A MULTI-OBJECTIVE CONTROL ALGORITHM: APPLICATION TO AN AMB MACHINE

Vincent Tamisier
ESTACA, Control & Systems Laboratory, 34 rue Victor Hugo, 92300 Levallois Perret, France
vtamisier@estaca.fr

ABSTRACT

We present here a multi-objective control algorithm based on the use of Youla parameterization and an independent Lyapunov function for each objective. We choose to consider two constraints: one with the H_2 norm and one with the H_∞ norm. The general optimisation problem is presented with the LMI expression of the constraints, followed by the presentation of the AMB machine model.

The model we use has been developed as follows: we consider that the rigid and flexible behaviour of the system are independent. Thus, we develop a nodal state space model of the flexible shaft, reduced into a modal model. The rigid equations of the system lead to a second state space model. The two parts are used conjointly in order to build a complete model of the AMB suspended system.

We introduce the matrix manipulations that lead to the final LMIs and the general algorithm as detailed in [6] and [7]. The controller we obtain is analysed and compared to the initial one.

INTRODUCTION

Modern robust control techniques often demand compromises among different or even conflicting objectives. Most of the time, we force the controller to satisfy simultaneously different performance and robustness objectives which are imposed on different channels of the closed-loop plant.

Some discussion about multi-objective control were first introduced by Boyd & Barrat ([1]), Dorato ([2]), and Khargonekar & Rotea ([3]). In particular, the mixed H2/H∞ problem has received many attentions.

Some convex optimisation formulations have been derived but such methods are generally conservative: they use a single common Lyapunov function for each synthesis objective and a change of variable which simultaneously affects this Lyapunov function and the controller ([4]), or they use infinite dimensional optimisation ([5]). More recently, Youla parameterisation has proved that it could be useful to reduce this conservatism ([6], [7]).

The subject of our study is to present a multi-objective control algorithm based on the Youla parameterisation, using an independent Lyapunov function for each objective (see [6]). We use a change of variables on these functions but not on the controller, and an observer-based structure which allows to reduce the degree of the controller. The solution is obtained using LMI optimisation that is now a computationally tractable framework. It consists in minimising a linear combination of the different objectives ([7]).

This multi-objective control method is then applied to an Active Magnetic Bearing (AMB) rotating machine (air compressor). The model of the machine is first presented, along with the method developed to build it, based on the joint consideration of the rigid and flexible part of the rotor. The Youla parameterisation then gives specific properties to the system. The observer-based structure allows to reduce the degree of the controller. The optimisation of the Youla parameter Q is at the end expressed as a LMI problem.

Contrary to usual approaches, the proposed method allows to choose different Lyapunov functions for each objective without loosing convexity. This is a crucial point to reduce the conservatism. Indeed each objective can be considered independently.

NOTATIONS AND DEFINITIONS

The plants considered here are finite dimensional discrete LTI systems. The closed-loop system is represented by the diagram of figure 1.

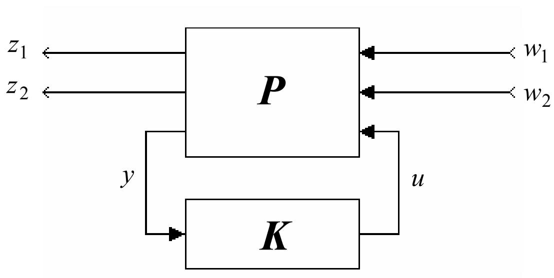

FIGURE 1: Closed-loop representation

u denote the control, y the measured output, w_i the external inputs, z_i the controlled outputs. The transfer functions T_i from w_i to z_i are used to specify two different robustness or performance objectives.

