

# Modelling the Electromechanical Interactions in a Null-Flux EDS Maglev System

Jeroen de Boeij<sup>1,2</sup>, Maarten Steinbuch<sup>2</sup> and Hector Gutiérrez<sup>1</sup>

<sup>1</sup>Florida Institute of Technology

Department of Mechanical & Aerospace Engineering, Melbourne, FL 32901

<sup>2</sup>Eindhoven University of Technology

Department of Mechanical Engineering, Eindhoven, The Netherlands

hgutier@fit.edu

## ABSTRACT

The fundamental electromechanical interactions in a passive null-flux EDS maglev system are mediated by the voltages induced in the levitation coils by the sled magnets, and by the forces exerted on the sled as a result of the induced currents. This paper presents a reliable and compact method to calculate these interactions by using analytical expressions of the magnetic field of a permanent magnet. The proposed approach provides a highly efficient and numerically stable approach to the computation of the flux induced in the levitation coils as well as the induced voltages and currents and lift, drag and guidance forces acting on the sled's magnets. The analytical model is compared to a simplified algebraic model suitable for real-time control of EDS Maglev suspension dynamics. Both models are compared with measurements and show good predictive quality.

## 1 INTRODUCTION

Development of a robust, reliable EDS maglev suspension requires a thorough understanding of the interaction between the magnets in the levitated vehicle and the null-flux coils. This paper presents a model of the voltages and currents induced in the null-flux coils of an open-loop EDS system as the launch vehicle runs along the track. From these, the forces acting on the vehicle are predicted. Coupled electromagneto-mechanical systems such as an EDS magnetic suspension have been typically modelled by finite element methods, using numerical integration of Maxwell's equations to estimate the magnetic flux induced in the levita-

tion coils. Currently existing packages that can solve the coupled electromagnetic, circuit and motion equations demand a large computational effort and can be inaccurate due to limitations in mesh size and target speed. In this paper, a closed-form representation of the magnetic field is used to calculate the induced flux, voltages, currents and forces. This representation results in a computational method that is far simpler and more accurate than FEM methods, since no numerical solution of the field differential equations is required. Furthermore, a simplified algebraic model that is compact enough to be used in real-time control is presented and compared to the proposed analytical model. This simpler model results in algebraic expressions for the flux through a null-flux coil and its time-derivative that can be solved even faster than the proposed analytical model. Both models are tested by comparing simulated model predictions with dynamic measurements of current and 3-axial force in a one magnet, one coil system.

## 2 TECHNICAL BACKGROUND

Several models have been proposed to describe the interaction between the levitation coils and sled magnets in null-flux systems. He and Rote [3, 4, 7] developed models using dynamic circuit theory, where the track dynamics are represented by equivalent circuit equations in matrix form. Matrix parameters can include time and space dependencies, so these models can be highly accurate provided a robust method to estimate the time-varying parameters is available. The computing time required by this ap-

proach is, however, prohibitive for real-time applications considering the large number of parameters to be estimated. A simpler approach proposed by Davie, [5, 6] does not attempt to analyze the entire system but only the interaction of a limited number of coils with a limited number of sled magnets. The analysis produces frequency-domain expressions which can be used to calculate time-averaged characteristics of the EDS maglev dynamics, very useful for design purposes but not suitable for real-time control. A compact mathematical model developed by De Boeij and Gutiérrez [8] describes the 3-DOF sled dynamics of a null-flux EDS system by approximating the field of the sled magnets as time-invariant parameters to be estimated, and the field distribution as uniform over the levitation coils, hence estimating flux as a simple function of the geometry of the coils and the sled's position. The required parameters are estimated off-line from input-output data, and the model is simple enough to be useful for real-time control. This approach is nevertheless limited since it only calculates lift forces, and it can't model the dependency of induced voltage and current with the distance between the plane of the magnets and the plane of the coils. In this paper the model described in [8] is extended to calculate forces in all three directions and is compared to a proposed more accurate analytical closed-form model. Finally, both models are compared with measurements. Fast computation of the three force components acting on each one of the sled's magnets opens the path to real-time control of the multi-DOF suspension dynamics of an EDS Maglev system.

