A New Modeling Technique and Control System Design of Flexible Rotor Using Active Magnetic Bearings for Motion and Vibration Control

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ABSTRACT

This paper proposed a new modeling technique and control system design for a flexible rotor system using active magnetic bearings (AMB).

The purpose of research is to pass through critical speed and fulfill high-speed rotation of the flexible rotor system. To achieve this purpose, it is necessary to control not only vibration but also motion. In order to express motion and vibration of the flexible rotor, an extended reduced order physical model is applied. Furthermore, a new controller combined PID with LQ control is proposed. Effectiveness of the modeling method and the controller is verified through simulations and experiments.

Keywords: Magnetic bearing, Modeling, Motion and vibration control, Flexible rotor, PID and LQ control.

1. INTRODUCTION

In recent years, AMB systems have been applied to various machines such as grinding machines, vacuum pumps and energy storage flywheel systems. In general, these control systems include a notch-filter in order to suppress the flexible mode or consider only the control of the rigid modes. Furthermore, in the case of an unforeseen accident, because the transfer function changes, the notch-filter doesn't work to suppress the flexible mode and then a so-called "spillover instability" may invite, due to the existence of vibration modes. In such cases, the vibration modes cannot be neglected in the controller design procedure. Then, the vibration control of higher order flexible modes is necessary. The flexible rotor should be modeled as a multi degree-of-freedom structure, as the notch-filter is not added to the controller, so that the vibration modes can be taken into consideration in the control system design procedure. According to the idea presented above, it is difficult to exactly identify the vibration modes of the flexible rotor, because in the case where the flexible rotor-AMB systems are precisely considered, the equivalent mass, stiffness and the orthogonal modal matrix become complex in shape. One of the authors had presented a method for identifying such complex flexible systems using an iterative modification of the modal matrix. Nevertheless, this method has not been applied to such a complicated system as a flexible rotor.

In this paper, the controller design procedure for the flexible rotor-AMB system is considered. The modeling technique is applied in order to obtain an exact multi-degree-of-freedom model of the flexible rotor-AMB system. Utilizing the obtained model, a state equation of the system model is composed and a feedback controller is designed using PID and LQ control laws. It is important to use different control methods for controlling both the rigid and flexible modes. That is, the feedback control of the rigid mode is designed using PID control law, whereas the feedback control of the flexible modes is designed using LQ control law. As the system model includes the multi-degree-of freedom structure model, the controller is designed to achieve simultaneous motion and vibration control. The effectiveness of the controller is confirmed through simulations and experiments.

2. CONTROL OBJECT

Figure 1 shows a schematic diagram of the flexible rotor used as the control object in this research. Mass of this rotor is about 5.369[kg]. This research considers only the dynamics of the flexible rotor in radial directions, because the PID controller alone controls the axial direction without the flexible mode.

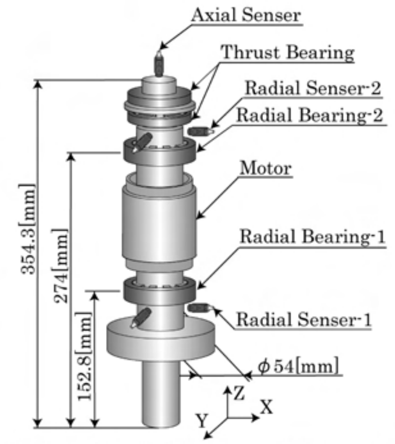


Fig.1 Schematic of flexible rotor-AMB system

3. EXTENDED REDUCED ORDER PHYSICAL MODEL

3.1 Design procedure of extended physical model

Design procedure of the extended reduced order physical model (simply extended physical model) is acted out three dimensions to apply generally. In extended physical model we consider designed conditions to represent motion and vibration. These conditions are shown below (from (a) to (d)). Masses and inertia moments are calculated to meet these designed conditions.

a) Agreement of motion energy in vibration mode
b) Keep orthogonal quality of vibration mode
c) Conservation of momentum in vibration mode (angular momentum of designed mass on each axis is equal to zero) :
d) Conservation of angular momentum in vibration mode (angular momentum of designed mass on each axis is equal to zero) can be transformed and combined to give the following.