We choose for the state-space representations of the plant P and controller K the following form detailed in equation (1) and (2). We assume $D_{yu} = 0$.

$$P = \left(\begin{array}{c|ccc} A & B_1 & B_2 & B_u \\ \hline C_1 & D_{11} & D_{12} & D_{1u} \\ C_2 & D_{21} & D_{22} & D_{2u} \\ C_y & D_{y1} & D_{y2} & D_{yu} \end{array} \right) \tag{1}$$

$$K = \left(\begin{array}{c|c} A_K & B_K \\ \hline C_K & D_K \end{array} \right) \tag{2}$$

Interconnection of two plants will be noted by the Redheffer star product. In particular, the closed loop system of figure 1 is noted $P * K$. The specifications and objectives under consideration in this paper are H_∞ and H_2 norm constraints, used to express frequency domain specifications. They are considered below with an LMI formulation and can be used in the proposed multi-objective control approach.

H_∞ performance is useful to enforce robustness and to express frequency domain specifications such as bandwidth, low-frequency gain, … (see [8]). The H_∞ norm of the transfer function $T_1(z)$ from w_1 to z_1, is defined by:

$$\| T_1 \|_\infty = \sup_{\substack{w_1[k] \in L_2 \\ w_1[k] \neq 0}} \left(\frac{\| z_1[k] \|_2}{\| w_1[k] \|_2} \right) \tag{3}$$

Lemma 1 [3]: the H_∞ norm of $T_1(z)$ is lower than γ_1 if and only if there exists a real matrix $X_1 = X_1{}^T > 0$ such that:

$$\begin{pmatrix} -X_1 & X_1 A & X_1 B & 0 \\ A^T X_1 & -X_1 & 0 & C_1{}^T \\ B_1{}^T X_1 & 0 & -\gamma_1 I & D_{11}{}^T \\ 0 & C_1 & D_{11} & -\gamma_1 I \end{pmatrix} < 0 \tag{4}$$

H_2 performance is useful for handling stochastic aspects such as measurement noise or random disturbance. The H_2 norm of the transfer function $T_2(z)$ from w_2 to z_2, is defined by:

$$\| T_2 \|_2 = \sqrt{\int_0^1 trace\left(T_2(e^{2\pi j\nu})^* T_2(e^{2\pi j\nu}) \right) d\nu} \tag{5}$$

Lemma 2 [9]: the H_2 norm of $T_2(z)$ is lower than γ_2 if and only if there exist real matrices $X_2 = X_2{}^T > 0$ and $Y = Y^T > 0$ such that following conditions (6), (7) and (8) hold:

$$\begin{pmatrix} -X_2 & X_2 A & 0 \\ A^T X_2 & -X_2 & C_2{}^T \\ 0 & C_2 & -I \end{pmatrix} < 0 \tag{6}$$

$$\begin{pmatrix} -X_2 & X_2 B_2 & 0 \\ B_2{}^T X_2 & -Y & D_{22}{}^T \\ 0 & D_{22} & -I \end{pmatrix} < 0 \tag{7}$$

$$trace(Y) < \gamma_2{}^2 \tag{8}$$

STATE SPACE MODEL OF AMB MACHINE

Complex rotors are usually modeled by means of Finite Element Method (FEM) to cope with the non-elementary shape of the shaft and of the rotating appendices connected to it. To account for the complex geometry, the discretization at the base of the FEM model is usually characterized by a high number of nodes and, correspondingly, of degrees of freedom. When AMB are concerned, a model is necessary to design the control law to stabilize the complete system represented by the shaft and the AMBs.

The first step consists in building a model for the flexible rotor that represent the state of each node. Its size N obviously depends on the number of nodes considered. Let n be the number of nodes, and p the number of degrees of freedom per node. The typical values to define the complete geometry of the rotor are $n = 50$ nodes, and $p = 4$ degrees of freedom per node. The following equation (9) is the mechanical equation of the flexible system.

$$M\ddot{X} + (D + \Omega G)\dot{X} + KX = BF \tag{9}$$

M, D, G, K are respectively the mass, damping, gyroscopic and stiffness matrix. X is the vector containing the np degrees of freedom. F represents the electromagnetic forces, and B the nodes where those forces are applied. Ω is the rotational speed. We also define the output equation (10), where C corresponds to the nodes we chose to observe.

