STIFFNESS ANALYSIS OF THE MAGNETIC BEARINGS SYSTEM

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ABSTRACT

The stiffness of the magnetic bearing system was studied on the basis of its non-linear mathematic model. How the controller will influence to the stiffness of the magnetic bearing system was studied first. One method was to change the parameter of the controller. In this way, the optimum parameter of the controller can be find out. The second method is to change the control algorithm. In this way, we select the PID controller and Fuzzy controller as the study objects. We discussed how these methods would influence to the stiffness of the system by simulation of computer. In order to enhance the stiffness of the system, the decoupling algorithm is studied in our control algorithm. Apart from these, the simulation experiment was made to illustrate the motion of the axis when the external disturbance was imposed on. To the same disturbance, the different control algorithm and different control parameters will deduce the different performance of stiffness and damping.

INTRODUCTION

Magnetic bearing is called as active magnetic bearing (AMB) or electromagnetic bearing. More attention was paid on it for its incomparable merits, such as no friction, long service life, without lubrication and no pollution, high rotational speed and high precision. Especially for its high rotational speed, it was used in high-speed grinder and advanced tool machine widely and got high efficiency and high precision in machining process. The application of the magnetic bearings in the high-speed machine tool was depended on the high stiffness of the bearings[1]. The study on the simulation of the magnetic bearings was based on the linear mathematic model. There are big differences between the simulation results and actual results for neglecting influence of the non-linear aspects and high order differential term. Therefore, we analyzed the influence of all kinds of parameters to the system based on the non-linear mathematic model[2], and the non-linear aspects are considered.

MAGNETIC BEARING SYSTEM MODEL

In y direction as shown in Figure 1, upon arrow as positive of the direction of magnetic force, according to the magnetic force formula, the differential control was adopted. The total force $F = F_1 - F_2 - mg$:

$$F = \frac{\mu_0 S N^2}{4}[(\frac{I_0 + i}{C_0 + y})^2 - (\frac{I_0 - i}{C_0 - y})^2] - mg \quad (1)$$

According to Newton's Second Law of Motion, the relationship of displacement and coil current:

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$$m\ddot{y} = \frac{\mu_0 S N^2}{4}[(\frac{I_0 + i}{C_0 + y})^2 - (\frac{I_0 - i}{C_0 - y})^2] - mg \quad (2)$$

The direction of displacement is the same as the direction of force.In the formula 2, y represents the offset of the center of axis, and its direction is upon arrow. The equilibrium point of the axis is center of the coordinate.

C0 -- the air gap of the equilibrium;

ys – the displacement of the upside of the axis measured by sensor, the origin of ys is the reference mounting plane, the direction of it is down arrow, the relationship of ys, C0 and y is: ys = C0 +y;

m – the mass of the axis;

Vs – the output voltage of the sensor;

Vr --the set value of voltage;

ek --the error of the voltage;

uk --the output voltage of the controller;

I0 -- the bias current;

i -- the control current;

The formula 2 discovered the non-linear relationship of displacement and current. It was dealt with linearization in normal method, and was linearized in the operating point by Taylor formula. But in this paper, it was dealt with the forth-order Runge-Kutta

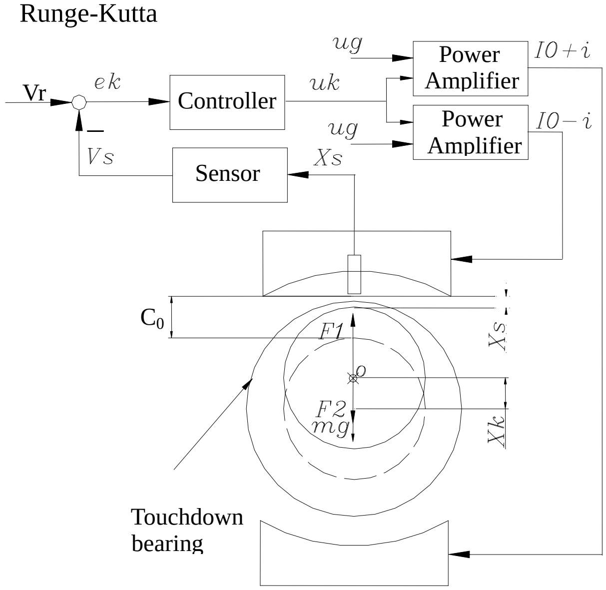


Fig1 The structure of magnetic suspension system

formula. Therefore, the internal non-linear performance of y and i was discovered and the simulation precision was improved correspondingly. The difference between theoretical model and actual system was shorten.

