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UNBALANCE COMPENSATION ON FEXIBLE ROTORS BY MAGNETIC BEARINGS USING TRANSFER FUNCTIONS

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ABSTRACT

This paper presents two methods for compensating the unbalance vibrations of flexible rotors by active magnetic bearings. In contrast to automatic balancing, for example by notch filter techniques, which lead to rotor spinning around its main axis of inertia the new method applies additional synchronous forces by the magnetic bearings to maintain the rotor to rotate around its geometric axis. The method is applicable on flexible rotor systems with many degrees of freedom which have several critical speeds within the operating speed range.

Two different approaches for estimating the magnetic unbalance compensation forces are discussed and compared: The first one estimates the compensation forces by an iterative testing technique. The second one estimates the compensation forces online from measured rotor vibrations at any rotor positions via speed depending transfer functions. The new method does not affect the stability and the performance of the magnetically levitated rotor system. It can be applied in stationary as well as in nonstationary operation states, e.g. during run-up and run-down.

The described active unbalance compensation method was successfully tested on a magnetically supported flexible rotor system passing several critical speeds without resonance phenomenon. In the experiments the rotor deflections at selected positions have been reduced by the compensation method significantly to less than 10% of the initial values.

INTRODUCTION

Normally, rotating machines which operate above critical speeds have to pass through resonances during run-up or run-down. Usually, the rotor vibration excited by unbalance are very large near the critical speeds. It can cause serious problems, sometimes even damage to the rotating machines.

Active magnetic bearings (AMBs) enable a number of notable advantages over conventional bearings, [3]. Active magnetic bearings do not only offer the supporting function, but also can be used as actuators for influencing the dynamic behavior of the supported rotor. Active methods are able to compensate large rotor vibrations not only near critical speeds, but also during the whole run-up and run-down process, [1, 2, 4].

In this paper, two methods for compensating unbalance responses of flexible rotors supported by AMBs are presented, which enable the rotor to pass through or to operate near natural frequencies without large unbalance responses at selected rotor positions. The first one estimates the compensation forces by an iterative testing technique [1]. This method is an extension of a method for simple rotors which has already applied successfully since some years. A second new approach estimates the compensation forces online from measured rotor vibrations at any rotor positions via pre-determined transfer functions of the rotor system.

UNBALANCE COMPENSATION BY ITERATIVE TESTING

To cancel the unbalance response at chosen rotor positions synchronous magnetic compensation forces are applied by magnetic bearings on the rotor, additionally to the supporting forces. Amplitude and phase of the compensation forces have to be estimated during operation. The synchronous time courses $\sin\varphi$ and $\cos\varphi$ are known by measuring the rotor revolution, e.g. by using an optical angular encoder and a standard integrated circuit.

A method to find amplitude and phase of the compensation forces by iterative testing was firstly realized by [1] on a simple rotor system as show in figure 1. The iterative search algorithm operates in a two step scheme: Initially the rotor deflection $\mathbf{r}_0$ is measured without any additional magnetic test forces, $\mathbf{F}_0=0$. In step one small arbitrary test forces $\mathbf{F}_i$ spinning synchronously with the rotor are added to the supporting AMB forces and the rotor response $\mathbf{r}_i$ is measured. In step two the test force is updated by computing

$$\mathbf{F}_{i+1} = \mathbf{F}_i - \mathbf{r}_i \frac{\mathbf{F}_i - \mathbf{F}_{i-1}}{\mathbf{r}_i - \mathbf{r}_{i-1}} \quad \text{for} \quad i = 1, 2, \ldots \quad (1)$$

After some iterations the test forces converge to the optimal unbalance cancellation forces and the rotor deflection is minimized. For simple rotor systems the convergence of the method is guaranteed for stationary as well as for non-stationary operating conditions. The time between applying the compensation force $\mathbf{F}_i$ and measuring the response $\mathbf{r}_i$ has to be long enough to guarantee the decay of transient vibrations. In figure 2 a run-up passing the first critical speed is shown without and with unbalance compensation.