### 3 MODELLING THE MAGNETIC FIELD AND LINKING FLUX

In this section both the algebraic model and the analytical model are presented. The models describe the magnetic field of a sled magnet and the flux it produces in a null-flux levitation coil.

#### Analytical Model Based on Closed Form Expressions of the Magnetic Field

Analytical expressions exist that describe the magnetic field density caused by a permanent magnet in any point in space  $(x,y,z)$  as a function of the magnet geometry and material properties. In this case a rectangular magnet is considered. The magnetic field can be computed [2]

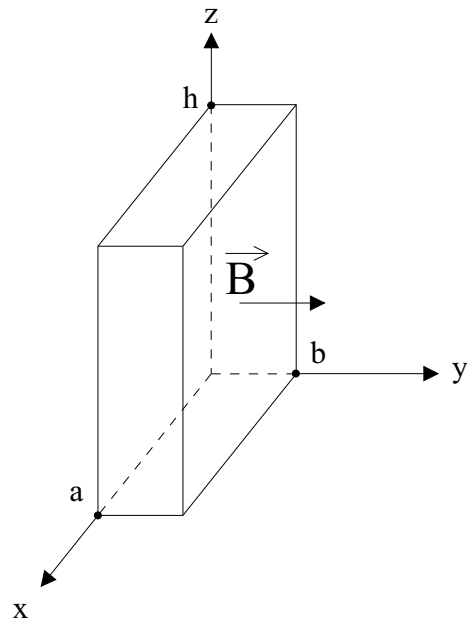


Figure 1: Coordinate system for magnetic field equations

using expressions 1, 2 and 3.

The coil orientation is defined as parallel to the x-z plane at a fixed distance  $y$  as shown in Figures 1 and 2. To calculate flux, the coil's surface is divided in a number of area elements. In each element the magnetic flux density perpendicular to the plane of the coil ( $B_y$ ) is calculated using equations 2 and 4. The flux linking each element is equal to the area of the element multiplied by the magnetic flux density  $B_y$ . In order to obtain accurate results a sufficient number of elements must be used. The number of elements is increased until very small changes result. The flux induced by other magnets not directly in front of the coil is considered negligible, as well as the flux induced by the nearby coils (i.e., the mutual inductance of two adjacent coils is near zero). The last two assumptions are supported both by existing literature and FEM simulations [1].

#### Algebraic Model of the Electromagnetic Interactions

The algebraic model is based on the following assumptions:

1. The  $y$ -component of the magnetic field of the permanent magnet is assumed uniform

$$H_x = \frac{-M}{4\pi} \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 (-1)^{i+j+k} \ln \left[ z_k + (x_i^2 + y_j^2 + z_k^2)^{1/2} \right] \quad (1)$$

$$H_y = \frac{-M}{4\pi} \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 (-1)^{i+j+k} \times \left\{ \tan^{-1} \left[ \frac{x_i y_j}{z_k (x_i^2 + y_j^2 + z_k^2)^{1/2}} \right] \tan^{-1} \left[ \frac{y_j z_k}{x_i (x_i^2 + y_j^2 + z_k^2)^{1/2}} \right] \right\} \quad (2)$$

$$H_z = \frac{-M}{4\pi} \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 (-1)^{i+j+k} \ln \left[ x_i + (x_i^2 + y_j^2 + z_k^2)^{1/2} \right] \quad (3)$$

$$\begin{aligned} x_1 = x & & y_1 = y & & z_1 = z \\ x_2 = x - a & & y_2 = y - b & & z_2 = z - h \end{aligned} \quad M = |M|$$

$$\vec{B} = \mu_0 \vec{H} \quad (4)$$

over its surface, with the magnetic field density ( $B_y$ ) a parameter to be estimated.