$$\begin{bmatrix} \boldsymbol{\Phi}_i^{T}\grave{}\boldsymbol{\Phi}_i \\ \boldsymbol{\Phi}_i^{T}\grave{}\boldsymbol{\Phi}_j \\ \boldsymbol{\Phi}^{T}\mathbf{I}_R\grave{}\mathbf{I}_X \\ \boldsymbol{\Phi}^{T}\mathbf{I}_R\grave{}\mathbf{I}_Y \\ \boldsymbol{\Phi}^{T}\mathbf{I}_R\grave{}\mathbf{I}_Z \\ (\grave{}\mathbf{I}_X^{T}\tilde{\mathbf{r}}_{Gm}\boldsymbol{\Phi})^{T}\mathbf{I}_R+\boldsymbol{\Phi}^{T}\mathbf{I}_\Theta\grave{}\mathbf{I}_X \\ (\grave{}\mathbf{I}_Y^{T}\tilde{\mathbf{r}}_{Gm}\boldsymbol{\Phi})^{T}\mathbf{I}_R+\boldsymbol{\Phi}^{T}\mathbf{I}_\Theta\grave{}\mathbf{I}_Y \\ (\grave{}\mathbf{I}_Z^{T}\tilde{\mathbf{r}}_{Gm}\boldsymbol{\Phi})^{T}\mathbf{I}_R+\boldsymbol{\Phi}^{T}\mathbf{I}_\Theta\grave{}\mathbf{I}_Z \end{bmatrix} \begin{bmatrix} \xi & \mathbf{0}_{3n\times 3n} \\ \mathbf{0}_{3n\times 3n} & \mathbf{I}_{3n\times 3n} \end{bmatrix} \begin{bmatrix} \breve{\mathbf{M}} \\ \breve{\mathbf{J}}' \end{bmatrix} = \begin{bmatrix} \boldsymbol{\mu}_i \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \quad (1)$$

Here,

$$\xi = \mathrm{diag}[\mathbf{I}_{31}\cdots\mathbf{I}_{31}]\ ,\ \mathbf{I}_{31} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$\breve{\mathbf{M}}$: Mass of inertia of each rigid body

$\breve{\mathbf{J}}'$: Moment of inertia of each rigid body

$\tilde{\mathbf{r}}_{Gm}$: Distance from center of gravity to design point

n : The number of rigid body element

$$\mathbf{I}_X = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ \mathbf{I}_Y = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ \mathbf{I}_Z = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\grave{}\mathbf{I}_X = \mathrm{diag}\left[\mathbf{I}_X \ \cdots \ \mathbf{I}_X\right](n\times n)$$

$$\grave{}\mathbf{I}_Y = \mathrm{diag}\left[\mathbf{I}_Y \ \cdots \ \mathbf{I}_Y\right](n\times n)$$

$$\grave{}\mathbf{I}_Z = \mathrm{diag}\left[\mathbf{I}_Z \ \cdots \ \mathbf{I}_Z\right](n\times n)$$

$$\mathbf{I}_R = \begin{bmatrix} \mathbf{I}_{3n} & \mathbf{0}_{3n} \\ \mathbf{0}_{3n} & \mathbf{0}_{3n} \end{bmatrix},\quad \mathbf{I}_\Theta = \begin{bmatrix} \mathbf{0}_{3n} & \mathbf{0}_{3n} \\ \mathbf{0}_{3n} & \mathbf{I}_{3n} \end{bmatrix}$$

$$\boldsymbol{\Phi} = \begin{bmatrix} \Phi_{R11} & \Phi_{R12} & \cdots & \Phi_{R1m} \\ \vdots & \vdots & & \vdots \\ \Phi_{Rn1} & \Phi_{Rn2} & \cdots & \Phi_{Rnm} \\ \Phi_{\Theta 11} & \Phi_{\Theta 12} & \cdots & \Phi_{\Theta 1m} \\ \vdots & \vdots & & \vdots \\ \Phi_{\Theta n1} & \Phi_{\Theta n2} & \cdots & \Phi_{\Theta nm} \end{bmatrix}$$

$\boldsymbol{\Phi}_R$: Mode element of translation direction

$\boldsymbol{\Phi}_\Theta$: Mode element of rotation

where $\tilde{\mathbf{r}}_{Gm}$ is a diagonally arranged matrix with a 3 • 3 sub-matrix which is a skew symmetric matrix associated with the position vector of the design points relative to center of gravity of the rotor in equilibrium.