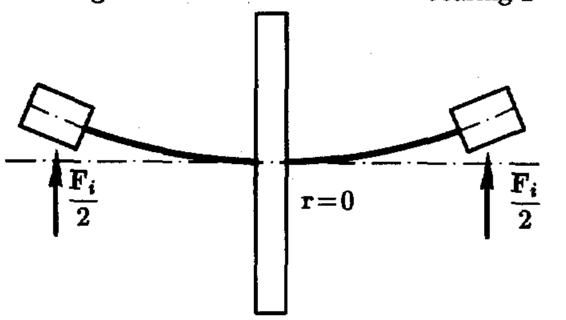

FIGURE 1: Schematic of the test rotor for the unbalance compensation with iterative searched forces

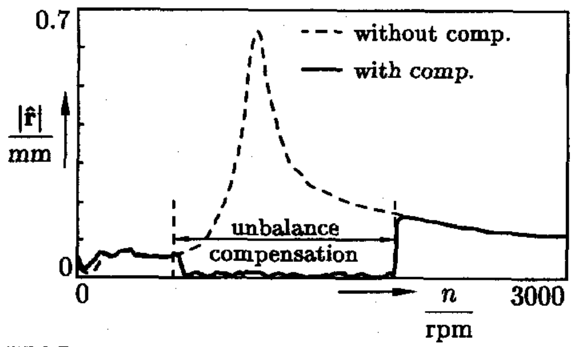

FIGURE 2: Vibration amplitude of the rotors disc during a run-up without and with unbalance compensation

For MDOF rotors the compensation forces in general can not be found with the simple relation (1). More complex search algorithms would be necessary.

UNBALANCE COMPENSATION USING TRANSFER FUNCTIONS

In contrast to the iterative search algorithm which does not guarantee the convergence a direct procedure for calculating the active unbalance compensation force is more reliable and easier to apply in practice.

The additional magnetic forces for compensating the unbalance vibrations can be computed directly using the transfer function of the rotor system. The transfer functions describe the linear interdependence between forces at the magnetic bearing positions and rotor deflections at those positions where the unbalance vibrations have to be minimized or suppressed. The transfer functions depend on speed. They were either pre-calculated from the theoretic model of the rotor or measured during operation. The synchronous unbalance compensation forces are computed online from the actual measured unbalance response using the inverse transfer functions. The compensation forces are calculated parallel but independent to the levitation forces. Levitation forces and unbalance compensation forces are applied simultaneously on the rotor by the AMBs.

The Test Rotor

The test rig for verifying the unbalance compensation method using transfer functions is shown in figure 3. The flexible rotor consists of a thin shaft and five rigid disks, two of them act as the journals of the AMBs. The rotor is supported by two AMBs and driven by a servo motor. Figure 4 shows the schematic configuration of this test rig.

The mathematical model of the rotor system is built up by the finite element method using beam and rigid disk elements and taking into account the gyroscopic effects of the disks. A 12-th order model is obtained,

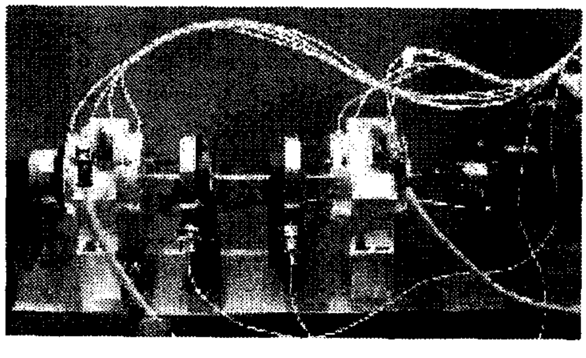

FIGURE 3: Picture of the rotor test rig

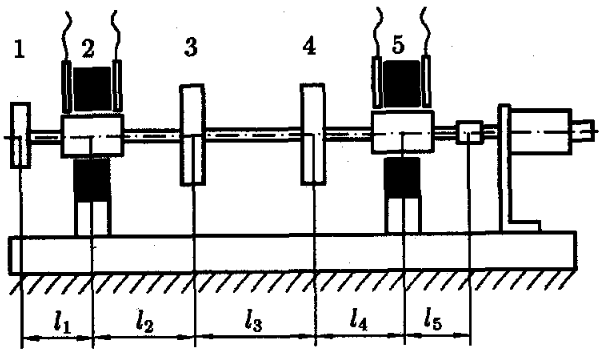

FIGURE 4: Configuration of the test rig

which can be summarized as

$$\mathbf{M}\ddot{\mathbf{q}} - j\Omega\mathbf{G}_d\dot{\mathbf{q}} + \mathbf{K}_0\mathbf{q} = \mathbf{f}_u - \mathbf{P}_m\mathbf{F}_m \qquad (2)$$