2. The flux through the levitation coil depends only on the x- and z-position of the magnet with respect to the coil. Angles are assumed to be small and of little influence.
3. The magnet stays always between the upper and lower edges of each null-flux coil.
4. The magnetic field components in the plane of the coil and the surface of the magnet are assumed to be functions of the x- and z-position of a sled magnet with respect to that coil ( $B_x$  and  $B_z$ , see Figure 1).

Using these assumptions, the flux linking the upper and lower part of the null-flux levitation coil can be defined as the overlap area of the coil and the magnet multiplied by the constant magnetic field density  $B_y$ . The net flux linking the coil is then calculated by subtracting the flux linking the lower part of the coil from the one linking the upper part of the coil.

If the position and velocity of one or multiple magnets are known in time, the total flux linking the null-flux coil and its time-derivative can be calculated using geometric relationships. The assumptions are necessary to keep the geometric relationships simple. The field components in the plane of the coil ( $B_x$  and  $B_z$ ) are fitted by polynomials of  $x$  and  $\delta$ . These functions are fitted in a least-square error sense from field values generated using the closed-form analytical expressions.

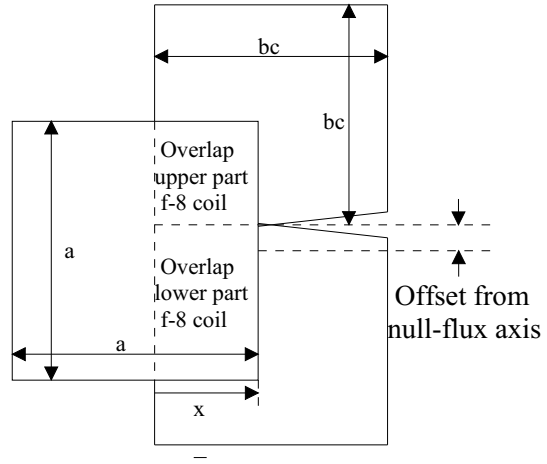


Figure 2: Layout of a magnet passing a null-flux coil

The induced voltage in a short circuited null-flux coil can be calculated from equation 5, where  $\Phi$  is the net flux linking the null-flux coil and  $N$  the number of turns.

$$V = -N \frac{d\Phi}{dt} \quad (5)$$

By solving the differential equation of the electrical circuit (6), the current in the coil is:

$$\frac{di}{dt} = \frac{1}{L} (V - Ri) \quad (6)$$

where  $L$  is the inductance of the null-flux coil and  $R$  its resistance.

The Lorentz forces acting on the coil are calculated using the corresponding the magnetic field equations. The general form of the Lorentz force acting on a infinitely small wire is:

$$\vec{dF} = \vec{idl} \times \vec{B} \quad (7)$$

Since the magnetic field component  $B_y$  in the algebraic model is approximated as constant over the coil's surface and the components  $B_x$  and  $B_z$  are polynomial functions of  $x$  and  $\delta$ , the forces in the *algebraic* model are computed using equations 8, 9 and 10, where  $x$  is the length of the horizontal wire at the center of the coil that overlaps with the magnet. The net drag force is zero when the magnet overlaps the coil in the horizontal direction, since the drag force acting on the right wire cancels the force acting on the left wire.

$$F_{lift} = 2NB_y ix \quad (8)$$

$$F_{guidance} = 2NB_x(x, \delta) i \delta \quad (9)$$

$$+ 2NB_z(x, \delta) ix \quad (10)$$

$$F_{drag} = 2B_y Ni \delta$$

Instead of these equations, the *analytical* closed-form model uses the general Lorentz force equation. Each coil loop is divided in differential segments of length  $dl$ , and the magnetic field density  $B$  is calculated at the center of each segment using equations 1, 2, 3 and 4. The force is then calculated using equation 7.

