SLIDING MODE CONTROLLER FOR ACTIVE CONTROL OF SURGE IN CENTRIFUGAL COMPRESSORS WITH MAGNETIC THRUST BEARING ACTUATION

Dorsa Sanadgol, Eric H. Maslen
University of Virginia, ROMAC Laboratories
Department of Mechanical and Aerospace Engineering
122 Engineer's Way, Charlottesville, VA 22904 USA
ds5bf@virginia.edu

ABSTRACT

A new method for active control of surge in unshrouded centrifugal compressors is developed using a magnetic thrust bearing to modulate the impeller tip clearance. Use of a sliding mode control method ensures that the system trajectories remain on the steady state compressor characteristic for a significantly extended operating range beyond the compressor surge point. The actuator authority required by this action is demonstrated to be well within expected capacities. Using a simulation based on the Greitzer model with adaptations, the stable mass flow rate was reduced from 0.5 kg/sec to 0.2 kg/sec.

INTRODUCTION

Surge and rotating stall degrade the performance of compressors at low mass flow rates and can cause significant damage to the compressor and the system that the machine is operating. Considerable research has been done on different control schemes to avoid these instabilities and to extend the stable operating range of the compressors for both axial and centrifugal machines.

The present work developes a new method for active control of surge in unshrouded centrifugal compressors using a magnetic thrust bearing as a servo actuator. If the compressor efficiency is sufficiently sensitive to axial position of the impeller, as in unshrouded centrifugal compressors, then it is possible to use the thrust actuator to actively affect flow instabilities. This capability arises because the blade tip clearance depends on axial rotor position. If this clearance is modulated with sufficient magnitude and speed, the induced pressure modulations can result in surge suppression.

The active nature of the magnetic bearing system makes the real-time static and dynamic positioning of the rotor possible. This allows straightforward modulation of the impeller tip clearance. Use of axial motion in centrifugal compressors distinguishes this work from that of Spakovsky et al.[1], who examined radial motion actuation of axial compressors. The ability to modulate the mean tip clearance, rather than to induce a circumferentially periodic perturbation is a significant advantage here.

The present work first establishes a theoretical model describing the sensitivity of centrifugal compressor parameters to tip clearance variations induced by axial motion of the impeller. This is based on available measured effects of tip clearance on compressor efficiency. The impeller is assumed to be unshrouded, generally enhancing this sensitivity. The Greitzer compressor model[2] is then modified to include the effects of tip clearance variation on the compressor characteristics.

The thrust bearing is modeled *a priori* as a third order Butterworth filter with a bandwidth of 70Hz to reflect reasonably achievable performance of this subsystem. A sliding mode controller is then developed[3] which is capable of substantially mitigating flow instabilities

NOMENCLATURE

Symbols

a	speed of sound	V	element volume
A	through flow area	γ	specific heat ratio
b	bade height	η	efficiency
k	gain	ξ	state vector
l	tip clearance	ϕ	boundary layer thickness
L	element length	ψ	compressor pressure ratio
m	mass flow rate		
P	pressure		
s	sliding function		
u	control input		

Subscripts

1	ambient	o	stagnation value
2	impeller exit	p	plenum
c	compressor	ss	steady state
eq	equivalent	th	throttle

using a magnetic thrust bearing to modulate the impeller tip clearance. The controller is designed with the objective that system trajectories remain on the sliding surface, with the steady state compressor characteristic curve defined as the sliding surface. This ensures zero steady state offset of the impeller, which maintains the efficiency of the compressor.

Results from the simulation of the nonlinear model for a single stage high-speed centrifugal compressor are presented, which show that the sliding mode control quickly suppresses mass flow and pressure oscillations associated with compressor surge and increases the stable operating range of the compressor significantly with reasonable control requirements. Planned experimental work is described in a companion paper by Buskirk et al.[4], detailing the experimental apparatus and its capabilities.

COMPRESSOR MODEL WITH TIP CLEARANCE ACTUATION

Using the same basic compression system as Greitzer[2], i.e. a compressor and its related ducting, plenum volume and the throttle valve, as shown in Fig. 1, Gravdahl[5] derived the model described by the set of equations (1). This set of equations is the same as Greitzer's model except that the states retain dimension. This avoids introducing additional nonlinearities into the system by normalization.

