilities. In *Proceedings of the 25th Conference on Decision and Control*, pages 2202–2207, December 1986.

[8] D. P. Looze and J. S. Freudenberg. Limitations of feedback properties imposed by open–loop right half plane poles. *IEEE Transactions on Automatic Control*, 36(6), June 1991.

[9] E. H. Maslen, D. T. Montie, and T. Iwasaki. Robustness limitations in self–sensing magnetic bearings. *ASME Journal of Dynamic Systems, Measurement, and Control*. (to appear).

[10] T. Mizuno and Y. Hirasawa. Self-sensing magnetic suspension using a hysteresis amplifier with a two-quadrant drive. In *Proceedings of the Sixth International Symposium on Magnetic Suspension Technology*, pages 76–80, October 2001.

[11] T. Mizuno, T. Ishii, and K. Araki. Self-sensing magnetic suspension using hysteresis amplifiers. *Control Engineering Practice*, 6:1133–1140, 1998.

[12] A. M. Mohamed et al. Modeling and robust control of self-sensing magnetic bearings with unbalance compensation. In *Proceedings of the 1997 IEEE Internatioanl Conference on Control Applications*, October 1997.

[13] D. T. Montie. *Performance Limitations and Self–Sensing Magnetic Bearings*. PhD thesis, University of Virginia, 2003.

[14] D. T. Montie and E. H. Maslen. Experimental self-sensing results for a magnetic bearing. In *Proceedings of the ASME International Gas Turbine Institute Turbo Expo*, June 2001.

[15] N. Morse, R. Smith, B. Paden, and J. Antaki. Position sensed and self-sensing magnetic bearing configurations and associated robustness limitations. In *Proceedings of the IEEE Conference on Decision and Control Including the Symposium on Adaptive Processes*, volume 3, pages 2599–2604, 1998.

[16] M. D. Noh and E. H. Maslen. Self-sensing magnetic bearings using parameter estimation. *IEEE Transactions on Instrumentation and Measurement*, 46(1):45–50, February 1997.

[17] A. Schammass and H. Bleuler. Experimental results on self-sensing AMB using a three-state PWM amplifier. In *Proceedings of the Eighth International Symposium on Magnetic Bearings*, pages 289–292, August 2002.

[18] Alexandre Schammass. *A Self-sensing Active Magnetic Bearing: Modulation Approach*. PhD thesis, Swiss Federal Institute of Technology Lausanne - EPFL, 2003.

[19] K. K. Sivadasan. Analysis of self-sensing active magnetic bearings working on inductance measurement principle. *IEEE Transactions on Magnetics*, 32(2), March 1996.

[20] N. M. Thibeault and R. Smith. Robustness comparison of a magnetic bearing system in various measurement configurations. In *Proceedings of the American Control Conference*, volume 5, pages 3002–3003, IEEE, Piscataway, NJ, USA, 2000.

[21] Nancy Morse Thibeault and Roy Smith. Robustness comparison of a magnetic bearing system in various measurement configurations. In *Proceedings of the American Control Conference*, volume 5, pages 3002–3003. IEEE, Piscataway, NJ, USA,00CB36334, 2000.

[22] D. Vischer and H. Bleuler. A new approach to sensorless and voltage controlled AMBs based on network theory concepts. In *Proceedings of the Second International Symposium on Magnetic Bearings*, pages 301–306, July 1990.

[23] D. Vischer and H. Bleuler. Self-sensing active magnetic levitation. *IEEE Transactions on Magnetics*, 29(2):1276–1281, March 1993.