## 4 EXPERIMENTAL SETUP

The experimental setup consists of a rotating arm with a null-flux coil attached to one end. The arm is driven by a servo controlled DC motor at constant speed. Induced current (or voltage) is measured and sampled at 32 kHz, and the data is sent to a computer in ground through a wireless transmitter attached to the arm. A permanent magnet in a frame fixed in ground is mounted facing the null-flux coil. The magnet frame is mounted to a triaxial force load cell. A laser displacement sensor is used to measure the offset of the magnet with respect to the null-flux axis of the figure-8 coil, which is defined as the height at which no net voltage is induced in the figure-8 coil. A schematic of the setup is shown in Figure 3.

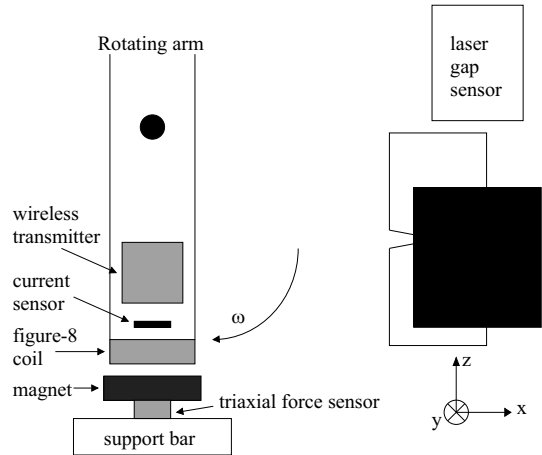


Figure 3: Experimental setup for voltage, current and force measurement

experiment	parameter	value
1	$\delta$	8.5 mm
	$g_y$	8.2 mm
	$v$	1.5 m/s
2	$\delta$	8.6 mm
	$g_y$	10.2 mm
	$v$	7.7 m/s
3	$\delta$	8.4 mm
	$g_y$	9.0 mm
	$v$	2.2 m/s

Table 1: Parameter values

## 5 EXPERIMENTAL RESULTS

In this section experimental results and model simulation results are presented. In each experiment the distance to the null-flux axis ( $\delta$ ), the speed of the rotating arm ( $v$ ) and the gap between the coil and the magnet ( $g_y$ ) are different. The parameters for each experiment are shown in Table 1. Results for experiment 1 are presented on Figures 4 to 7, experiment 2 on Figures 8 to 11 and experiment 3 on Figures 12 to 15.

## 6 DISCUSSION

The measured current is predicted accurately by both models in all three experiments. Predicted forces closely follow measured values, although structural vibrations that couple with force measurements (specially at high speeds)

