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ROBUST SWITCHING CONTROL STRATEGY FOR A CLASS OF SECOND ORDER UNCERTAIN SYSTEMS WITH APPLICATION IN ACTIVE CONTROL OF UNBALANCING VIBRATION FOR MAGNETIC BEARING

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ABSTRCT

This paper is concerned with a problem about robust stabilization control of second-order linear switching discrete systems with coefficient uncertainties, which are appeared in control problem for robust rejection of unbalancing vibration of magnetic bearing system in which full information about system model is not known. At first we utilize supervisory control theory to propose a novel robust stabilization switching control strategy for the class of system. Then the proposed approach is applied to a problem of active control of periodic unbalancing vibrations of active magnetic bearing. The experimental results show that the proposed algorithm is effective for suppression of periodic disturbance and robust to model uncertainties.

INTRODUCTION

Many physical systems are hybrid in the sense that they involve interaction between discrete and continuous dynamics. Ones are inherent, examples are mechanical systems with backlash, dead zones, and static friction, or electrical systems with switches; another ones are introduced artificially when applying supervisor based multiple model switching control to plants which are uncertain or operate under uncertain and/or varying environmental conditions. Therefore recently a remarkable attention has been paid to study on hybrid dynamical systems especially switching systems [1-7]. In [1,2] a new concept so called multiple Lyapunov function is introduced to derive out results about stability of switching systems. For example in [7] the problem on analysis of stability for switching systems consisting of a number of linear subsystems has been cast into a convex optimization problem in terms of linear matrix inequalities (LMI) which can be effectively solved by numerical methods. It is another important problem how to find appropriate switching rule that stabilizes plant to be controlled at origin. To our knowledge, researches on this design problem have just been conducted, so a few of relevant results have been published. In [8] robust stabilizing method of hybrid systems in which controllers are switched based on certain rule was proposed. In [9] switched systems consisting of subsystems with unstable foci were studied and stabilizing conic switching laws for such systems were introduced. In particular necessary and sufficient conditions for asymptotic stabilizability are derived for such systems. In this paper we study a class of Second order system about which just a part of information is known and propose a novel robustly asymptotically stabilizing switching control law for such systems. Then we apply this method to problem of suppressing periodic disturbances. We inject additive control input whose frequency is the same as the periodic disturbance into the system and update Fourier coefficients α and β of additive control periodically based on certain rule so as to cancel the periodic disturbance. Finally the novel algorithm is implemented with DSP and as applied to real-time active control of unbalancing vibrations of active magnetic bearing closed loop system about which just a part of information is known. Simulation and experimental results showed that the proposed novel algorithm is effective for problem of active control of periodic vibrations and noises for some plants about which just a part of information is known.

PROBLEM FOMULATION AND SUPERVISOR BASED SWITCHING CONTROLLER DESIGN

Consider second order uncertain nonlinear discrete time system of the following form.

$$
\begin{aligned}
x_1(k+1) &= (1-u_1(k+1)cos(\theta)-u_2(k+1)sin(\theta))x_1(k)\\
&+(u_1(k+1)sin(\theta)-u_2(k+1)cos(\theta))x_2(k)\\
x_2(k+1) &= -(u_1(k+1)sin(\theta)-u_2(k+1)cos(\theta))x_1(k)\\
&+(1-u_1(k+1)cos(\theta)-u_2(k+1)sin(\theta))x_2(k)
\end{aligned}
\tag{1}
$$

where: $[x_1,x_2]^T \in R^2$ is a state vector with unknown initial value,, $[u_1,u_2]^T \in R^2$ is the control input. For sake of simplicity , here suppose $[u_1,u_2]^T \in \{-u_0,u_0\}\times\{-u_0,u_0\}$. θ is unknown parameter of system (1). The problem is to design appropriate control law $[u_1,u_2]^T$ that makes system (1) globally robustly asymptotically stable. The following theorem describes the solution of the problem.

Theorem 1

The following switching control law based on supervisor makes system (1) enter into asymptotic stable state within at most four steps.

$$
\begin{aligned}
&u_1(k+1)=-u_1(k)sgn(V(k)-V(k-1))\\
&u_2(k+1)=u_2(k)
\end{aligned}
\quad \text{if} \quad k=1,3,\ldots
\tag{2a}
$$

$$
\begin{aligned}
&u_2(k+1)=-u_2(k)sgn(V(k)-V(k-1))\\
&u_1(k+1)=u_1(k)
\end{aligned}
\quad \text{if} \quad k=2,4,\ldots
\tag{2b}
$$

where: $V(k)=x_1^2(k)+x_2^2(k)$, set a appropriate positive number to $V(0)$, $\text{sgn}(x)=\begin{cases}1 & x\ge 0\\ -1 & x<0\end{cases}$,

$u_1(1)=u_0$ or $u_1(1)=-u_0$, $u_2(1)=u_0$ or $u_2(1)=-u_0$,here, u_0 is a constant number satisfying $0<u_0<1$.