$$Y = CX \tag{10}$$

This corresponds to an order $N = 2np$ of the resulting state-space model, for which the state vector δ is defined as follows (11).

$$\delta = \begin{bmatrix} \dot{X} \\ X \end{bmatrix} \tag{11}$$

The nodal state-space model is defined by the following set of equations (12) and (13).

$$\dot{\delta} = \begin{pmatrix} -M^{-1}(D+\Omega G) & -M^{-1}K \\ I & 0 \end{pmatrix} \delta + \begin{pmatrix} B \\ 0 \end{pmatrix} F \tag{12}$$

$$Y = \begin{pmatrix} C & 0 \end{pmatrix} \delta \tag{13}$$

The main drawback of this model is its order (usually $N = 400$), that makes it uneasy or even heavy to use for calculation. Moreover, such an accuracy concerning all the nodes is not necessary.

Therefore a modal reduction of the system with the modal state vectors given by the FEM software is used. The so-called modes are the square roots of the eigenvalues of the $M^{-1}K$ matrix. Let Φ be the matrix composed of the eigenvectors associated to the m flexible modes we want to observe. A new state vector μ of size m as described in equation (14) is used.

$$X = \Phi\mu \tag{14}$$

The equations (9) and (10) then become (15) and (16) :

$$M\Phi\ddot{\mu} + (D+\Omega G)\Phi\dot{\mu} + K\Phi\mu = BF \tag{15}$$

$$Y = C\Phi\mu \tag{16}$$

The multiplication of the equation (15) by Φ^T on the left gives equation (17).

$$\Phi^T M\Phi\ddot{\mu} + \Phi^T (D+\Omega G)\Phi\dot{\mu} + \Phi^T K\Phi\mu = \Phi^T BF \tag{17}$$

A matrix of eigenvectors Φ is chosen as to obtain $\Phi^T M\Phi = I$. Let χ defined by equation (18) be the state vector of the modal state-space model represented by equations (19) and (20).

$$\chi = \begin{pmatrix} \dot{\mu} \\ \mu \end{pmatrix} \tag{18}$$

$$\dot{\chi} = \begin{pmatrix} -\Phi^T (D+\Omega G)\Phi & -\Phi^T K\Phi \\ I & 0 \end{pmatrix} \chi + \begin{pmatrix} \Phi^T B \\ 0 \end{pmatrix} F \tag{19}$$

$$Y = \begin{pmatrix} C\Phi & 0 \end{pmatrix} \chi \tag{20}$$

The order of this modal state-space model is $2m$. A flexible modal model for the rotor is thus obtained, of a reasonable order. The rigid part of the system composed of the rotor and the AMBs has to be added in order to complete the model. The description of the rigid behaviour of the system depends on its geometry, the positions of the actuators and the positions of the sensors [10].

Any movement of a rigid rotor inside its AMBs can be represented as a combination of a translation movement and a tilting movement. Consider the system composed of a bar, two AMBs and the corresponding sensors.

The bearings generate the forces F_1 and F_2. The displacements of the rotor on the detectors are called x_1 and x_2, while x is the displacement of the center of gravity G. α is the angle of rotation of the rotor during a tilting movement around G. L_{b1}, L_{b2}, L_{d1}, L_{d2} are respectively the distances from G to the first and second bearings, and to the first and second detectors. Let J be the axial moment of inertia of the rotor, and M its mass.