where $\mathbf{q} = \{r_1, \varphi_1, ..., r_6, \varphi_6\}^T$ with $r_i = z_i + jy_i$ and $\varphi_i = \varphi_{zi} + j\varphi_{yi}$ represents the complex state vector (displacement and slope) of the rotor cross-sections at the nodes. The mass matrix $\mathbf{M}$ is composed by the mass matrix of the shaft and the mass matrix of the disks. $\mathbf{K}_0$ is the stiffness matrix of the shaft, $\mathbf{G}_d$ is the skew-symmetric gyroscopic matrix of the disks and Ω is the rotor speed. The unbalance forces are represented by the vector $\mathbf{f}_u$, while $\mathbf{F}_m$ is the vector of the active forces of the two AMBs, whose positions are described by a position matrix

$$\mathbf{P_m} = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}^T .$$

If the AMBs are controlled by a PD-controller the active magnetic levitation forces are given by

$$\mathbf{F}_m = \begin{pmatrix} F_{m1} \\ F_{m2} \end{pmatrix} = \begin{pmatrix} k_{m1}r_2 + d_{m1}\dot{r}_2 \\ k_{m2}r_5 + d_{m2}\dot{r}_5 \end{pmatrix}, \qquad (3)$$

where k_{m1}, k_{m2} are the stiffness and d_{m1}, d_{m2} the damping coefficients of the magnetic bearings.

With PD-controllers and without compensation forces the equation of the whole rotor system including magnetic bearings has the general form

$$\mathbf{M}\,\ddot{\mathbf{q}}_u + \mathbf{B}\,\dot{\mathbf{q}}_u + \mathbf{K}\,\mathbf{q}_u = \mathbf{f}_u \qquad (4)$$

with $\mathbf{B} = -j\Omega\mathbf{G}_d + \mathbf{D}_m$ and $\mathbf{K} = \mathbf{K}_0 + \mathbf{K}_m$. $\mathbf{K}_m$ and $\mathbf{D}_m$ are the stiffness and damping matrix of the magnetic bearings controlled by PD-controllers.

In steady-state operation the unbalance force as well as the unbalance response are harmonic,

$$\mathbf{f}_u = \hat{\mathbf{f}}_u\, e^{j\Omega t}, \qquad \mathbf{q}_u = \hat{\mathbf{q}}_u\, e^{j\Omega t}. \qquad (5)$$

The vector $\hat{\mathbf{f}}_u$ contains the complex amplitudes of the unbalance forces and $\hat{\mathbf{q}}_u$ those of the rotor deflections.

The transfer matrix of total rotor system is given by

$$\mathbf{H}(\Omega) = \left(-\Omega^2\mathbf{M} + j\Omega\mathbf{B} + \mathbf{K}\right)^{-1}. \qquad (6)$$

The complex unbalance response amplitudes $\hat{\mathbf{q}}_u$ are related to the complex unbalance force amplitudes $\hat{\mathbf{f}}_u$ by the linear equation

$$\hat{\mathbf{q}}_u = \mathbf{H}(\Omega)\,\hat{\mathbf{f}}_u\,. \qquad (7)$$

Compensation Method

To compensate the unbalance response, additional forces $\mathbf{F}_{mc} = [F_{mc1}, F_{mc2}, \cdots, F_{mcp}]^T$ are applied on the rotor by the magnetic bearings. In general the number of degrees of freedom of the rotor is larger then the number of magnetic bearings. Thus, the unbalance response $\hat{\mathbf{q}}_u$ can be cancelled only on a few chosen rotor positions described by the position matrix $\mathbf{P}_c$,

$$\hat{\mathbf{q}}_c = \mathbf{P}_c\,\hat{\mathbf{q}}\,. \qquad (8)$$

Synchronous compensation forces of the AMBs generate at the positions described by $\mathbf{P}_c$ the rotor vibration amplitudes

$$\hat{\mathbf{q}}_{mc} = \mathbf{P}_c\mathbf{H}(\Omega)\mathbf{P}_m\hat{\mathbf{F}}_{mc} = \mathbf{H}_{comp}(\Omega)\hat{\mathbf{F}}_{mc}\,. \qquad (9)$$