Proof of Theorem 1 will use the following Theorem 2. So we firstly state the following lemma before proving Theorem 2.

Lemma 1 If $\Phi \in \left\{\begin{bmatrix} a & b\\ -b & a\end{bmatrix} \middle| a\in R, b\in R\right\}$, then eigenvalues of Φ lie all inside unit circle or all on unit circle or all outside unit circle. If eigenvalues of Φ lie all inside unit circle, then

$$\Phi^T\Phi < I \tag{3}$$

If eigenvalues of Φ lie all on unit circle or all outside unit circle, then

$$\Phi^T\Phi \ge I \tag{4}$$

Proof

Because modulus of two eigenvalues of Φ are same and are equal to $\sqrt{a^2+b^2}$, then obviously the first statements of the lemma holds.

$$\Phi^T\Phi=\begin{bmatrix} a & b\\ -b & a\end{bmatrix}^T\begin{bmatrix} a & b\\ -b & a\end{bmatrix}=\begin{bmatrix} a^2+b^2 & 0\\ 0 & a^2+b^2\end{bmatrix} \tag{5}$$

If eigenvalues of Φ lie all inside unit circle that is modulus of two eigenvalues of Φ satisfy $\sqrt{a^2+b^2}<1$, then from (5), obviously (3) holds. If eigenvalues of Φ lie all on unit circle or all outside unit circle, that is modulus of two eigenvalues of Φ satisfy $\sqrt{a^2+b^2}\ge 1$, then from (5), obviously (4) holds. □

Theorem 2

Consider following second order discrete time switching linear system

$$x(k+1)=\Phi_{\sigma(k+1)}x(k) \tag{6}$$

where

$\Phi_p \in \left\{\begin{bmatrix} a & b\\ -b & a\end{bmatrix} \middle| a\in R, b\in R, a,b \text{ are unknown}\right\} p\in P$, σ is a mapping function which maps nonnegative integer number $\{k|k\in\{0,1,2,\cdots\}\}$ to finite index set $P=\{1,2,\ldots,N\}$.

Suppose at least one member belonging to $\{\Phi_p : p\in P\}$ is asymptotic stable, then following supervisor based switching control law:

$$\begin{aligned} \sigma(k+1) &= \sigma(k) \quad & V(k)-V(k-1)<0 \\ \sigma(k+1) &= \sigma(k)+1 \quad & V(k)-V(k-1)\geq 0 \end{aligned} \tag{7}$$

where, $V(k)=x^T(k)x(k)$, we might as well set $\sigma(1)=1$.

at most through N steps can make system (6) switch to an asymptotic stable system and exponentially asymptotically converges to equilibrium 0 along on the trajectory of the asymptotic stable system.

Proof of Theorem 2

Suppose $\sigma(k+1)=p$, Φ_p is not asymptotic stable, then from lemma 1, modulus of two eigenvalues of Φ are both larger than one or equal to one, moreover

$$\Phi_p^T\Phi_p \geq I$$

holds.

Select Lyapunov candidate function $V(k)=x^T(k)x(k)$, then for any $x(k)\in R^2$,

$$\begin{aligned} V(k+1) &= x^T(k+1)x(k+1) \\ &= x^T(k)\Phi_p^T\Phi_p x(k) > x^T(k)x(k) = V(k) \end{aligned} \tag{8}$$

Hence, from (7)

$$\sigma(k+2)=\sigma(k+1)+1=p+1$$

system (6) switches from mode p to mode $p+1$. So one can say that provided at the k'th step, $\sigma(k+1)=p$ and Φ_p, $p\in P$ is not asymptotic stable, then it is sure for system (6) to switch.

In addition, because there is at least a member in set $\{\Phi_p : p\in P\}$ which is asymptotic stable, we might as well assume Φ_q, $q\in P$ to be an asymptotic stable member, then from lemma 1

$$\Phi_q^T\Phi_q < I$$

holds. For any $x(k)\in R^2$,

$$\begin{aligned} V(k+1) &= x^T(k+1)x(k+1) \\ &= x^T(k)\Phi_q^T\Phi_q x(k) < x^T(k)x(k) = V(k) \end{aligned} \tag{10}$$

is valid.

