A LQG CONTROLLER DESIGN FOR AN AMB-FLEXIBLE ROTOR SYSTEM

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ABSTRACT

Because active magnetic bearing (AMB) has many particular virtues, it might be the best way to support the motor rotor and gas turbine rotor in the 10MW high temperature gas-cooled test module reactor (HTR-10). The two rotors are designed to pass through the first two bending critical speed (BCS) to reach the working point. In this paper, firstly, aiming at the small experiment setup, more accurate mathematic model of AMB flexible rotor system than the rigid one heretofore is formulated with subsection discrete method (SDM). Then some valuable results, such as, eigen-frequencies changing with supporting stiffness, eigen-shapes and gyroscopic effect, etc., are drawn in figures. Finally, a controller with linear quadratic Gaussian (LQG) control synthesis adding some phase compensators is designed after some simulations. In actual experiments, the rotor could not only run up to 340 Hz, above the first BCS, but also stay at that speed stably and safely for a long time.

INTRODUCTION

Active magnetic bearing (AMB) is a kind of novel type, high performance bearing, suspending the rotor with electro-magnetic force made by the currents in the bearing coils. AMB system has a lot of virtues, for example, no contact, no lubricant and active damping, the rotor could rotate in higher speed and pass through several critical speeds [1,2]. Because of the above special advantages of AMB system, it found a practicable way that the two rotors, motor rotor and gas turbine rotor in the 10MW high temperature gas-cooled test module reactor (HTR-10), could be supported by AMB without lubrication and contact to meet some special desires. And in order to fulfill the reactor output power, they must pass through the first two bending critical speeds (BCS) to reach the operating speed. So, as known, this AMB-flexible rotor system model in the form of state space would be more complex than one of the rigid system.

To verify the whole AMB system practicability and gain some experience in passing through BCS, a small experiment setup similar to the HTR-10 motor rotor, is built up. As follows, in TABLE 1, some important coefficients are given.

TABLE 1: Coefficients of the small setup

Rotor Mass	6.128kg
Rotor Length	613mm
Radial Moment of Inertia	0.148kg m^2
Polar Moment of Inertia	0.00379kg m^2
Air Gap	0.4mm
Coils	300n
Pole Area	320mm^2
Inductance	45.2mH

The small experiment system is controlled by a digital controller on the real-time linux system, and the sampling time is 100 μs , as illustrated in FIGURE 1.

FIGURE 1: Whole setup structure

In this paper, it has 5 sections. In the first section, a method named subsection discrete method (SDM) is introduced to formulate the mathematic model of the

AMB-flexible rotor system; in the second section, some modal analysis results, for example, eigen-frequencies changing with supporting stiffness, gyroscopic effect and eigen-shapes, are drawn, illustrated in pictures; in the third section, some controllers designed with LQG method based on the above mathematic model are introduced. In the same time, some simulation analysis is carried out; in the fourth section, some actual experiments are carried out, such as static suspending, speeding up to pass through the first BCS, etc., and a more practicable controller with phase compensators is formulated; in the last section, based on the above work, some valuable conclusions are drawn, which would be as a guidance for the further study work.

SUBSECTION DISCRETE METHOD

In the AMB-flexible rotor system, the rotor would pass through BCS, and the resonance effect must be carefully taken into account. So the system state quantities and the order of the system model are more than the rigid one. For a simple example, the modal information whose eigen-frequency is small than the operating speed, has to be contained in the system model. It has many software to formulate system model, such as ANASYS, MARC, MARDIN. Here it would present a method named SDM to formulate more accurate mathematic model of the AMB-flexible rotor system [3].

Firstly, it splits the rotor into many segments and treats the rotor with some disks. These disks, with equivalent mass and without stiffness, are connected with girder of equivalent stiffness and without mass. According to some relevant physics theorems, some equations are formulated, as shown in FIGURE 2.