To cancel the total rotor vibration $\hat{\mathbf{q}}_c = \hat{\mathbf{q}}_{uc} + \hat{\mathbf{q}}_{um}$ at the desired positions demands

$$\hat{\mathbf{q}}_{mc} = -\hat{\mathbf{q}}_{uc} \qquad (10)$$

and thus

$$\hat{\mathbf{q}}_{uc} = -\mathbf{H}_{comp}(\Omega)\,\hat{\mathbf{F}}_{mc}\,. \qquad (11)$$

The matrix $\mathbf{H}_{comp}(\Omega)$ depends on rotor speed and comprises the influence coefficients that describe the relation between the AMB forces and the rotor vibrations at the cancellation positions. The dimension of $\mathbf{H}_{comp}(\Omega)$ depends on the number of AMBs and the number of cancellation positions which are in general much smaller than the number of degrees of freedom. The magnetic forces necessary to compensate the unbalance response at the desired positions can be calculated from the measured unbalance response $\hat{\mathbf{q}}_{uc}$ by inverting equation (11). For inverting equation (11) three cases must be distinguished:

If the number of AMBs is equal to the number of compensation positions, $\mathbf{H}_{comp}(\Omega)$ can simply be inverted,

$$\hat{\mathbf{F}}_{mc} = -\mathbf{H}^{-1}_{comp}(\Omega)\,\hat{\mathbf{q}}_{mc}\,. \qquad (12)$$

If the number of AMBs is greater than the number of compensation positions, $\mathbf{H}_{comp}(\Omega)$ has to be supplemented by appropriate restraints before inverting it.

If the number of AMBs is smaller than the number of compensation positions the total compensation can not be achieved. However, the unbalance response is minimized using the pseudo-inverse of $\mathbf{H}_{comp}(\Omega)$, possibly by applying weighting factors.

EXPERIMENTAL VERIFICATION

The new unbalance compensation method has been implemented and tested on the test rig shown in figure 3. A dSPACE system is used for controlling the bearings and computing the compensation forces by using Matlab. In the shown test case the unbalance vibrations of disc 3 and disc 4 were compensated (see figure 4), hance the corresponding position matrix describing the compensation position is

$$\mathbf{P}_c = \begin{bmatrix} 0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0 \end{bmatrix} .$$