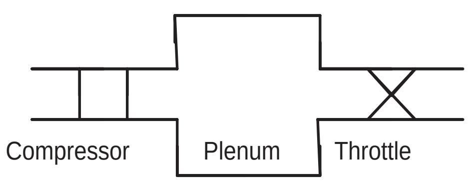

Figure 1: Sketch of the basic compression system

$$\dot{P}_p = \frac{a_{o1}^2}{V_p}\left(m_c - m_{th}\right) \tag{1a}$$

$$\dot{m}_c = \frac{A}{L_c}\left(\psi P_{o1} - P_p\right) \tag{1b}$$

the throttle mass flow rate is given by

$$m_{th} = k_{th}\sqrt{P_p - P_{o1}}.$$

Results from the simulation of this model have shown good agreement with experimental data [5, 6].

Compressor performance and efficiency are highly dependent on the clearance between the blade tips and the adjacent shroud. As the tip clearance increases, the induced leakage reduces the energy transfer from impeller to fluid and decreases the exit velocity angle, which consequently produces a substantial loss in pressure rise, efficiency and surge margin. Graf et al.[7] studied the effects of different tip clearance values on a four-stage low speed axial compressor and showed that increased tip clearance caused a decrease in peak pressure rise and efficiency and an increase in stalling mass flow rate.

Variation of the compressor characteristics due to the tip clearance is usually expressed as a variation in efficiency. Senoo and Ishida [8] developed a simple model for the leakage loss in centrifugal compressors. Remarkably good predictions of the loss in pressure ratio and efficiency were produced for a number of compressors. The decrement on efficiency was found to be proportional to the ratio of clearance to blade height at the impeller outlet provided that this ratio is less than 0.1. The trend is expressed as

$$\frac{\Delta\eta}{\eta} = \frac{\Delta l}{4b_2} \tag{2}$$

Applying a quasi-steady approximation for the effects of tip clearance, based on Senoo's relation (2), the compressor pressure ratio can be expressed as

$$\psi = \left(1 + \frac{\psi_{ss}^{\frac{\gamma-1}{\gamma}} - 1}{1 - \frac{0.25u}{b_2}}\right)^{\frac{\gamma}{\gamma-1}} \tag{3}$$

where $u = \Delta l$. Since the compressor is driven by a speed-controlled electric motor, effects of rotor speed variations are ignored and the following model is derived

$$\dot{P}_p = \frac{a_{o1}^2}{V_p}\left(m_c - m_{th}\right) \tag{4a}$$

$$\dot{m}_c = \frac{A}{L_c}\left[\left(1 + \frac{\psi_{ss}^{\frac{\gamma-1}{\gamma}} - 1}{1 - \frac{0.25u}{b_2}}\right)^{\frac{\gamma}{\gamma-1}} P_{01} - P_p\right] \tag{4b}$$

Note that this model explicitly assumes that the only effect of axial rotor motion is to modulate the instantaneous compression efficiency which is effectively the compressor pressure gain.

SLIDING MODE CONTROL

Sliding mode control deals with the problem of getting the state of a system to track a specific time varying trajectory, or to satisfy a specific functional constraint in the presence of model inaccuracies. Consider the single input dynamic system

$$x^{(n)} = f(\xi, u; t)$$

where the notation $x^{(n)}$ denotes the ith time derivative of x and

$$\xi = \left[\begin{array}{cccc} x & \dot{x} & \ldots & x^{(n-1)} \end{array}\right]^T$$

the tracking error vector $\tilde{\xi}$ is

$$\tilde{\xi} = \xi - \xi_{ss} = \left[\begin{array}{cccc} \tilde{x} & \dot{\tilde{x}} & \ldots & \tilde{x}^{(n-1)} \end{array}\right]^T$$

Define a time-varying *sliding function*, $s(\xi;t)$, as

$$s(\xi;t) = \left[\prod_{i=1}^{n-1}\left(\frac{d}{dt} + \lambda_i\right)\right]\tilde{x}$$

where λ_i's are strictly positive constants.