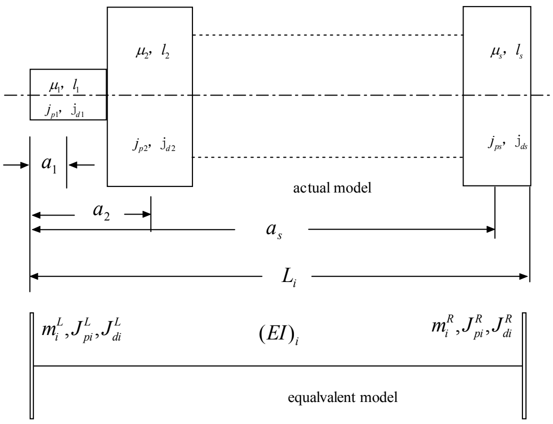

FIGURE 2: Principle of SDM

Let,

$$m_i^R=\sum_{k=1}^{s}\frac{(\mu l a)_k}{L_i}\,,\qquad m_i^L=\sum_{k=1}^{s}\frac{[\mu l(L_i-a)]_k}{L_i}=\sum_{k=1}^{s}(\mu l)_k-m_i^R\,,$$

$$J_{pi}^R=\sum_{k=1}^{s}\frac{a_k^2}{a_k^2+(L_i-a_k)^2}j_{pk}l_k\,,\qquad J_{pi}^L=\sum_{k=1}^{s}\frac{(L_i-a_k)^2}{a_k^2+(L_i-a_k)^2}j_{pk}l_k$$

$$J_{di}^R=\sum_{k=1}^{s}\frac{a_k^2}{a_k^2+(L_i-a_k)^2}\left[j_d l+\frac{1}{12}\mu l^3-\mu l a(L_i-a)\right]_k$$

$$J_{di}^L=\sum_{k=1}^{s}\frac{(L_i-a_k)^2}{a_k^2+(L_i-a_k)^2}\left[j_d l+\frac{1}{12}\mu l^3-\mu l a(L_i-a)\right]_k \qquad (1)$$

Where, μ_k , j_{pk} and j_{dk} are separately mass density, polar moment inertial and radial moment inertial of unit length, l_k is the length of every cylinder and a_k is the length from the center of the cylinder to the left side of this segment, k=1,2,…,s. More detail representation could be found in [4];

After some arithmetic deduction, a high order mathematic model could be expressed as follows

$$[M]\{\ddot{Y}\}+\Omega[G]\{\dot{Y}\}+[K]\{Y\}=\{P\},\qquad y=CY \qquad (2)$$

Where, M is mass matrix, K stiff matrix, G gyroscopic effect matrix, P input force matrix, Ω rotational speed, C output matrix, Y system state quantities and y sensors output signal.

Compared with different ways of how to split the flexible rotor and reduce the order of the model with modal reduction method, a more accurate and comfortable model is got. More detail above procedure could be referred to [5].

MODAL ANALYSIS

When a low order model is formulated with the above reduction model, many useful results can be concluded with modal analysis method. To be expressed more visually and clearly, they are all depicted in figures.

1. Eigen-frequencies vs. AMB Supporting Stiffness

As known, the rigid eigen-frequencies change very much with the AMB supporting stiffness, but the bending eigen-frequencies don't change much at some given range, depicted in FIGURE 3.

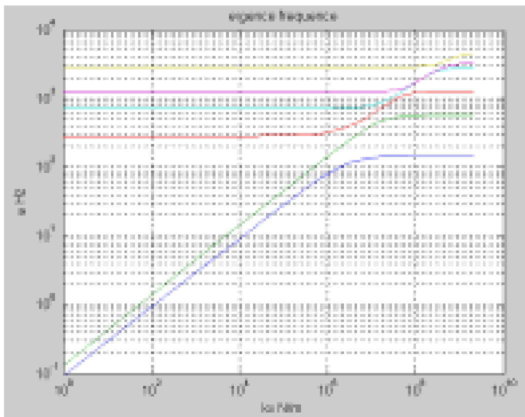

FIGURE 3: Eigen-freqs. vs. stiffness

2. Gyroscopic effect

When the rotor, with symmetric structure, is rotating, the same two values of every eigen-frequency at condition of the static suspension will be split into two different values. The one, which is forward whirl, is bigger than it, and the other, which is backward whirl, is less. In FIGURE 4, it depicts the above phenomena with the same supporting stiffness, 3×10^4 N/m, at the both side of AMBs.