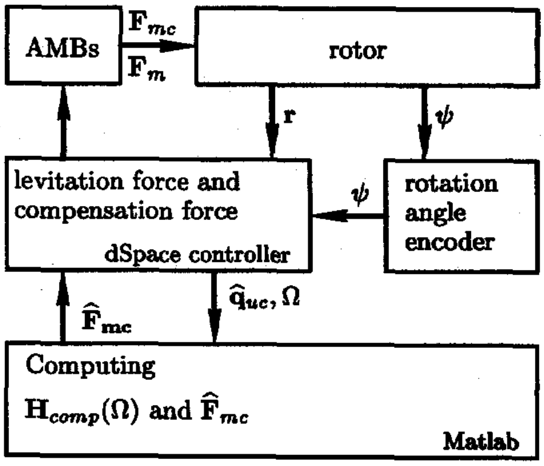

FIGURE 5: Process chart of the control structure with unbalance compensation

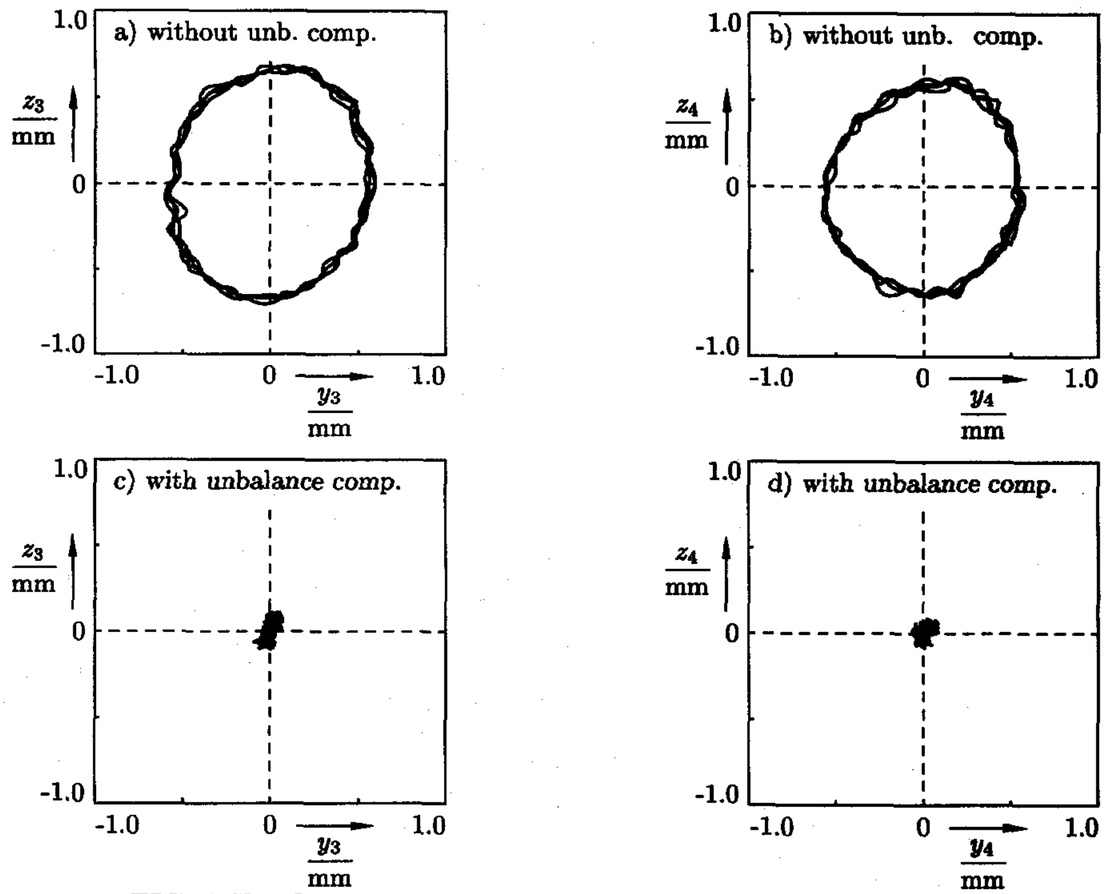

FIGURE 6: Measured rotor orbits at 950 rpm (near the first critical speed);
a) disc 3, without unbalance compensation b) disc 4, without unbalance compensation
c) disc 3, with unbalance compensation d) disc 4, with unbalance compensation