From (7), $\sigma(k+2)=\sigma(k+1)=q$ so system (6) does no switch. Hence one can say that provided at the k'th step, $\sigma(k+1)=q$ and Φ_q, $q\in P$ is asymptotic stable, then it is sure that system (6) does not switch no longer and will remains at the asymptotic stable member. Since $q\leq N$, so proof is completed. □

Proof of Theorem 1: Suppose u_1, u_2 take value in set $\{-u_0, u_0\}$, then system (1) cab be rewritten as discrete time switching linear system of the following form:

$$x(k+1)=\Phi_{\sigma(k+1)}x(k) \tag{11}$$

where σ is a mapping function which maps nonnegative integer number $\{k | k\in\{0,1,2,\cdots\}\}$ to finite index set $P=\{1,2,3,4\}$.

$$\Phi_1 = \begin{bmatrix} 1-u_0(cos(\theta)+sin(\theta)) & u_0(sin(\theta)-cos(\theta)) \\ -u_0(sin(\theta)-cos(\theta)) & 1-u_0(cos(\theta)+sin(\theta)) \end{bmatrix}$$

$$\Phi_2 = \begin{bmatrix} 1-u_0(cos(\theta)-sin(\theta)) & u_0(sin(\theta)+cos(\theta)) \\ -u_0(sin(\theta)+cos(\theta)) & 1-u_0(cos(\theta)-sin(\theta)) \end{bmatrix}$$

$$\Phi_3 = \begin{bmatrix} 1-u_0(-cos(\theta)+sin(\theta)) & -u_0(sin(\theta)+cos(\theta)) \\ u_0(sin(\theta)+cos(\theta)) & 1-u_0(-cos(\theta)+sin(\theta)) \end{bmatrix}$$

$$\Phi_4 = \begin{bmatrix} 1+u_0(cos(\theta)+sin(\theta)) & u_0(-sin(\theta)+cos(\theta)) \\ -u_0(-sin(\theta)+cos(\theta)) & 1+u_0(cos(\theta)+sin(\theta)) \end{bmatrix} \tag{12}$$

Define set $\Theta_k=\{\theta | |s_i|<1, s_i\in ch(\Phi_k), i=1,2\} k\in P$, through simple calculation of eigenvalues of $\Phi_k(k\in P)$, it is known that eigenvalues of $\Phi_k(k\in P)$ is either conjugate eigenvalues or eigenvalues with multiplicity 2. So their modulus is just either both larger than one or equal one or less than one. Moreover sets $\Theta_k(k\in P)$ can be given as follows:

$$\Theta_1=\{\theta | \pi/4-\theta_0<\theta<\pi/4+\theta_0 \ ,$$

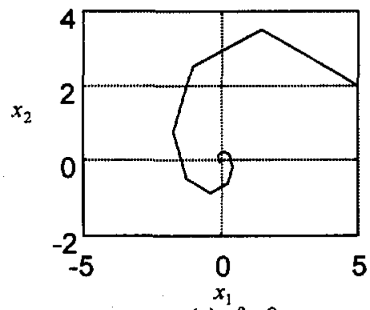
$$\Theta_2=\{\theta | 7\pi/4-\theta_0<\theta<7\pi/4+\theta_0\} \tag{13a}$$

$$\Theta_3 = \{\theta | 3\pi./4 - \theta_0 < \theta < 3\pi./4 + \theta_0\},$$

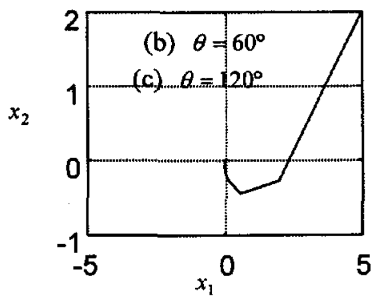
$$\Theta_4 = \{\theta | 5\pi./4 - \theta_0 < \theta < 5\pi./4 + \theta_0\} \qquad (13b)$$

where $\theta_0 = \cos^{-1}(u_0 / \sqrt{2})$.