After writing down the mechanical equations, we obtain for the rigid behaviour the following state space model:

$$\frac{d}{dt}\begin{pmatrix} x \\ \dot{x} \\ \alpha \\ \dot{\alpha} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ \dot{x} \\ \alpha \\ \dot{\alpha} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ M^{-1} & M^{-1} \\ 0 & 0 \\ L_{b1}J^{-1} & L_{b2}J^{-1} \end{pmatrix} \begin{pmatrix} F_1 \\ F_2 \end{pmatrix} \tag{21}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & L_{d1} & 0 \\ 1 & 0 & L_{d2} & 0 \end{pmatrix} \begin{pmatrix} x \\ \dot{x} \\ \alpha \\ \dot{\alpha} \end{pmatrix} \tag{22}$$

The final model for the rotor equipped with its magnetic bearings is the combination of two terms : the rigid part of the magnetically suspended body, and the expression of the flexible behaviour of the rotor.

Some other terms of the closed-loop are also taken into account to increase the accuracy of the model. A model for the amplifiers, anti-aliasing filters, detectors, smoothing filters, and of course the numerical controller are then included too. So, this augmented model represents as accurately as possible the behaviour of the system.

In order to validate the modelling method we have built, we apply it to an air turbine machine. The fifty nodes that compose the FE model are specified. The structure obtained for the complete system is particularly adapted to the multi-objective control method we developed.

MULTI-OBJECTIVE CONTROL

The matrix inequalities that have been presented are not linear in decision variables if the closed loop system is considered. In order to perform a synthesis via convex optimisation, it is necessary to transform the initial problem. A combination of the different tools presented in this section leads to the synthesis algorithm given below. The 3 tools are the following:

- The Youla parameterization gives specific properties to the system,
- The observer-based structure allows to reduce the degree of the controller,
- The optimisation of the Youla parameter is expressed as an LMI problem.

Contrary to usual approaches, the proposed method allows to choose different Lyapunov functions for each objective without loosing convexity; this is a crucial point to reduce the conservatism. Indeed each objective can be considered independently.

The set of all stabilizing controllers for P can be parameterized [11] as $K = J * Q$ (see figure 2) where the Youla parameter Q is any stable system. More precisely, if we consider a plant model Sys1. Suppose there exists matrices K_c and K_f of appropriate dimensions such that $A - BK_c$ and $A - K_f C$ are stable. Then the set of all stabilising controllers for Sys1 can be parameterized as $K = J * Q$, where $Q \in RH_\infty$ and ([11]):

$$J = \begin{pmatrix} A - B_u K_c - K_f C_y & K_f & B_u \\ -K_c & 0 & I \\ -C_y & I & 0 \end{pmatrix} \qquad (23)$$

Any stabilising controller can be expressed as the interconnection of an observer-based structure and a Youla parameter Q. Suppose an initial controller has been designed using a classical mono objective method. Such a controller can then be expressed as $K = J * Q$ with J of minimum degree (i.e. the degree of P).

Finally the Youla parameterization gives a specific form to each channel state space representation of G as follows in equation (23).

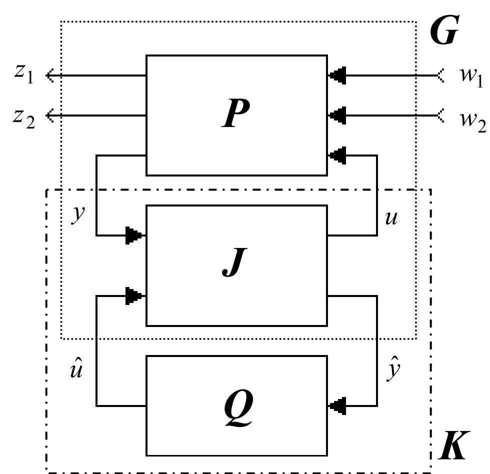

FIGURE 2: Closed loop structure including Youla parameterization

$$G = \left(\begin{array}{cc|cc|c} A_1 & A_3 & B_{11} & B_{21} & \hat{B}_u \\ 0 & A_2 & B_{12} & B_{22} & 0 \\ \hline C_{11} & C_{12} & D_{11} & D_{12} & D_{1u} \\ C_{21} & C_{22} & D_{21} & D_{22} & D_{2u} \\ \hline 0 & \hat{C}_y & D_{y1} & D_{y2} & 0 \end{array}\right) \qquad (24)$$