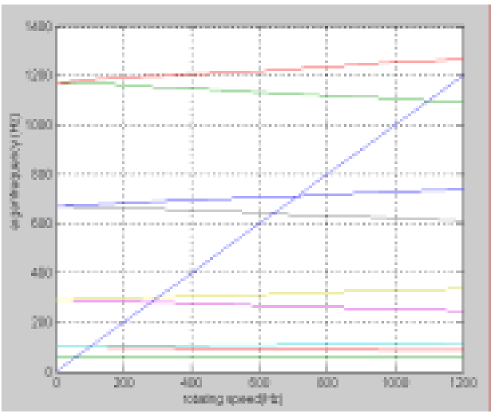

FIGURE 4: Gyroscopic effect

3. System Obserbvability and Controllability

When designed an AMB-flexible rotor system, system observability and controllability must be early taken care of. In a bad case, if AMBs or sensors are at the nodes of any eigen-mode, the vibration of the eigen-frequency can't be restrained easily especially when passing them. In FIGURE 5, it illustrates the first six eigen-shapes. From left to right, four vertical lines respectively represent sensor A, AMB A, AMB B and sensor B. And it also illustrates system observability and controllability qualitative [1].

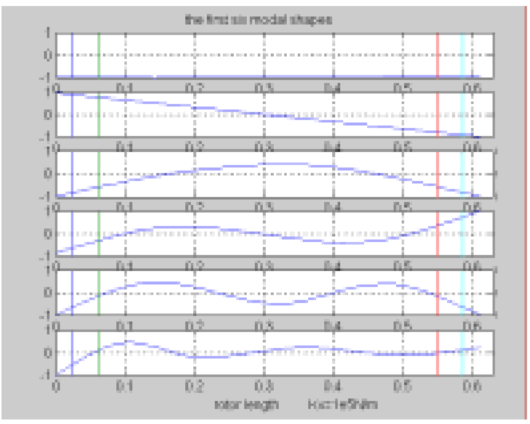

FIGURE 5: Eigen-shapes

From FIGURE 5, it draws a conclusion that the designed system has good observability and controllability for passing through the fist two BCSs.

CONTROLLER DESIGN

There are many methods to design controller, such as, Hiroyuki Fujiwara introduced simple mode control and phase lead in [6]; Hideo Shida explored a controller with LQR and PD in [7]. In this section, it mainly introduces a LQG method to design a controller decoupling x and y vertical planes. After a process of iteratively changing the weighting factors, a controller with input weighting matrix R and state weighting matrix Q is designed. Where, R=diag(1,1), Q=diag(159,159,159, 159,32,32,16,16).

When given 3×10^4N/m supported stiffness, the first four eigen-frequencies, two rigid eigen-frequencies and two smaller bending eigen-frequencies, are 21.3，32.3，303.5，and 706.4 Hz in both theory calculation and actual experiment measurement[8].

In some actual rotating experiments, it was found that at some special speeds, the rotor amplitude became too large. In order to restrain the amplitude, some phase compensators are added into the above designed LQG controller.

For example, to restrain the amplitude around 300Hz, a compensator, expressed in frequency domain form, e.g. for 300Hz, $tf_3 = \frac{tf_{31} + tf_{32}}{2}$, is added to the above controller. Where,

$$tf_{ij} = \frac{as^2 + bs + c}{s^2 + ds + e}, \text{ (i=3,7; j=1,2.)} \tag{3}$$

In TABLE 2, the phase compensators coefficients around 300 and 700 Hz are given.

TABLE 2: Coefficients of phase compensator tf_{ij}

	a	b	c(10^6)	d	e(10^6)
tf_{31}	0.9993	87.51	3.599	99.49	3.841
tf_{32}	0.9993	87.5	2.993	99.49	3.235
tf_{71}	0.9907	286.7	16.44	477.2	18.45
tf_{72}	0.9727	201.6	19.27	754.8	23.11

EXPERIMENT

To verify the above results, some actual experiments were carried out. Both the two controllers with and without phase compensators could make the flexible rotor suspend stably. When running up, the amplitude of the rotor, without any compensator, was increasing until hitting the sustainer bearing. When with the controller with phase compensators around 300 and 700, the amplitude was decreased obviously even at BCS. To our test setup, the flexible rotor supported by AMB could pass through the first BCS safely and smoothly. Even so, the test rig could rotate at the first BCS for a long time without any abnormal phenomenon, as depicted in FIGURE from 6 to 9.