THE INFLUENCE OF PID CONTROLLER TO STIFFNESS

Controller Model

non-complete differential PID control algorithm was adopted to restrain the high-frequency noise. In the actual PID control circuit, the bandwidth of the differential term was limited by importing the low-pass filter part. The mathematic expression is

$$G_c(s) = K_p(1 + \frac{1}{T_i s} + \frac{1 + T_d s}{1 + \frac{T_d}{K}s})$$

$$= K_p(1 + K + \frac{1}{T_i s} + \frac{1 - K}{1 + \frac{T_d}{K}s}) \quad (3)$$

Here Kp ----- proportional gain;

Ti ----- integral time constant;

Td ----- differential time constant;

K ------ decay constant;

So the equation (3) can be dispersed and expressed as increment:

$$u(k) = u(k-1) + \Delta u(k)$$

$$= u(k-1) + \Delta u_p(k) + \Delta u_i(k) + \Delta u_d(k) \quad (4)$$

The Influence Of Controller's Parameter To The System Stiffness

In y direction of 1-DOF as example, the axis was imposed on a external force wf = mg/2 when the axis is on the operating point, whose direction is the same as gravitation. The performance of system stiffness and damping was simulated to find out the influence of controller's parameters. The output response displacement can be expressed by the equations:

$$x(t) = Ce^{-\sigma t}\cos(\omega t - \varphi) \quad (5)$$

Here

$$\omega = \sqrt{k/m - d^2/4m^2} \qquad (6)$$
$$\sigma = d/2m$$

k --- system stiffness; d --- system damping; ω --- disturbance frequency; σ decay rate constant。

The parameters of the magnetic bearings system were shown in table 1.

Table 1 parameters of the magnetic bearings system

Description	Sym.	Value	Dim.
Area of pole shoe	S	600.279	mm^2
Coil	N	100	circle
Air gap	C_0	0.4	mm
Mass of axis	m	8.5	Kg
Bias current	I_0	3.0	A

1. The influence of Kp to the system stiffness and damping

When Ti equals to 0.02 and Td equals to 0.02, the displacement curves of the axis were shown as figure 2 with different Kp values. Kp equals to 0.8,1.0and 1.5 respectively.

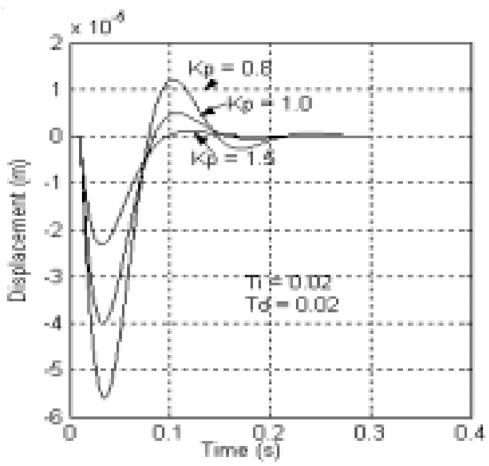


Fig 2 The influence of parameter Kp

From figure 2, we can deduce that the system response speed would be accumulated with the bigger of Kp, and the same time, the system overshoot would be reduced. Under this circumstance, the value of σ reduced and ω almost unchanged.

Therefore, the damping of system reduced and the stiffness of system increased from the formula (6).

2. The influence of Ti to the system stiffness and damping

When Kp equals to 1.0 and Td equals to 0.02, the displacement curves of the axis were shown as figure 3 with different Ti values. Ti equals to 0.01,0.02 and 0.05 respectively.