Sliding mode control reduces the n^{th} order tracking problem to a 1^{st} order stabilization problem in s. Keeping scalar s at zero is achieved by choosing the control input u such that

$$\frac{d}{dt}\left(\frac{1}{2}s(\xi;t)^2\right) \leq -\mu|s(\xi;t)| \tag{5}$$

where μ is a strictly positive constant. The *sliding condition*, equation (5), can be expanded as

$$\text{sign}\left[s(\xi;t)\right]\dot{s}(\xi;t) \leq -\mu. \tag{6}$$

This sliding condition implies that trajectories off the *sliding surface* move towards the surface and, once on the surface, trajectories remain on the sliding surface.

SLIDING MODE SURGE CONTROL

The control objective in surge mitigation is to force the system trajectories to follow the nominal compressor characteristic. This characteristic represents an equilibrium in phase space parameterized by throttle setting: surge arises because this equilibrium is not everywhere stable. Therefore, we define the sliding surface to be the steady state compressor characteristic

$$s = \psi - \psi_{ss}. \tag{7}$$

The system dynamic while in sliding mode can be expressed as

$$\dot{s} = 0. \tag{8}$$

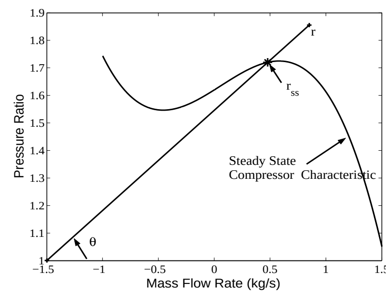

Figure 2: Polar coordinates

The equivalent control, which is the continuous control law that would maintain $\dot{s} = 0$, can be obtained by solving equation (8) for the control input u.

The equivalent control resulting from the sliding function introduced in equation (7) requires division by $\frac{d\psi}{dm}$ which is zero at the peak of the compressor characteristic. To avoid this problem, polar coordinates with the origin at (α, β) as shown in Fig. 2 are used. The new sliding function is defined as

$$s = (r - r_{ss})\sin\theta. \tag{9}$$

The equivalent control law resulting from this new sliding function is

$$u_{eq} = 4b_2\left(1 - \frac{\psi^{\frac{\gamma-1}{\gamma}} - 1}{\psi_{eq}^{\frac{\gamma-1}{\gamma}} - 1}\right)$$

where

$$\psi_{eq} = P - \frac{r^3 - (m-\alpha)\left[r_{ss}(m-\alpha) + \frac{\partial r_{ss}}{\partial\theta}(P-\beta)\right]}{\frac{AP_{o1}}{L_c}(P-\beta)\left[r_{ss}(m-\alpha) + \frac{\partial r_{ss}}{\partial\theta}(P-\beta)\right]}\dot{P} \tag{10}$$

We can rewrite $\dot{s}$ in terms of ψ_{eq} as

$$\dot{s} = \frac{r_{ss}(m-\alpha) + \frac{\partial r_{ss}}{\partial\theta}(P-\beta)}{r^2}(\psi - \psi_{eq})$$

so that the sliding condition (6) reduces to

$$\text{sign}(s)\frac{r_{ss}(m-\alpha) + \frac{\partial r_{ss}}{\partial\theta}(P-\beta)}{r^2}(\psi - \psi_{eq}) \leq -\mu. \tag{11}$$

The above sliding condition is guaranteed to be satisfied if ψ is chosen to be

$$\psi = \psi_{eq} - k\ \text{sign}(s). \tag{12}$$

Therefore

$$k \geq \frac{r^2 \mu}{r_{ss}(m-\alpha) + \frac{\partial r_{ss}}{\partial \theta}(P-\beta)} \tag{13}$$

in which μ is calculated from the following condition

$$\mu \leq \frac{|s(t=0)|}{t_{reach}} \tag{14}$$

Here, t_{reach} is the time required to reach the sliding surface $s = 0$ when $\xi(t = 0)$ is different from $\xi_{ss}(t = 0)$, which is bounded as shown above.