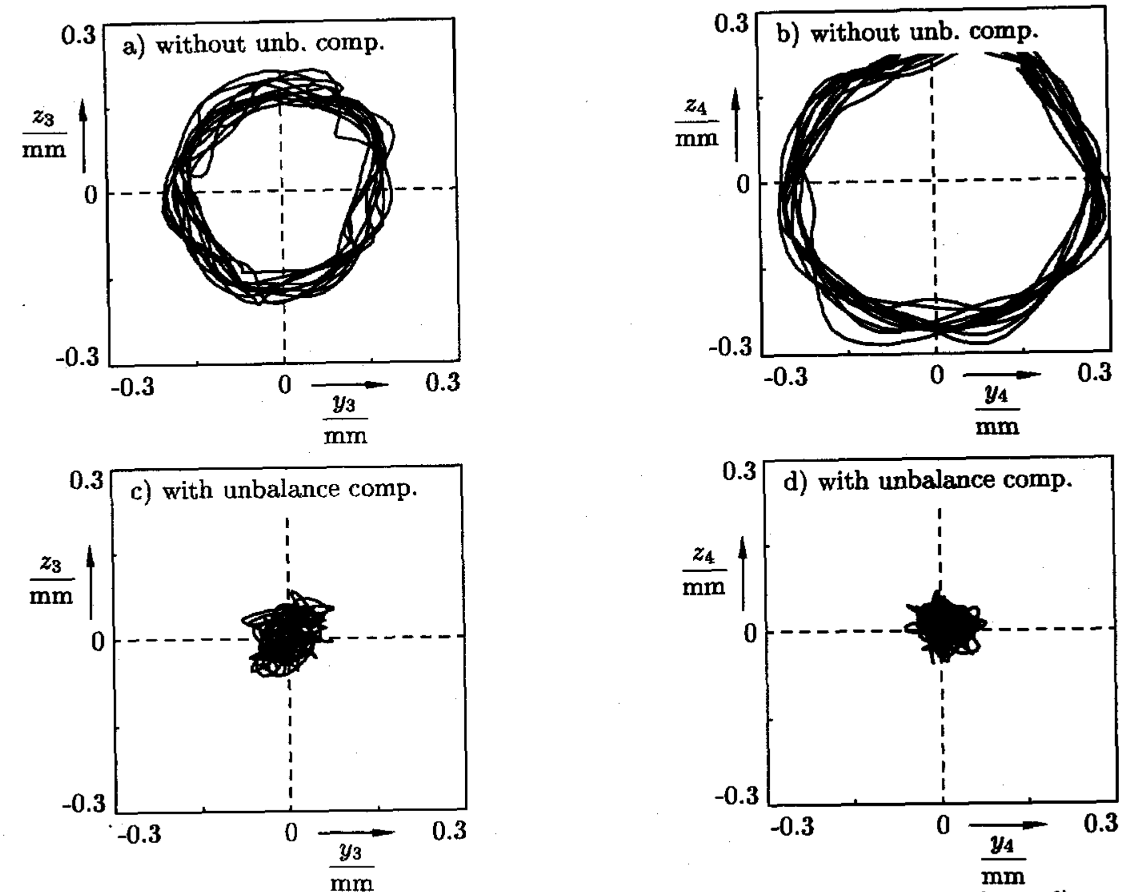

FIGURE 7: Measured rotor orbits at 3600 rpm (near the fourth critical speed);
a) disc 3, without unbalance compensation b) disc 4, without unbalance compensation
c) disc 3, with unbalance compensation d) disc 4, with unbalance compensation

The AMB controller acquires additionally the rotor deflections $\underline{\mathbf{r}}_{uc}$ at disc 3 and disc 4, the rotor speed Ω and the rotation angle ψ. Parallel to the levitation control the dSPACE system executes the unbalance compensation algorithm. For that it computes the speed depending matrix of influence coefficient

$$\mathbf{H}_{comp}(\Omega) = \begin{bmatrix} h_{5,3}(\Omega) & h_{5,9}(\Omega) \\ h_{7,3}(\Omega) & h_{7,9}(\Omega) \end{bmatrix}$$

from the system model extracting the elements specified by the position matrices $\mathbf{P}_m$ and $\mathbf{P}_c$. Then, it computes the complex amplitudes of the compensation forces $\underline{\mathbf{F}}_{mc}$ using the measured unbalance response $\underline{\mathbf{r}}_{uc}$. The compensation forces are applied on the rotor by the AMBs simultaneous to the levitation forces, controlled and synchronized to the rotation angle. The control structure is shown in figure 5.

The results of the unbalance compensation are shown in figures 6 and 7 for two different rotor speeds near the first and the fourth critical speed.

At rotor speed 950 rpm (which is near the first critical speed) the compensation method reduced the unbalance response at discs 3 and 4 significantly by 90% (figure 6). The residual vibrations are very small and do not longer contain synchronous components (figures 6c and 6d). At rotor speed 3600 rpm (which is near the fourth critical speed) the unbalance response is reduced by the magnetic compensation forces to comparable very small values.

SUMMARY AND OUTLOOK

The two presented compensation methods for minimizing the unbalance response of magnetically suspended flexible rotors worked very well on the test rig. The first compensation method suitable for simple rotors only is easy to realize and dose not require explicit knowledge of the dynamic properties of the rotor system. The second method operates also for complex rotor systems in the whole speed rage, but it requires an exact rotor model. From the experimental results, it is obvious to see that the unbalance excited vibrations of the flexible rotor could be reduced significantly by the proposed active compensation method.

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