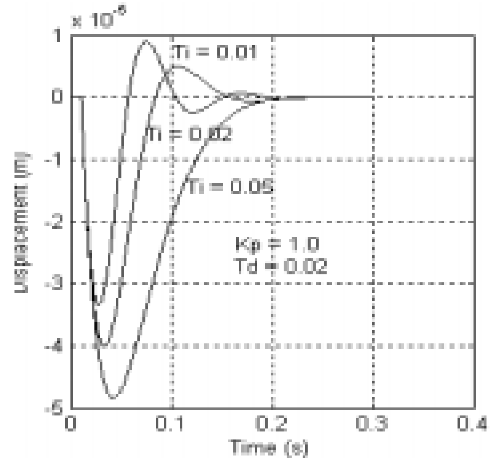


Fig3 The influence of parameter Ti

The main function of integral term is to eliminate the static error of the system. With the bigger of integral time constant (Ti), the influence of the integral term will be weakened. The value of σ increased and ω increased too. Therefore, the damping of system increased and the stiffness of system increased more.

3. The influence of Td to the system stiffness and damping

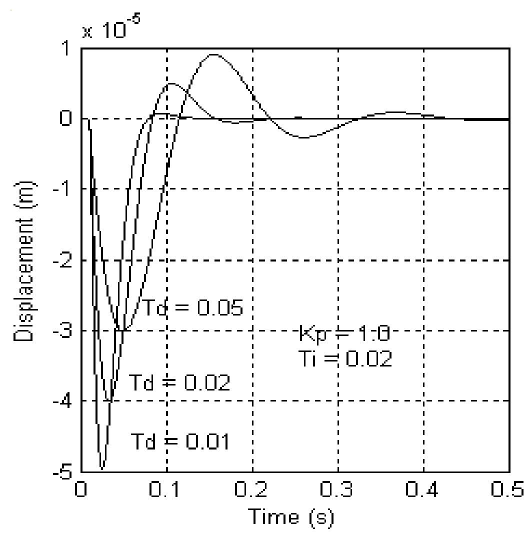


Fig 4 The influence of parameter Td

When Kp equals to 1.0 and Ti equals to 0.02, the displacement curves of the axis were shown as figure 4 with different Td values. Td equals to 0.01,0.02 and 0.05 respectively.

The main function of differential term was to improve the performance of control object with inertia. With the increasing of Td, The value of σ reduced and ω increased too. Therefore, the damping of system reduced, whether the stiffness of system increased or not decided by the scalar of σ and ω.

THE INFLUENCE OF FUZZY+PID TO THE SYSTEM PERFORMANCE

The Suspension Performance of the Fuzzy+PID

It's known to all that the dynamic performance of Fuzzy controller is wonderful comparing with its static

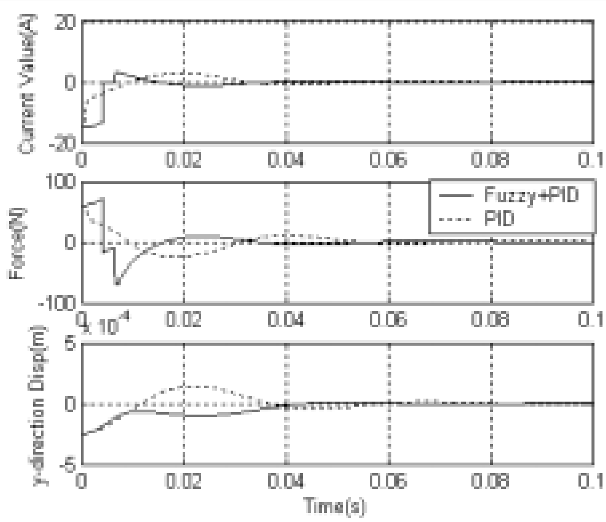


Fig 5　The compare of Fuzzy+PID and PID

performance. The input of the controller only includes the error and the change of the error, so it can be equivalence to a parameter-variable PD controller. It's hard to eliminate the static error without the integral part. According to the different displacement, Fuzzy or PID will be selected as the control algorithm. the merits of these two strategies are combined perfectly.