The multi-objective control problem is now to find an output feedback Q for the system G, such that the objectives listed before are satisfied. Consider the i^{th} objective (H_2 or H_∞), from w_i to z_i and the Lyapunov function X partitioned into:

$$X_i = \begin{pmatrix} W_i & Z_i \\ Z_i^T & Y_i \end{pmatrix} \qquad (25)$$

According to $A = \begin{pmatrix} A_1 & A_2 \\ 0 & A_3 \end{pmatrix}$

Matrix inequalities (4) and (6) to (8) applied to the closed loop system $G * Q$ are non linear on the decision variables Q and X_i. They are therefore going to be transformed by a change of variables and congruence transformations.

$$\left(\begin{array}{cc|cc|c|c} -R_1 & 0 & A_1R_1 & A_1S_1 - S_1A_2 + A_3 + \hat{B}_u Q \hat{C}_y & B_{11} + \hat{B}_u Q D_{y1} - S_1 B_{12} & 0 \\ 0 & -T_1 & 0 & T_1A_2 & T_1B_{12} & 0 \\ \hline * & * & -R_1 & 0 & 0 & R_1^T C_{11}^T \\ * & * & 0 & -T_1 & 0 & C_{12}^T + C_y^T Q^T D_{1u}^T + S_1^T C_{11}^T \\ \hline * & * & * & * & -\gamma_1 I & D_{11}^T + D_{y1}^T Q^T D_{1u}^T \\ \hline * & * & * & * & * & -\gamma_1 I \end{array}\right) < 0 \quad (29)$$

$$\left(\begin{array}{cc|cc|c} -R_2 & 0 & A_1R_2 & A_1S_2 - S_2A_2 + A_3 + \hat{B}_u Q \hat{C}_y & 0 \\ 0 & -T_2 & 0 & T_2A_2 & 0 \\ \hline * & * & -R_2 & 0 & R_2^T C_{21}^T \\ * & * & 0 & -T_2 & C_{22}^T + \hat{C}_y^T Q^T D_{2u}^T + S_2^T C_{21}^T \\ \hline * & * & * & * & -I \end{array}\right) < 0 \quad (30)$$

$$\left(\begin{array}{cc|c|c} -R_2 & 0 & B_{21} + \hat{B}_u Q D_{y2} - S_2 B_{22} & 0 \\ 0 & -T_2 & T_2 B_{22} & 0 \\ \hline * & * & -Y & D_{22}^T + D_{y2}^T Q^T D_{2u}^T \\ \hline * & * & 0 & -I \end{array}\right) < 0 \quad (31)$$

$$trace(Y) < \gamma_2 \quad (32)$$

Consider the following bijective change of variable:

$$\begin{pmatrix} W_i & Z_i \\ Z_i^T & Y_i \end{pmatrix} \rightarrow \begin{pmatrix} R_i & S_i \\ S_i^T & T_i \end{pmatrix} \quad (26)$$

$$\begin{pmatrix} R_i & S_i \\ S_i^T & T_i \end{pmatrix} = \begin{pmatrix} W_i^{-1} & -W_i^{-1} Z_i \\ -Z_i^T W_i^{-1} & Y_i - Z_i^T W_i^{-1} Z_i \end{pmatrix} \quad (27)$$

We define:

$$M_i = \begin{pmatrix} R_i & 0 \\ S_i^T & I \end{pmatrix} \quad (28)$$

The congruence transformations of the matrix inequalities lead to LMI formulation (see [6]). The H_∞ performance can be expressed as follows, after pre and post multiply equation (4) by $diag(M_1\ M_1\ I\ I)$, (6) by $diag(M_2\ M_2\ I)$ and (7) by $diag(M_2\ I\ I)$, we obtain the LMIs (29) to (32).