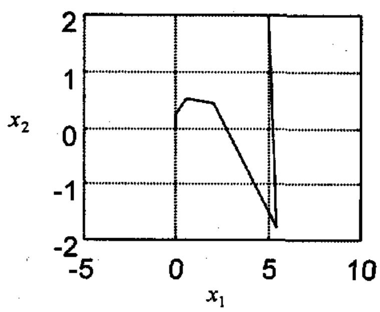
In the case $u_0 < 1$, it is easy to verify $\Theta_1 \supset [0, \pi/2]$, $\Theta_2 \supset [3\pi/2, 2\pi]$, $\Theta_3 \supset [\pi/2, \pi]$, $\Theta_4 \supset [\pi, 3\pi/2]$, and $\Theta_1 \cup \Theta_2 \cup \Theta_2 \cup \Theta_2 = [0, 2\pi]$. Hence, $\forall \theta, \theta \in [0, 2\pi]$, at least $\exists k, k \in P$, so that eigenvalues of $\Phi_k (k \in P)$ lie inside unit circle. eigenvalues of unstable $\Phi_k (k \in P)$ belonging to $\{\Phi_k | \ k \in P\}$ lie either both on unit circle or both outside unit circle. So from Theorem 2, supervisor based switching control law (7) makes system (11) asymptotic stable after at most four steps, that is, supervisor based switching control law (2a) and (2b) makes system (1) enter asymptotic stable state through at most four steps. The proof is completed. □



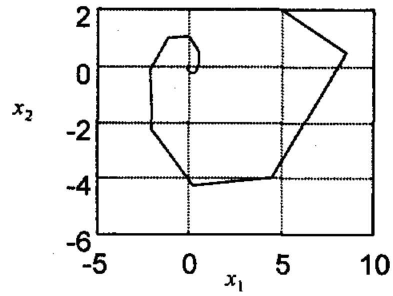
(a) θ =0



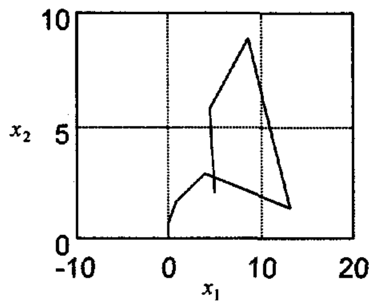
(b) θ =60°



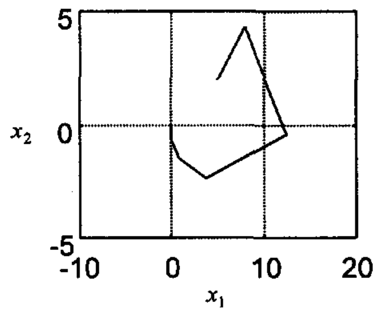
(c) θ =120°



(d) θ =180°



(e) θ =240°



(f) θ =300°

Fig. 1 Phase trajectories of system (1), (2)

SIMULATION

In section 2, we used theory of stability analysis and stabilizing switching logic rule design for hybrid system to stabilization problem of a class of second order uncertain nonlinear discrete time system and proposed a novel supervisor based robustly stabilizing switching control law and proved that resulted closed loop system is asymptotic stable. In order to verify effectiveness of proposed method and robustness against uncertainty of system parameter θ and give intuitive understanding of the proposed method, for second order discrete time system (1) with control law (2), in which θ takes different value in $[0,2\pi]$, a various simulations are conducted. A few simulation results when θ takes a few representative values in set are shown in Fig.1. The simulation results show correction of theoretic result given in section 2.

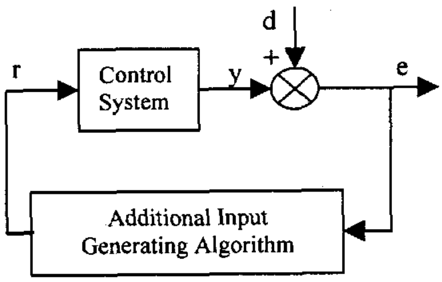


Fig. 2 Schematic diagram of periodic disturbance active control

THEIR APPLICATION--- ACTIVE CONTROL OF UNBALANCING VIBRATION FOR MAGNETIC BEARING

Schematic block diagram of periodic disturbance active control is shown in Fig.2. Where r is additive control input aiming at rejecting periodic disturbance at output. Suppose that plant to be controlled is single input single output linear stable system whose transfer function is G(s) and correspondingly whose frequency response $G(j\omega)=A(\omega)e^{j\theta}$. Here assume that the upper bound of $A(\omega)$ is known and equal to A, but no any information is known about phase properties of system. Now problem is how to produce appropriate additive control input r so that steady output of system robustly asymptotically converges to zero regardless of periodic disturbance d at output.