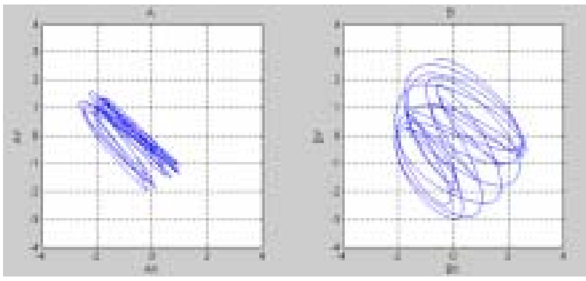

FIGURE 6: Axial center orbits at 310Hz

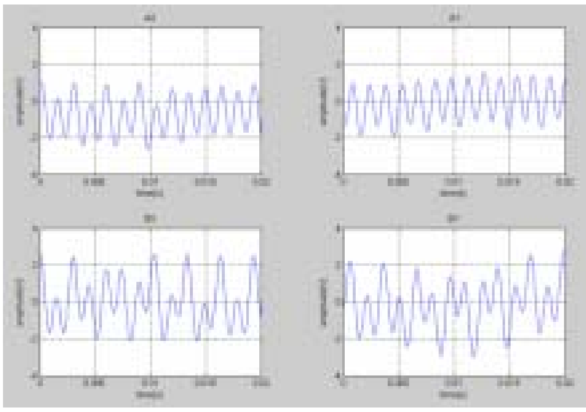

FIGURE 7: Four radial sensor signals at 310Hz

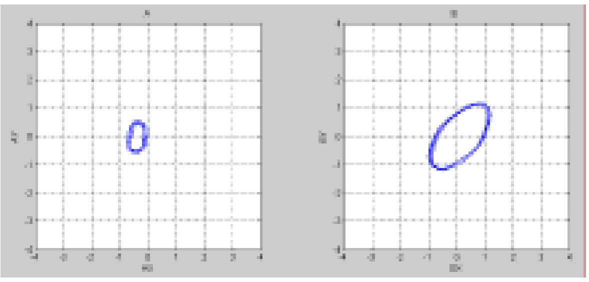

FIGURE 8: Axis center orbits at 340Hz

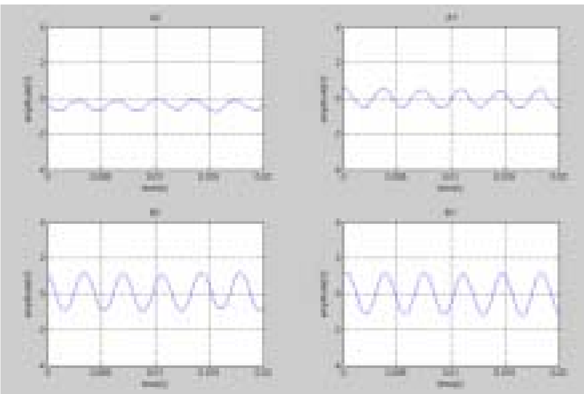

FIGURE 9: Four radial sensor signals at 340Hz

Note that in the above figures of the actual experiments, signal magnitude is Voltage unit in horizontal scale, and every voltage means 0.06mm. In FIGURE 8 and 9, the speed, 340Hz, is above the first BCS and as shown, the rotor vibration magnitude decreased more clearly than at the first BCS.

CONCLUSION

In this paper, it introduces an AMB-flexible rotor system how to pass through the first BCS with a LQG controller. After simulation and actual experiments, some conclusion could be drawn: SDM could be used to formulate the mathematic model of the AMB-flexible rotor system; for flexible rotor setup design, modal analysis is very important before setup manufactured and assembled; with this setup, a LQG controller could make the flexible rotor suspended stably; when adding some phase compensators to the above controller, the rotor could pass through the first BCS smoothly. Even when reaching BCS, the rotor could rotate stably with acceptable small amplitude. For further work, it verified that a LQG controller added with some phase compensators could make the AMB-flexible rotor system pass through the first BCS.

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