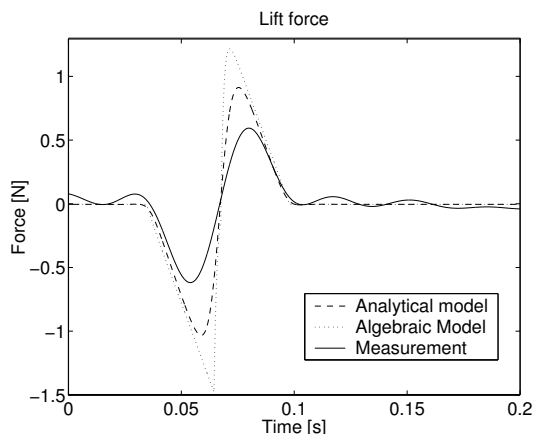


Figure 4: Lift force experiment 1

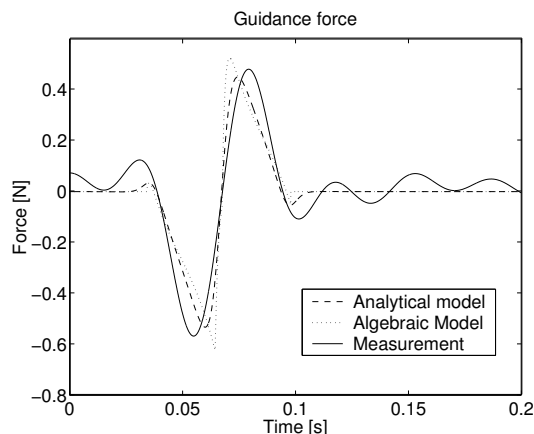


Figure 6: Guidance force experiment 1

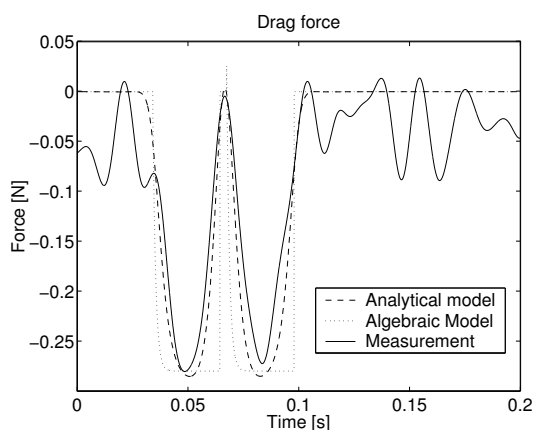


Figure 5: Drag force experiment 1

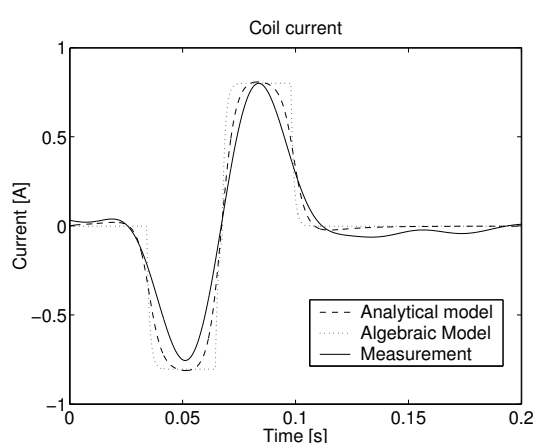


Figure 7: Coil current experiment 1

makes accurate force measurements difficult, i.e. large amplitude vibrations (Figures 8, 9 and 10) are due to structural vibrations of the test setup, and not a filtering artifact. According to the models, the two peaks in the drag force curve should have the same amplitude. The small difference in amplitude in the measurements is caused by the fact that the coil and the magnet are not perfectly parallel. Prediction errors from the algebraic model are related to the relatively large gap between the coil and the magnet, i.e., the prediction capability of the algebraic model improves when smaller gaps are considered, as discussed in [8]. The accuracy of the algebraic model can easily be improved by making the field density components  $B_y$ ,  $B_x(x, \delta)$  and  $B_z(x, \delta)$  functions of the distance between the surface of the magnet and the plane of the coil,  $y$ . In any case, the assumptions of the algebraic model are more accurate when

the distance between the coil and the magnet is small. Computer simulations of the algebraic model predicted also that  $B_x(x, \delta)$  and  $B_z(x, \delta)$  can be simplified to  $B_x(\delta)$  and  $B_z(x)$ .

## 7 CONCLUSIONS

Both the algebraic and the analytical closed-form model are capable of describing the electromechanical interactions in a null-flux EDS system. The models are much more computationally efficient than other existing methods such as FEM analysis and Dynamic Circuit Theory. Comparisons between simulations and measurements show good agreement. The algebraic model is fast enough to be used in real time control of the EDS Maglev multi-DOF vehicle dynamics.