The mass $\mathbf{M}_A$ and the moment of inertia $\mathbf{J}_A$ of the rigid body are calculated to be equal to real mass $\mathbf{M}_{real}$ and real moment of inertia $\mathbf{J}_{real}$ of the system in total.

$$\mathbf{M}_A = \mathbf{M}_{real} - \boldsymbol{\eta}^T\mathbf{m}\boldsymbol{\eta} \quad (2)$$

$$\mathbf{J}'_A = \mathbf{J}'_{real} - \left(\boldsymbol{\eta}^T\mathbf{J}'\boldsymbol{\eta} + \boldsymbol{\eta}^T\tilde{\mathbf{r}}_{Gm}^T\mathbf{M}\tilde{\mathbf{r}}_{Gm}\boldsymbol{\eta}\right) \quad (3)$$

$$\boldsymbol{\eta} = \left[\mathbf{I}_1 \ \cdots \ \mathbf{I}_n\right]^T$$

Matrix $\boldsymbol{\eta}$ is unit matrix with as many number of the 3• 3.

The stiffness matrix is obtained from the following equation.

$$\mathbf{K} = \boldsymbol{\Phi}^{-T}\boldsymbol{\mu}\boldsymbol{\omega}^2\boldsymbol{\Phi}^{-1} \quad (4)$$

$\boldsymbol{\omega}$: Natural frequency of each mode

3.2 Extended reduced order physical model of the rotor-actuator system

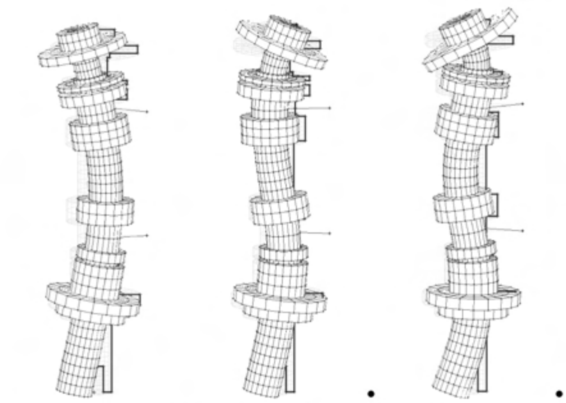
In this section, the experimental rotor system is modeled using the extended physical modeling technique shown in above section. In this study, rigid two modes and flexible one mode are considered to control. Therefore, a number of rigid body element is three. But the storyboard number of rigid body element is 9 (n=9) in order to make clearly understandable vibration characteristic. Here, in making model, the following conditions are defined.

Reference rigid body that expresses motion (rigid mode) is labeled as G0. And rigid body elements that express vibration (flexible mode) are labeled as B1~B6

We consider the model of rotor at two-dimension. The G0 and B1~B6 have 2 degrees-of-freedom (2DOF) in X-axis direction and Y-axis rotation, respectively.

Each rigid body element between rigid body element and reference rigid body are connected with the spring each other.

Figure 2 shows the mode shapes of the rotor and their natural frequencies obtained by FEM. These mode shapes are analyzed with the rotor imaginarily supported by servo stiffness of PID controller. Here, the first 2nd modes are considered to make the model.



1stmode • • 2ndmode • • 3rd mode

1105.2[Hz] • • 2807.5[Hz] • • 4517.6[Hz]

Fig.2 Vibration mode shapes

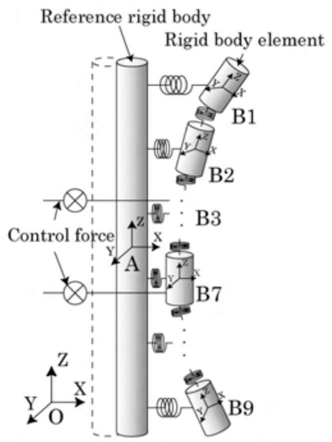


Fig.3 Extended physical model

Figure 3 shows the extended model of the control object using our method. As a modeling point, the top and the bottom of the rotor (B1, B6), the supported points of the actuators in the thrust direction (B2, B4) and the center of gravity of the rotor (B3) are selected and the rest of the rigid body element fits in the position between B4 and B6 (B5).

The parameters of each rigid body element are shown in Where, element G0 is reference rigid body. In this study, the control for the flexible rotor is of particular interest.

3.3 Equations of motion

In this section, the equation of motion is derived using the constraint addition method. Firstly, the equations of motion of the reference rigid body (A) with no constraint and havinB3 DOF are given by

$$\mathbf{M}_{OA}\dot{\mathbf{V}}_{OA} = \mathbf{F}_{OA} \quad (5)$$

$$\mathbf{J}'_{OA}\boldsymbol{\Omega}'_{OA} + \tilde{\boldsymbol{\Omega}}'_{OA}\mathbf{J}'_{OA}\boldsymbol{\Omega}'_{OA} = \mathbf{N}'_{OA} \quad (6)$$