So we have shown that the multi-objective problem depends linearly on the variables R_1, S_1, T_1, R_2, S_2, T_2, Y, γ_1, γ_2^2 and Q.

ALGORITHM AND APPLICATION

The previous results lead to a multi-objective control design algorithm based on the Youla parameterization:

- *Initial synthesis*: design of an initial controller using conventional techniques
- *Observer-based structure parameterization*: we obtain the Youla parameterization with J described by (23).
- *Convex optimisation*: choose the order of the Youla parameter Q and solve the multi-objective control problem using the LMIs (29) to (31)
- *Controller reconstruction*: the final controller is obtained by interconnecting J and Q: $K = J * Q$

We use a conventional controller (PID design) as initial controller. Once we have chosen the optimisation weithings, we obtain the following controller $K = J * Q$:

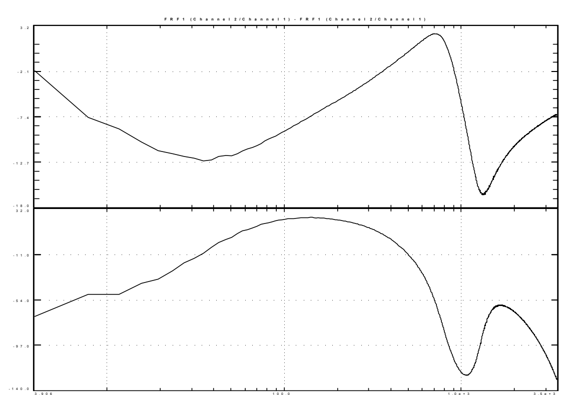

FIGURE 3: Controller for axis 13

We obtain the following open-loop for the first axis on the turbine.

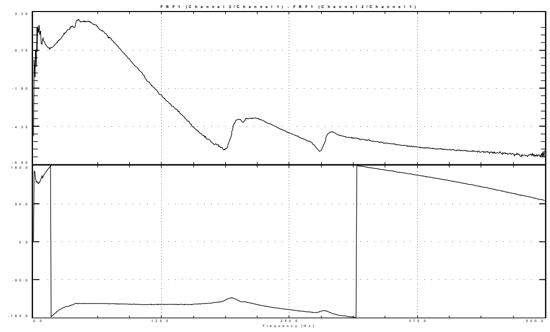

FIGURE 4: Bode diagram of the open loop for axis 1

The controller obtained have been tested with uncertainties added on the rigid and on the flexible modes. The criterion on the H_2 norm of the closed loop transfer function helps reducing the effect of the unbalance, but the time simulations and experiments are better with a specific unbalance control algorithm, like AVR (see [12]).

In this particular case, the performance comparison of the controller obtained and the PID controller designed by a specialist is not amazing, but what is interesting in the approach proposed is the quality and reliability reached with the multi-objective algorithm. If the constraints change, for example because of the gyroscopic effects, we only have to go through the two last steps of the algorithm.

CONCLUSION

The objective of this study was to apply a multi-objective control algorithm to an AMB machine. We used the Youla parameterization and LMI formulation for the objectives. We first build a model of a 2-AMBs machine by considering separately the rigid and flexible behaviour of the machine. This model is reduced as much as possible using a modal description of the bending part.

We then present the general multi-objective optimisation algorithm. Each step is based on matrix manipulations and convex optimization. Besides matrix manipulation are numerically well conditioned and convex optimization can be solved in polynomial time. So it leads to computationally tractable problems.

This method offers advantages over existing methods. It allows to reduce the conservatism by using a particular Lyapunov function for each objective. It is not necessary to inverse the changes of variables. Note however that increasing the number of decision variables can turn to numerical problems when the plant order or the number of objectives is large.

The optimization method have been applied to the model with specific constraints and weightings, and gave a controller validated on the machine. The results are not only satisfactory, but they also open up to various fields. Our objective is now to optimise the order of the Youla parameter Q and apply gain scheduling techniques in order to adapt Q to the change of rotational speed.

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