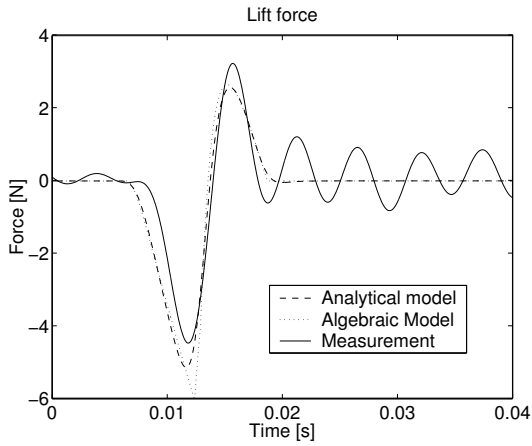


Figure 8: Lift force experiment 2

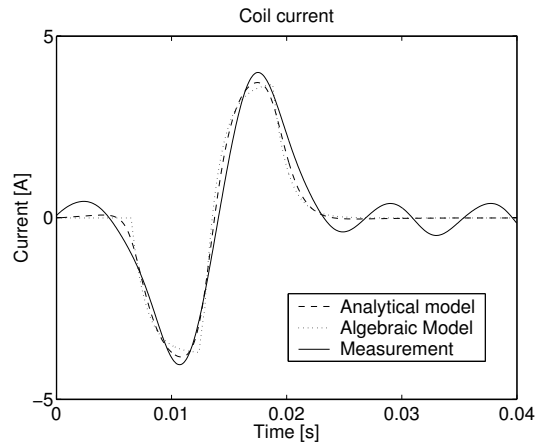


Figure 11: Coil current experiment 2

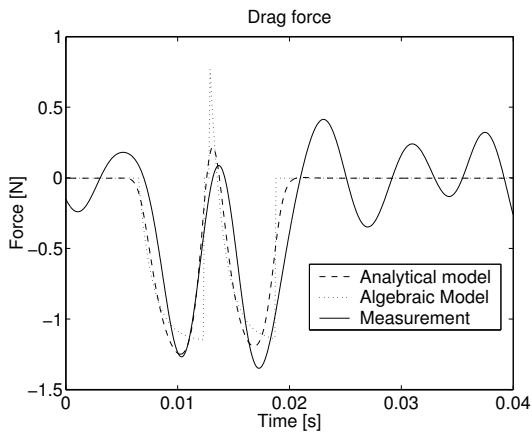


Figure 9: Drag force experiment 2

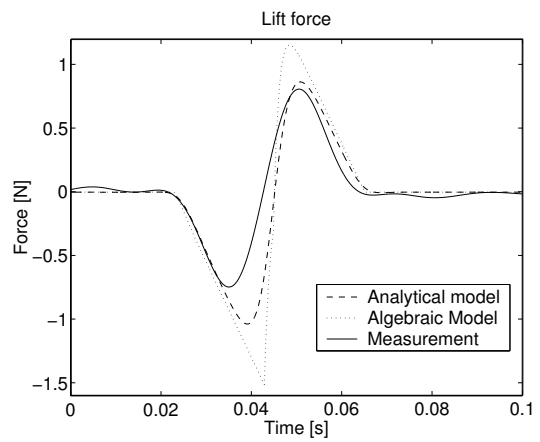


Figure 12: Lift force experiment 3

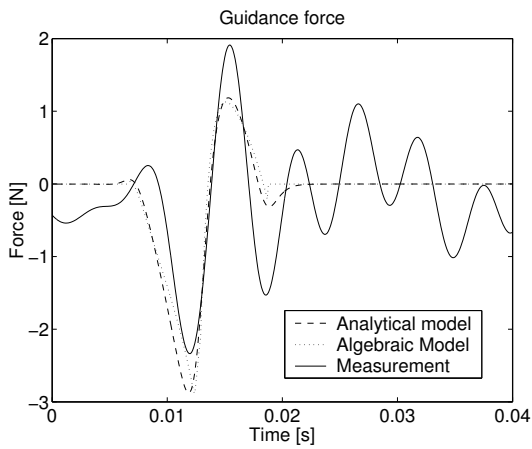


Figure 10: Guidance force experiment 2

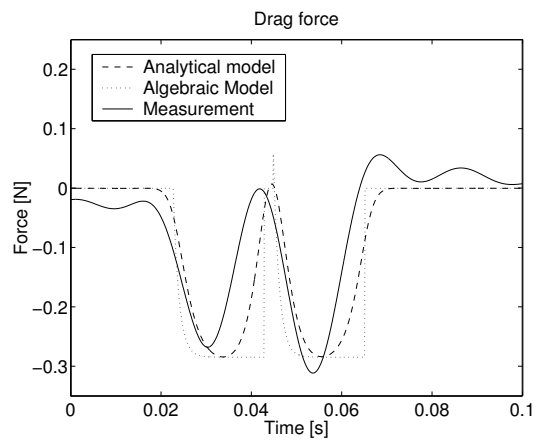


Figure 13: Drag force experiment 3

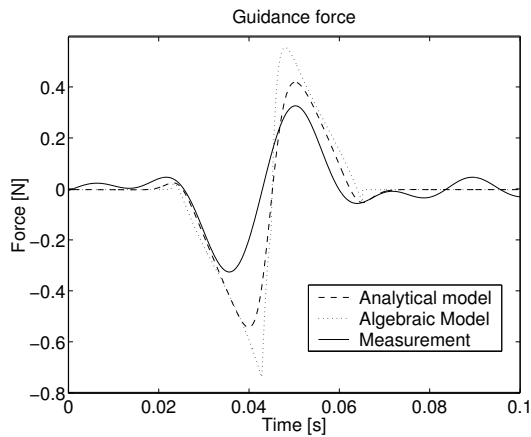


Figure 14: Guidance force experiment 3

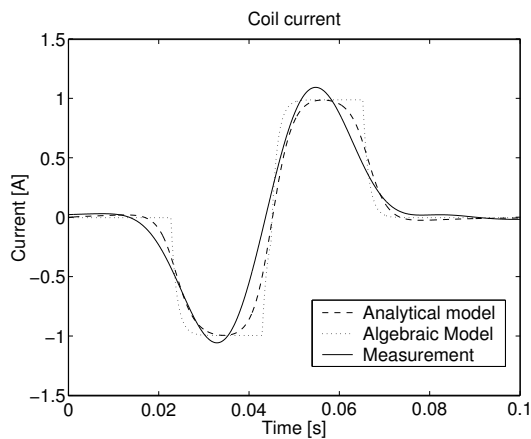


Figure 15: Coil current experiment 3

## References

- [1] Dill, J. and Meeker, D.: "Maglifter trade-off study and subscale system demonstrations", NAS-98069-1362, Foster-Miller, Inc, Waltham MA, December 2000.
- [2] Thome, Richard J. and Tarrh, John M.: "MHD and Fusion Magnets", John Wiley & Sons Inc, 1982