Similarly, unconstrained equations of motion of the rigid body elements (B1~B6) are given by

$$\mathbf{J}'_{OB}\boldsymbol{\Omega}'_{OB} + \tilde{\boldsymbol{\Omega}}'_{OB}\mathbf{J}'_{OB}\boldsymbol{\Omega}'_{OB} = \mathbf{N}'_{OB} \quad (7)$$

The generalized speed $\mathbf{H}$ for the unconstrained system is given by

$$\mathbf{H} = \begin{bmatrix} \mathbf{V}_{OA} \\ \boldsymbol{\Omega}'_{OA} \\ \mathbf{V}_{OB} \\ \boldsymbol{\Omega}'_{OB} \end{bmatrix} \quad (8)$$

Mass matrix $\mathbf{M}^{H}$ and external force matrix $\mathbf{F}^{H}$ are given by

$$\mathbf{M}^{H} = \begin{bmatrix} \mathbf{M}_{OA} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{J}'_{OA} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{M}_{OB} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{J}'_{OB} \end{bmatrix} \quad (9)$$

$$\mathbf{F}^{H} = \begin{bmatrix} \mathbf{F}_{OA} \\ \mathbf{N}'_{OA} \\ \mathbf{F}_{OB} \\ \mathbf{N}'_{OB} \end{bmatrix} \quad (10)$$

Using equations (8) ~ (10) equation of motion for the unconstrained system can be written as

$$\mathbf{M}^{H}\dot{\mathbf{H}} = \mathbf{F}^{H} \quad (11)$$

Secondly, Positions of the rigid body elements A with respect to the global coordinate system O are given by

$$\mathbf{R}_{OB} = \boldsymbol{\eta}_{B}\mathbf{R}_{OA} + \grave{}\mathbf{C}_{OA}\mathbf{R}_{AB} \quad (12)$$

Here,

$$\boldsymbol{\eta}_{B} = \begin{bmatrix} \mathbf{I}_{3} & \cdots & \mathbf{I}_{3} \end{bmatrix}^{T}$$

Transformation matrix when reference rigid body A is rotated at an angle of θ about the Y-axis of the global coordinate system O is given by

$$\mathbf{C}_{OA} = \mathbf{C}_{OA1}\mathbf{C}_{A1A2}\mathbf{C}_{A2A}$$
$$= \begin{bmatrix} c\theta_{A1A2}c\theta_{A2A} & -c\theta_{A1A2}s\theta_{A2A} & s\theta_{A1A2} \\ s\theta_{OA1}s\theta_{A1A2}c\theta_{A2A} + c\theta_{OA1}s\theta_{A2A} & -s\theta_{OA1}s\theta_{A1A2}s\theta_{A2A} + c\theta_{OA1}c\theta_{A2A} & -s\theta_{OA1}c\theta_{A1A2} \\ -c\theta_{OA1}s\theta_{A1A2}c\theta_{A2A} + s\theta_{OA1}s\theta_{A2A} & c\theta_{OA1}s\theta_{A1A2}s\theta_{A2A} + s\theta_{OA1}c\theta_{A2A} & c\theta_{OA1}c\theta_{A1A2} \end{bmatrix} \quad (13)$$

$$\grave{\mathbf{C}}_{OA} = \mathrm{diag}\left[\mathbf{C}_{OA} \quad \cdots \quad \mathbf{C}_{OA}\right]$$

Time derivative of equation (12) is given by

$$\mathbf{V}_{OB} = \boldsymbol{\eta}_{B}\mathbf{V}_{OA} - \grave{\mathbf{C}}_{OA}\tilde{\mathbf{R}}_{AB}\boldsymbol{\Omega}'_{OA} + \grave{\mathbf{C}}_{OA}\boldsymbol{\Phi}_{R}\mathbf{h} \quad (14)$$

Here,

$$\boldsymbol{\Phi}_{R} = \begin{bmatrix} \boldsymbol{\Phi}_{R11} & \boldsymbol{\Phi}_{R12} & \boldsymbol{\Phi}_{R13} & \boldsymbol{\Phi}_{R14} \\ \vdots & \vdots & \vdots & \vdots \\ \boldsymbol{\Phi}_{R91} & \boldsymbol{\Phi}_{R92} & \boldsymbol{\Phi}_{R93} & \boldsymbol{\Phi}_{R94} \end{bmatrix}$$

Positions of the rigid body elements B with respect to the reference rigid body coordinate system A are given by

$$\mathbf{R}_{AB} = \boldsymbol{\Phi}_{R}\mathbf{q} + \mathbf{R}_{ABZcont} \quad (15)$$