In Figure 5, it shows the compare of the Fuzzy+PID and PID in the suspension of the spindle. And in Table 2, it illustrated that the performance of these two strategies.

Table 2　The compare of Fuzzy+PID and PID

Description	Sym.	Fuzzy+PID	PID	Dim.
Peak-value time	tp	0.0511	0.0221	s
Overtake	yt	4.38	58.10	%
Regulation time	ts	0.0410	0.0702	s
Rising time	tr	0.0325	0.0035	s
Delay time	td	0.0027	0.0029	s

The Restrain to Disturbance of the Fuzzy+PID

In the time of 0.2s, the disturbance was added to the spindle. the PID part plays the major role in this period. So in Figure 6, we can find that the restrain capacity of these two strategies nearly to be same.

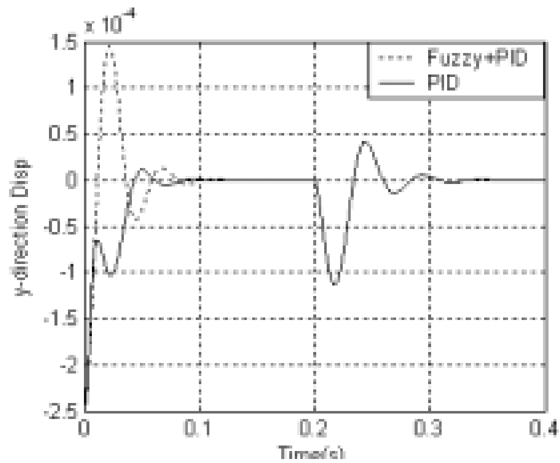
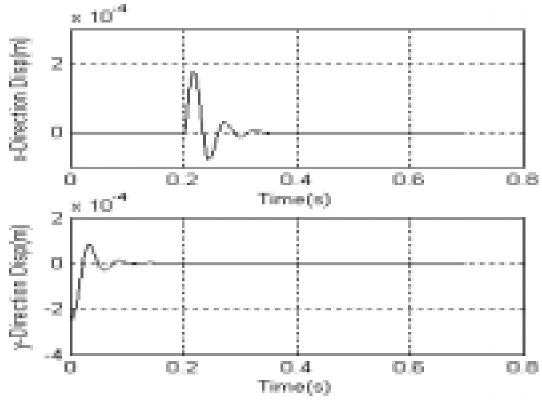


Fig6 Disturbance simulation of Fuzzy+PID

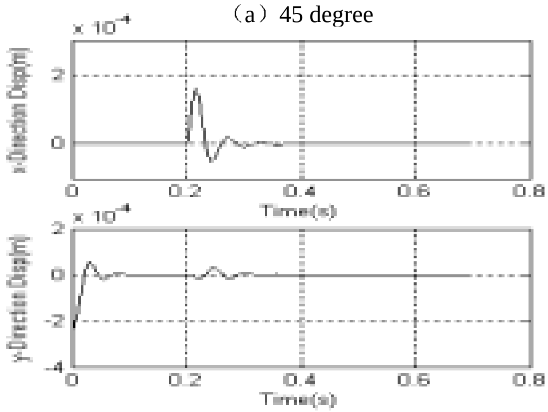
THE INFLUENCE OF MAGNETIC POLES POSITION TO THE SYSTEM PERFORMANCE

When the magnetic poles were placed in different position (the angle with horizon are 45 degree and 90 degree), we analyzed the suspension and the restrain to disturbance performance. A external force was added to the spindle in horizon direction to simulate the grind force. In Figure 7 (a) and (b) shows the simulation results of 45 degree and 90 degree.

From Figure 7 we can deduce that there is no displacement in the y-direction when placed in 90 degree and x-direction and y-direction are all disturbed in 45 degree. So it was consistent with the actual situation in 90 degree. And from another hand, the restrain capacity to disturbance was weaker than 45 degree.