The feedback control law selected in (12) is discontinuous across s which leads to chattering[3]. The control discontinuity can be smoothed in a thin boundary layer neighboring the switching surface, leading to a trade–off between control bandwidth and perfect tracking which results in a guaranteed upper bound on tracking error. Therefore the control law (12) is replaced with

$$\psi = \psi_{eq} - k\text{sat}\left(\frac{s}{\phi}\right) \tag{15}$$

in which ϕ is the boundary layer thickness: the bound on tracking error.

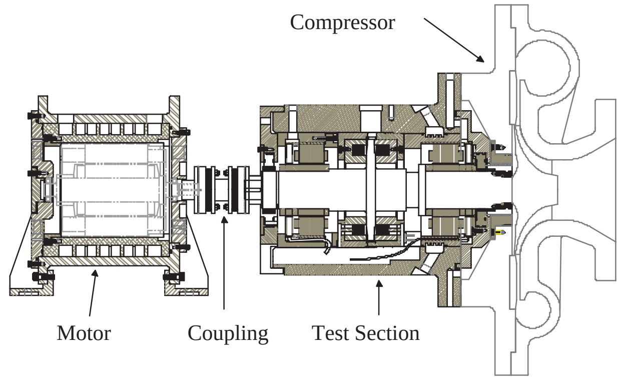

Figure 3: Centrifugal compressor test rig assembly

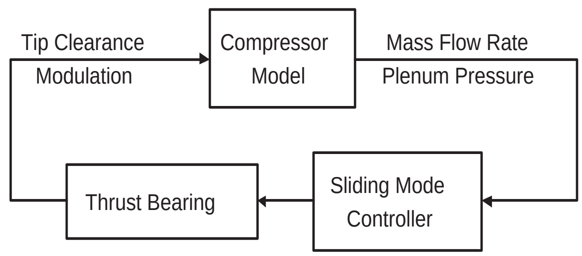

Figure 4: Schematic of the control system

COMPRESSOR TEST RIG

A compressor test facility is currently under construction with the objective of experimental investigation of this solution. The layout of the test rig is shown in Fig. 3. The compressor, rated approximately 55kW at 23000rpm, is a single stage, unshrouded centrifugal compressor provided by Kobe Steel. The design pressure ratio is 1.7 at the design mass flow rate of 2500m^3/h. The compressor is driven by a 3-phase induction motor rated 95kW at 23000rpm. The overhung single-stage centrifugal compressor is supported by active magnetic bearings. The test rig is equipped with two 12-pole, E-core, radial magnetic bearings and a magnetic thrust bearing with a load capacity of 6600N and a force slew rate of 1.3$\times$10^{6}N/s. To avoid material contact, the test rig is equipped with auxiliary bearings that restrict the maximum displacement of the impeller. For the current case, the maximum modulation is limited to 50% of the impeller tip clearance. More detail about the compressor test rig and its capabilities is described in a companion paper by Buskirk et al. [4].

RESULTS

The schematic of the control algorithm applied to this system is shown in Fig. 4. The control input resulting from the sliding mode control method, equations (15) and (3), is derived assuming that tip clearance servoing is instantaneous and the servo force is unbounded, i.e.: no servo dynamics have been included in the sliding mode analysis. Reducing the servo bandwidth (from infinity) in itself degrades the stability. To reflect the physical limitations of the actuator, the magnetic thrust bearing servo system is modeled as a third order butterworth filter with a bandwidth of 70Hz. Therefore, servo dynamics are considered in the simulations of the system, as is shown in Fig. 4.

Results from the simulation of the nonlinear model for the single stage high-speed centrifugal compressor are shown in Figs. 5, 6 and 7. Effects of sudden changes downstream of the compressor are modeled as sudden changes in discharge throttle opening. In order to show the effectiveness of the sliding mode control strategy, the system trajectories are shown for two different cases: with and without controller. Figure 5 shows the system behavior without active control, when the throttle valve is modulated from a stable point (60% throttle open) to a point well beyond the surge point (20% throttle open). As the operating point passes the surge point, the pressure and mass flow oscillations grow, eventually reversing the mass flow rate and the compressor enters the deep surge cycle.