Here,

$\mathbf{q}$: modal coordinates

Time derivative of equation (15) is given by

$$\mathbf{V}_{AB} = \boldsymbol{\Phi}_{R}\dot{\mathbf{q}} = \boldsymbol{\Phi}_{R}\mathbf{h} \quad (16)$$

$\mathbf{h}$: modal speeds

The angular velocity of rigid body element B with respect to the global coordinate system O is given by

$$\boldsymbol{\Omega}'_{OB} = \mathbf{C}^{T}_{AB}\boldsymbol{\eta}_{B}\boldsymbol{\Omega}'_{OA} + \boldsymbol{\Omega}'_{AB} \quad (17)$$

The angular velocity of rigid body element B with respect to the reference rigid body system A is given by

$$\boldsymbol{\Omega}'_{AB} = \boldsymbol{\Phi}_{\theta}\dot{\mathbf{q}} = \boldsymbol{\Phi}_{\theta}\mathbf{h} \quad (18)$$

As equation (14) ~ (18) is laid out, the velocity and the angular velocity of the rigid body elements B are given by

$$\mathbf{V}_{OB} = \boldsymbol{\eta}_{B}\mathbf{V}_{OA} - \grave{\mathbf{C}}_{OA}\tilde{\mathbf{R}}_{AB}\boldsymbol{\Omega}'_{OA} + \grave{\mathbf{C}}_{OA}\boldsymbol{\Phi}_{R}\mathbf{h} \quad (19)$$

$$\boldsymbol{\Omega}'_{OB} = \mathbf{C}^{T}_{AB}\boldsymbol{\eta}_{B}\boldsymbol{\Omega}'_{OA} + \boldsymbol{\Phi}_{\theta}\mathbf{h} \quad (20)$$

The force acting on each rigid body elements given by

$$\mathbf{F}^{H} = \begin{bmatrix} \mathbf{F}_{OA} \\ \mathbf{N}'_{OA} \\ \mathbf{F}_{OB} \\ \mathbf{N}'_{OB} \end{bmatrix}$$

$$= \begin{bmatrix} -\mathbf{D}_{Z}m_{OA}g \\ \boldsymbol{\eta}^{T}_{B}\grave{\tilde{\mathbf{R}}}_{AB}\left(\mathbf{k}_{Rsp}\mathbf{R}_{AB} + \mathbf{k}_{R\theta sp}\boldsymbol{\Phi}_{\theta}\mathbf{q}\right) \\ -\grave{\mathbf{D}}_{Z}\mathbf{m}_{OB}g \\ -\mathbf{C}^{T}_{AB}\mathbf{c}_{\theta Rsp}\mathbf{V}_{AB} \end{bmatrix} \cdots$$
$$\begin{bmatrix} +\mathbf{C}_{OA}\boldsymbol{\eta}^{T}_{B}\left(\mathbf{c}_{Rsp}\mathbf{V}_{AB} + \mathbf{c}_{R\theta sp}\boldsymbol{\Phi}_{\theta}\mathbf{h} + \mathbf{k}_{Rsp}\mathbf{R}_{AB} + \mathbf{k}_{R\theta sp}\boldsymbol{\Phi}_{\theta}\mathbf{q}\right) \\ +\boldsymbol{\eta}^{T}_{B}\left(\mathbf{c}_{\theta Rsp}\mathbf{V}_{AB} + \mathbf{c}_{\theta sp}\boldsymbol{\Phi}_{\theta}\mathbf{h} + \mathbf{k}_{\theta Rsp}\mathbf{R}_{AB} + \mathbf{k}_{\theta sp}\boldsymbol{\Phi}_{\theta}\mathbf{q}\right) \\ -\grave{\mathbf{C}}_{OA}\left(\mathbf{c}_{Rsp}\mathbf{V}_{AB} + \mathbf{c}_{R\theta sp}\boldsymbol{\Phi}_{\theta}\mathbf{h} + \mathbf{k}_{Rsp}\mathbf{R}_{AB} + \mathbf{k}_{R\theta sp}\boldsymbol{\Phi}_{\theta}\mathbf{q}\right) \\ -\mathbf{C}^{T}_{AB}\left(\mathbf{c}_{\theta sp}\boldsymbol{\Phi}_{\theta}\mathbf{h} + \mathbf{k}_{\theta Rsp}\mathbf{R}_{AB} + \mathbf{k}_{\theta sp}\boldsymbol{\Phi}_{\theta}\mathbf{q}\right) \end{bmatrix}$$
$$- \begin{bmatrix} \mathbf{0} \\ \tilde{\boldsymbol{\Omega}}'_{OA}\mathbf{J}'_{OA}\boldsymbol{\Omega}'_{OA} \\ \mathbf{0} \\ \grave{\tilde{\boldsymbol{\Omega}}}'_{OB}\mathbf{J}'_{OB}\boldsymbol{\Omega}'_{OB} \end{bmatrix} \quad (21)$$