(a) 45 degree



(b) 90 degree

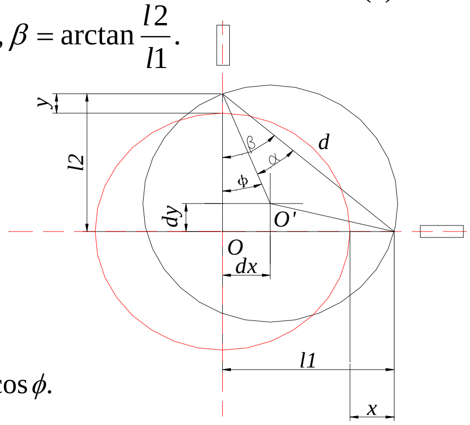
Fig7 Disturbance simulation in different degrees

THE INFLUENCE OF DECOUPLING METHOD TO THE SYSTEM PERFORMANCE

In Figure 8 it was shown that the output value of the sensor is not consistent of the center of axis. This was caused by the displacement coupling.

$$\cos\alpha = \frac{d}{2r}, \alpha = \arccos\frac{d}{2r}. \qquad (7)$$

$$\tan\beta = \frac{l2}{l1}, \beta = \arctan\frac{l2}{l1}.$$



$$\phi = \beta - \alpha$$

$$dx = r\sin\phi,$$

$$dy = l2 - r\cos\phi.$$

Fig8 Displacement decoupling algorithm

The Suspension Simulation

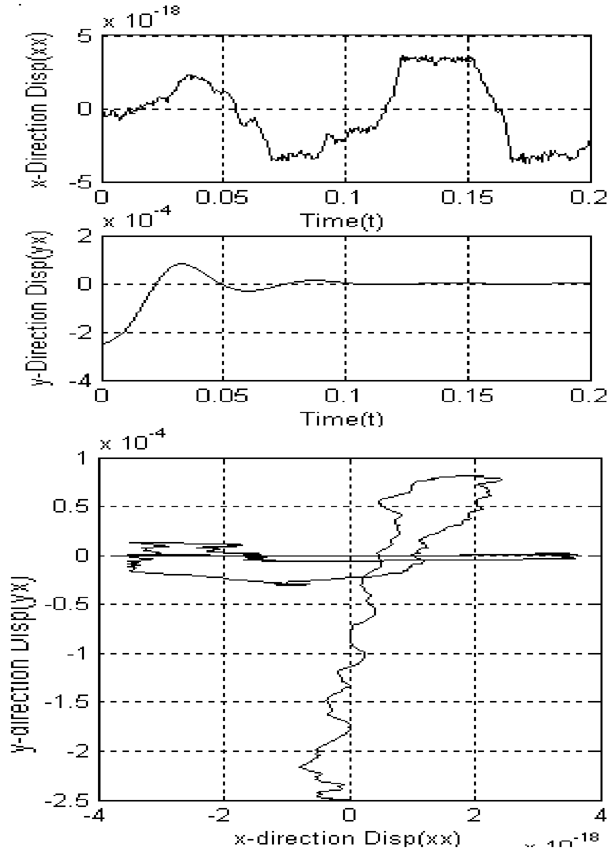


Fig9 Suspension simulation after decoupling in 90°

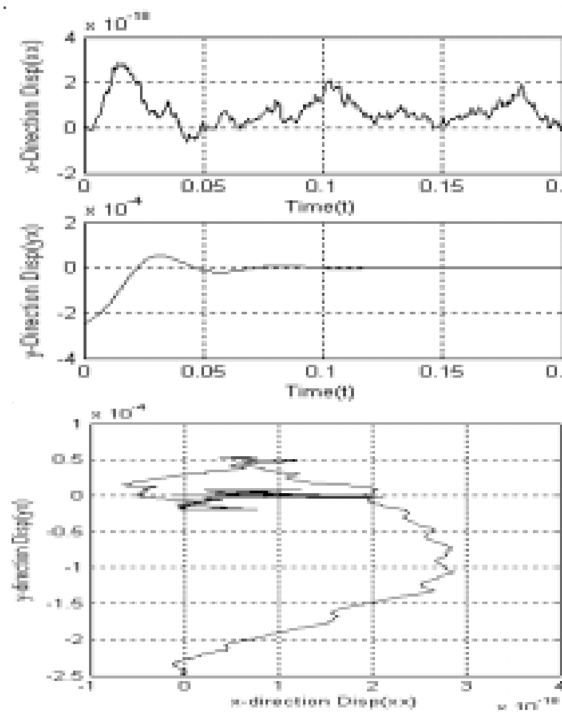


Fig10 Suspension simulation after decoupling in 45°

Under static circumstance, the suspension curve of the spindle in different magnetic pole placement was simulated. From Figure 9 and Figure 10, the farther distance of the sensor and the spindle, the more serious of the influence to the performance. The radian which the probe of sensor covered becoming larger with the increasing of the distance.