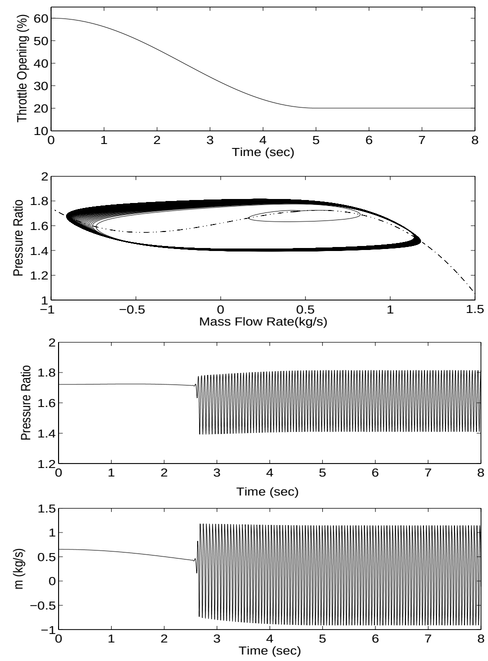

Figure 5: Transient response of the compressor WITH NO control

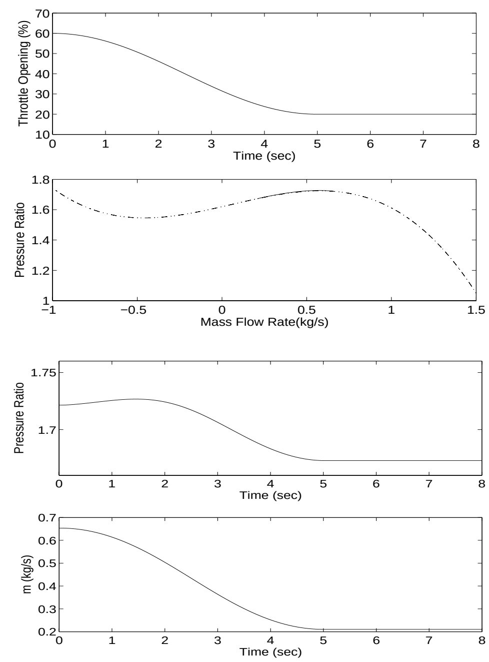

Figure 6: Transient response of the compressor WITH control

Figure 6 shows the state trajectories with the sliding mode controller activated. The throttle position is varied in the same manner as in the previous case to show the effects of the controller. The steady state compressor characteristic is shown in the second plot with a dashed line. The transient compressor characteristic is plotted with a solid line, which follows the steady state characteristic as a result of tip clearance actuation. The pressure and mass flow oscillations associated with compressor surge are quickly suppressed as shown in the next two plots. The stable mass flow rate is reduced from 0.5 kg/sec to 0.2 kg/sec.

The control performance and requirements are shown in Fig. 7. The sliding surface is kept at zero, which ensures zero steady state offset of the impeller, therefore maintaining the efficiency of the compressor. The main benefit of this control scheme is that it leads to a significantly extended operating region with very small control requirements. The control gain and tip clearance required to stabilize surge are plotted next. With tip clearance modulation less than 20% of the nominal impeller clearance, the pressure and mass flow oscillations are eliminated and the stable range of the compressor is considerably increased. The required servo force and servo force slew rate are also shown, which are well within the physical limitations of the system.

SUMMARY AND CONCLUSION

This paper presents a new method for active surge control in unshrouded centrifugal compressors using a magnetic thrust bearing to axially modulate the position of the impeller. A fluid dynamical model including a quasi-static model of the effect of tip clearance on compressor characteristic is introduced. Using the sliding mode method, a controller is designed with the objective that system trajectories remain on the compressor characteristic curve. It is shown that this control method is capable of increasing the stable range of operation extensively: in the present example which uses realistic compressor design parameters, minimum stable mass flow rate was reduced from 0.5 kg/sec to 0.2 kg/sec. Actuator slew rate, peak force and maximum tip clearance excursion requirements are well within the typical capacities of such systems: typically less than 20 percent of the available limits.