Here,

$$\mathbf{D}_{Z} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \grave{\mathbf{D}}_{Z} = \begin{bmatrix} \mathbf{D}_{Z} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{D}_{Z} \end{bmatrix}$$

$$\mathbf{k}_{sp} = \begin{bmatrix} \mathbf{k}_{Rsp} & \mathbf{k}_{R\theta sp} \\ \mathbf{k}_{\theta Rsp} & \mathbf{k}_{\theta sp} \end{bmatrix} \quad (\mathbf{k}_{R\theta sp} = \mathbf{k}_{\theta Rsp})$$

$$\mathbf{c}_{sp} = \begin{bmatrix} \mathbf{c}_{Rsp} & \mathbf{c}_{R\theta sp} \\ \mathbf{c}_{\theta Rsp} & \mathbf{c}_{\theta sp} \end{bmatrix} \quad (\mathbf{c}_{R\theta sp} = \mathbf{c}_{\theta Rsp})$$

Stiffness matrix $\mathbf{k}_{sp}$ and Damping matrix $\mathbf{C}_{sp}$ is made out by the extended physical model method.

Finally, the equation of motion for the constrained system will be derived. The generalized speed $\mathbf{S}$ for the constrained the system is given by

$$\mathbf{S} = \begin{bmatrix} \mathbf{V}_{OA} \\ \boldsymbol{\Omega}'_{A} \\ \mathbf{h} \end{bmatrix} \quad (22)$$

Equation in which $\mathbf{H}$ is expressed by $\mathbf{S}$ is given by

$$\mathbf{H} = \mathbf{H}_{S}\mathbf{S} + \mathbf{H}_{\bar{S}} \quad (23)$$

Here,

$$\mathbf{H}_{S} = \begin{bmatrix} \mathbf{I}_{3} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{3} & \mathbf{0} \\ \boldsymbol{\eta}_{B} & -\grave{\mathbf{C}}_{OA}\tilde{\mathbf{R}}_{AB} & \grave{\mathbf{C}}_{OA}\boldsymbol{\Phi}_{R} \\ \mathbf{0} & \mathbf{C}^{T}_{AB}\boldsymbol{\eta}_{B} & \boldsymbol{\Phi}_{\theta} \end{bmatrix} \quad (24)$$

$$\mathbf{H}_{\bar{S}} = \mathbf{0} \quad (25)$$

Time derivative of $\mathbf{H}_{s}$ is given by

$$\frac{d\mathbf{H}_S}{dt}=\begin{bmatrix}\mathbf{0} & \mathbf{0} & \\ \mathbf{0} & 0 & \\ \boldsymbol{\eta}_2 & \tilde{\mathbf{C}}_{OG0}\tilde{\Delta}\boldsymbol{\Omega}'_{OG0}\tilde{\Delta}\left(\tilde{\mathbf{d}}_X\boldsymbol{\Phi}_R\mathbf{q}_G+\mathbf{R}_{G0GZ}\right) & \\ \mathbf{0} & \mathbf{0} & \end{bmatrix}$$

$$\left.\begin{matrix}\mathbf{0}\\ \mathbf{0}\\ +\tilde{\mathbf{C}}_{OG0}\tilde{\Delta}\tilde{\mathbf{d}}_X\boldsymbol{\Phi}_R\mathbf{h}_G \quad \tilde{\mathbf{C}}_{OG0}\tilde{\Delta}\boldsymbol{\Omega}'_{OG0}\tilde{\mathbf{d}}_X\boldsymbol{\Phi}_R\\ \mathbf{0}\end{matrix}\right] \quad (26)$$

$$\frac{d\mathbf{H}_{\bar{S}}}{dt}=\mathbf{0} \quad (27)$$

Therefore, the equations of motion can be written as below using above these equations.

$$\mathbf{M}^S\dot{\mathbf{S}}=\mathbf{F}^S \quad (28)$$

Here,

$$\mathbf{M}^S=\mathbf{H}_S^T\mathbf{M}^H\mathbf{H}_S \quad (29)$$

$$\mathbf{F}^S=\mathbf{H}_S^T\left\{\mathbf{F}^H-\mathbf{M}^H\left(\frac{d\mathbf{H}_S}{dt}\mathbf{S}+\frac{d\mathbf{H}_{\bar{S}}}{dt}\right)\right\} \quad (30)$$

4. CONTORL SYSTEM DESIGN

In this section, the design of a control system for a flexible rotor-AMB system using the model obtained in the former section is carried out. Firstly, the PID control law that controls the two rigid modes (parallel and conical) is applied to the system model. Then, LQ control law that controls the flexible modes is applied.