The Disturbance Simulation

In Figure 11 and Figure 12, it shows that

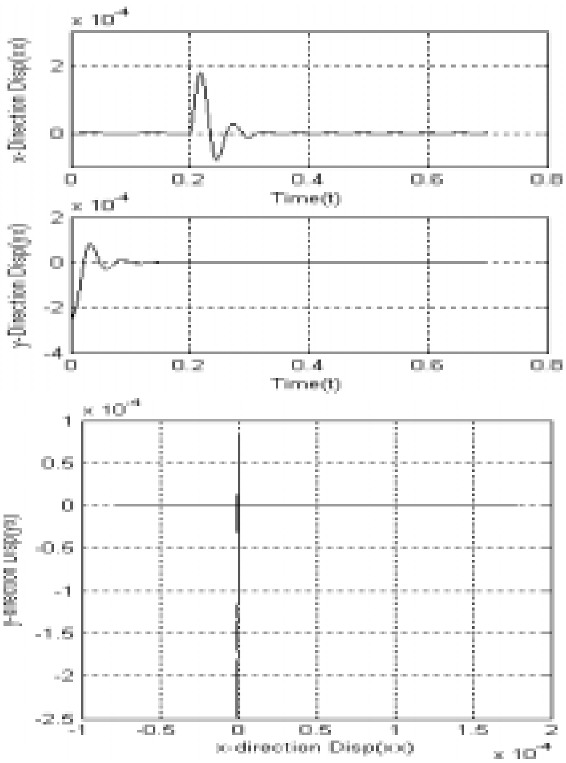


Fig11 Disturbance simulation after decoupling in 90°

decoupling algorithm play less role in the restrain to the disturbance. The main reason is that the radian which

the probe of sensor covered was so small that can be looked upon as plane when the diameter of sensor and the rotor is 5 mm and 78mm

CONCLUSIONS

The simulation result based on the non-linear mathematic model shows that the dynamic performance will be improved by adjusting the parameters of PID. The suspension performance is

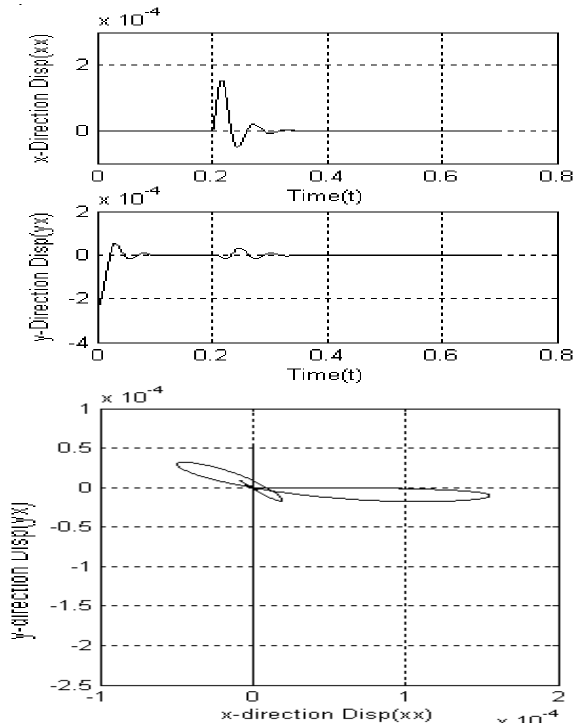


Fig12 Disturbance simulation after decoupling in 45°

improved when the Fuzzy+PID strategy was adopted.And analyzed the influence of the magnetic pole position and decoupling algorithm to the system performance.

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