The principal advantage of the proposed approach over conventional surge control methods lies in that, in machines already equipped with magnetic thrust bearings, this method can potentially be implemented by simply modifying controller software. This dispenses with the need to introduce additional hardware, permitting adaptation of existing machinery at virtually no cost.

Implementing the surge control theory on the test rig is the future step. Further work will examine effects of

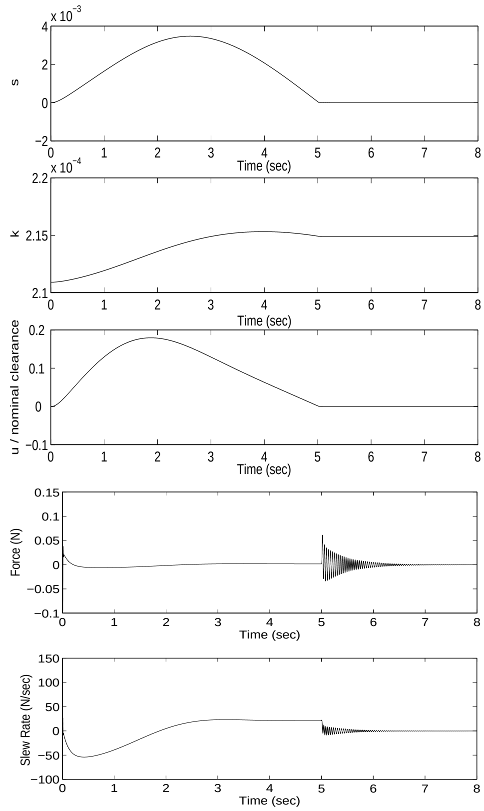

Figure 7: Sliding mode control requirements

adding the thrust bearing model to the sliding function and effects of time varying sliding surface boundary layer on the control performance. The important problem of estimating the mass flow rate from practical measurements such as pressure distributions and the impact of this estimation on achievable performance will be studied.

ACKNOWLEDGMENT

Authors gratefully acknowledge the help and advice of Hyeong-Joon Ahn from Seoul National University and Toshikazu Miyaji and Koichiro Iizuka form Kobe Steel, LTD for providing the compressor.

REFERENCES

[1] Spakovsky Z. S., Paduano J. D., Larsonneur R., Traxler A. and Bright M. M., 2000, “Tip Clearance Actuation with Magnetic Bearing for High Speed Compressor Stall Control”,ASME Turbo Expo.

[2] Greitzer E. M., 1976, “Surge and Rotating Stall in Axial Flow Compressors, Part I, II”, ASME Journal of Engineering for Power, Vol. 120, pp 190-217.

[3] Slotine, J-J. E. and Li, W., 1991, *Applied Nonlinear Control*, Prentice-Hall, Englewood Cliffs, NJ.

[4] Buskirk E.S., Sanadgol D., Maslen, E.H. and Brown N., 2004,“Centrifugal Compressor Test Facility with Active Magnetic Bearings”, The Ninth International Symposium on Magnetic Bearings.

[5] Gravdahl J.T. and Egeland O., 1999, *Compressor Surge and Rotating Stall*, Springer.

[6] Gravdahl J.T., Egeland O. and Vatland S.O., 2001, “Active Surge Control of Centrifugal Compressors Using Drive Torque”, Proceedings of 40th IEEE Conference on Decision and Control, pp 1286-1291.

[7] Graf M. B., Wong T. S., Greitzer E. M., Marble F. E., Tan C. S., Shin H. W. and Wisler D.C., 1998, “Effects of Nonaxisymmetric Tip Clearance on Axial Compressor Performance and Stability”, ASME Journal of Turbomachinery, Vol. 120, pp 661-648.

[8] Senoo Y., 1987, “Pressure Losses and Flow Field Distortion Induced by Tip Clearance of Centrifugal and Axial Compressors”, JSME International Journal, Vol. 30, No. 261, pp 375-385.

[9] Edwards, C. and Spurgeon S. K., 1998, *Sliding Mode Control: Theory and Application*, Taylor and Francis.

[10] Cumpsty N. A., 1989, *Compressor Aerodynamics*, Longman Scientific & Technical.