4.1 PID Control

Since the rotor-AMB system is essentially unstable, in this study, the system is stabilized using PID control. The two rigid modes are controlled by PID controller. Figure 4 shows the associated block diagram.

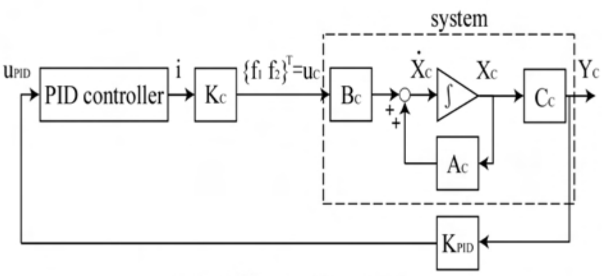


Fig.4 Block diagram

The transfer function of the PID controller is given by

$$\frac{\mathrm{i}}{\mathrm{u_{PID}}}=\left(\frac{\mathrm{T\alpha s+1}}{\mathrm{Ts+1}}\right)\left(\frac{\mathrm{T\alpha s+1}}{\mathrm{Ts+1}}\right) \quad (31)$$

where T and α are a time constant and phase lead factor as design parameters of the PID controller. This transfer function can be rewritten as

$$\frac{\mathrm{i}}{\mathrm{e}}\cdot\frac{\mathrm{e}}{\mathrm{u_{PID}}}=\frac{\mathrm{T^2\alpha^2s^2+2T\alpha s+1}}{\mathrm{T^2s^2+2Ts+1}} \quad (32)$$

Here, in figure 4 f_{11} and f_{22} mean that

$$f_{11}=K_c i_{11}\ ,\ f_{22}=K_c i_{22} \quad (33)$$

and

$$\mathbf{U}=\left\{u_{1PID} \quad u_{2PID}\right\}^T$$

According to the equation shown above,

$$f_{11}=2K_cT\alpha(1-\alpha)\dot{e}+K_c\left(1-\alpha^2\right)e+K_c\alpha^2u_{1PID} \quad (34)$$

$$f_{22}=2K_cT\alpha(1-\alpha)\dot{e}+K_c\left(1-\alpha^2\right)e+K_c\alpha^2u_{2PID} \quad (35)$$

4.2 Augment system

Figure 5 shows a block diagram of the system combined PID control with LQ control. The feedback gain vectors $\mathbf{K}_{PID}$ and $\mathbf{K}_{LQ}$ are obtained by applying LQ control law to the augmented system.

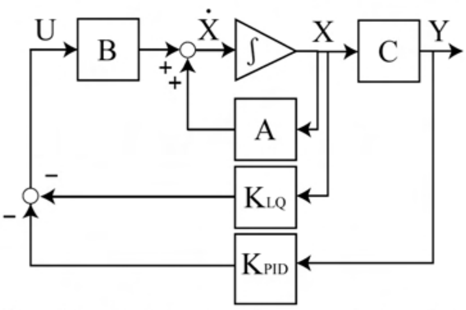


Fig.5 Block diagram of augmented system

Derived nonlinear equation (28) is linearized to obtain a state equation. The state equation and output equation that include PID control system into this equation are shown as follows.

$$\dot{\mathbf{X}}=\mathbf{AX}+\mathbf{BU} \quad (36)$$

$$\mathbf{Y}=\mathbf{CX} \quad (37)$$

Here,

$$\mathbf{X}_c=\left[\mathrm{V_{OA}} \quad \Omega'_{\mathrm{OA}} \quad \mathrm{h} \quad \mathrm{R_{OA}} \quad \Theta'_{\mathrm{OA}} \quad \mathrm{q}\right]$$

$$\mathbf{X}_h=\left[\ddot{e}_{x1} \quad \dot{e}_{x1} \quad e_{x1} \quad \ddot{e}_{y1} \quad \dot{e}_{y1} \quad e_{y1} \quad \ddot{e}_{x2} \quad \dot{e}_{x2} \quad e_{x2} \quad \ddot{e}_{y2} \quad \dot{e}_{y2} \quad e_{y2}\right]$$

$$\mathbf{X}=\left\{\mathbf{X}_c \quad \mathbf{X}_h\right\}^T$$

$$\mathbf{U}=\left\{u_{1xPID} \quad u_{1yPID} \quad u_{2xPID} \quad u_{2yPID}\right\}^T$$