- [3] He, J.L., Rote, D.M. and Coffey, H.T.: "Applications of the dynamic circuit theory to maglev suspension systems", IEEE Transactions on Magnetics, vol. 29, no. 6, pp 4153-4164, November 1993.
- [4] He, J. L.; Coffey, H.: Magnetic damping forces in figure-eight-shaped null-flux coil suspension systems, IEEE Transactions on Magnetics, V.33, No.5, Part 2, Sept. 1997.
- [5] Kent, R.D.: "Designing with Null flux coils", IEEE Transactions on Magnetics, vol. 33, no. 5, pp 4327-4334, September 1997.
- [6] Kent, R.D.: "Analysis of an electro dynamic maglev system", IEEE Transactions on Magnetics, vol. 35, no. 5, pp 4259-4267, September 1999.
- [7] Mulcahy, T.M., He, J.L., Rote, D.M. and Rossing, T.D.: "Forces on a Magnet Moving Past Figure-Eight Coils", IEEE Transactions on Magnetics, vol. 29 no. 6, pp 2947-2949, November 1993.
- [8] De Boeij, J., Gutiérrez, H.M., Agarwal, R., Steinbuch, M.: "Modeling the 3-DOF Dynamics of an Electrodynamic Maglev Suspension System with a Passive Sled", 3rd International Conference in Transport Systems & Telematics TST 2003, Katowice, Poland, November 2003.