A modal displacement is translated to physical displacement using the following equation

$$\mathbf{Z}=\mathbf{TX} \quad (38)$$

Here,

$$\dot{\mathbf{Z}}=\mathbf{TAT}^{-1}\mathbf{Z}+\mathbf{TBU}$$

$$\mathbf{Y}=\mathbf{T}^{-1}\mathbf{CZ}$$

Here,

$$\mathbf{X}_s = \begin{bmatrix} V_{OB1X} & V_{OB1Y} & V_{OB2X} & V_{OB2Y} \\ R_{OB1X} & R_{OB1Y} & R_{OB2X} & R_{OB2Y} \end{bmatrix.$$

$$\cdots\cdots\cdots \begin{matrix} V_{OB7X} & V_{OB7Y} & V_{OB9X} & V_{OB9Y} \\ R_{OB7X} & R_{OB7Y} & R_{OB9X} & R_{OB9Y} \end{matrix}\Bigg]$$

$$\mathbf{Z} = \left\{ \mathbf{X}_s \quad \mathbf{X}_h \right\}^T$$

5. COMPUTER SIMULATION

In this simulation, controlled results are shown by frequency responses using only PID control law and control law combined PID with LQ. The rotational frequency of the rotor is 145Hz in this simulation. Figure 6 shows frequency response obtained at rigid body element (B3) when the rotor is exited with impulse at the same point. In this figure, a red line shows the frequency response with the PID controller alone and a black line with PID and LQ. It is clearly shown that the rigid modes are well suppressed by the PID controller. However, the flexible modes are not suppressed. This is because PID control law is used for controlling the rigid modes only. It is evident shown that the 1st flexible mode is well and 2nd flexible mode is slightly suppressed applying the controller combined PID control law with LQ control law.

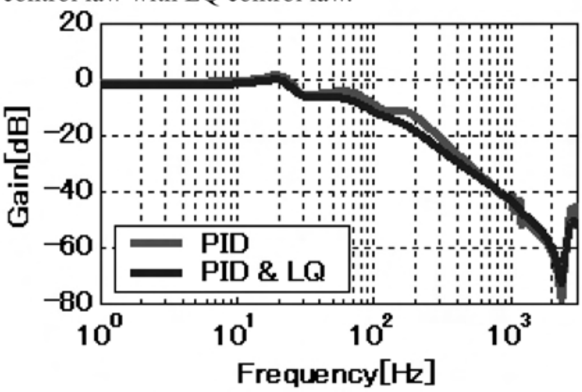


Fig.6 Frequency response

6. EXPERIMENTAL

6.1 Experimental devise

Displacements of the rotor measured by gap sensors are input to the controller through A/D converter. Velocities of control object are obtained by differentiating the displacement signals. Actuators driven by amplified output signals from D/A converter generate control forces.

6.1 Experimental Result

A rotation test are carried out by the experimental setup shown in FiB1. This experiment is done under the same condition as the simulation described above. An unbalance response was measured as shown in Figure 7 It is clearly shown that the amplitudes in rigid modes are minuteness. The experimental result showed that the control system was effective in the critical speed passage problem in rigid body modes.

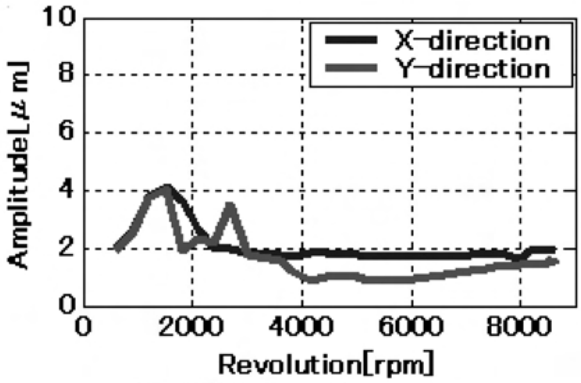


Fig.7 Unbalance Responses

7. CONCLUSIONS

In this paper, a controller design procedure for a flexible rotor-AMB system has been investigated. The proposed modeling technique was applied to obtain an exact multi degree of freedom model of the flexible rotor-AMB system. The extended physical model of the flexible rotor-AMB system can simultaneously express the motion and vibration. It was demonstrated through simulation and experiment that the controller by control law combined PID control law with LQ control law was effective for system stabilization and vibration and motion control of high order flexible modes.

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