Suppose periodic disturbance d at output is of following form (14).

$$d=\alpha_d\sin(\omega t)+\beta_d\cos(\omega t) \tag{14}$$

Additive control input r is taken the following form (15).

$$r=\alpha\sin(\omega t)+\beta\cos(\omega t) \tag{15}$$

Then steady output y produced from additive control input r and total steady output of system can be expressed as (16) (17) respectively.

$$y=A(\alpha\sin(\omega t+\theta)+\beta\cos(\omega t+\theta)) \tag{16}$$

$$e=A(\alpha\sin(\omega t+\theta)+\beta\cos(\omega t+\theta))+\alpha_d\sin(\omega t)+\beta_d\cos(\omega t) \tag{17}$$

Amplitudes of sine component $n_1(t)$ and cosine component $n_2(t)$ of total steady output e can be expressed as

$$\begin{aligned} n_1(t)&=0.5(A\alpha\cos(\theta)+\alpha_d-A\beta\sin(\theta)) \\ n_2(t)&=0.5(A\beta\cos(\theta)+\beta_d+A\alpha\sin(\theta)) \end{aligned} \tag{18}$$

If we update Fourier coefficients α and β of additive control input r periodically with period T in terms of (19), then (18) can be rewritten as (20).

$$\begin{aligned} \alpha(k+1)&=\alpha(k)-(\mu_1(k+1)n_1(k)+\mu_2(k+1)n_2(k)) \\ \beta(k+1)&=\beta(k)-(\mu_1(k+1)n_2(k)+\mu_2(k+1)n_1(k)) \end{aligned} \tag{19}$$

$$\begin{aligned} n_1(k+1)&=(1-0.5A(\mu_1(k+1)\cos(\Theta)+\mu_2(k+1)\sin(\Theta)))n_1(k) \\ &+0.5A(\mu_1(k+1)\sin(\Theta)-\mu_2(k+1)\cos(\Theta))n_2(k) \\ n_2(k+1)&=(1-0.5A(\mu_1(k+1)\cos(\Theta)+\mu_2(k+1)\sin(\Theta)))n_2(k) \\ &-0.5A(\mu_1(k+1)\sin(\Theta)-\mu_2(k+1)\cos(\Theta))n_1(k) \end{aligned} \tag{20}$$

Corollary

If and only if μ_0 takes number satisfying $0<\mu_0<2/A$, supervisor based switching control law (2a) and (2b) can make system (20) at most through four steps switch to an asymptotic stable system and exponentially asymptotically converges to equilibrium 0.

Proof: This corollary is directly derived from Theorem 1.

From the corollary it has been seen that the proposed active control algorithm of periodic disturbance can make output of system asymptotically converge zero. Moreover if μ_0 takes smaller number within $(0,\ 2/A)$, the control law can admit larger uncertainty to nominal gain of system, but maybe reduce decay rate of periodic disturbance.

Finally we implement the control algorithm with DSP and carry out active control of unbalancing vibration for magnetic bearing experimentally. The experimental results are shown in Fig.3.

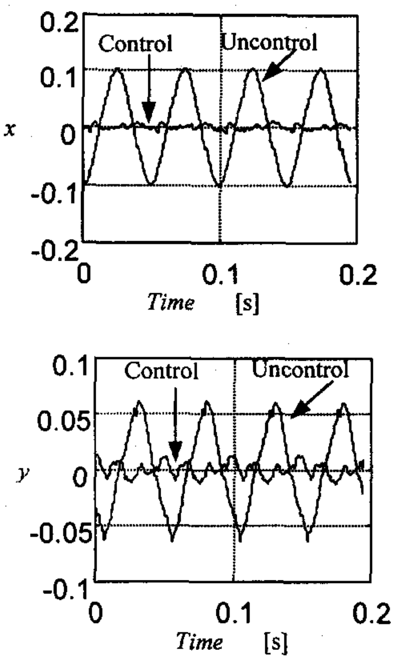


Fig. 3 Experimental results of active control of unbalancing vibration for magnetic bearing

CNCLUSIONS

In this paper we study asymptotically stabilizing problem of a class of second order uncertain discrete time system. This problem arises in periodic disturbance adaptively suppression problem for system about whose model information is known just partly. We have proposed a novel adaptive robust switching control law using hybrid system control theory and theoretically proved asymptotic stability of the closed loop system using the control law. Finally we implement the control algorithm with DSP and carry out active control of unbalancing vibration for magnetic bearing experimentally. Experimental results show that effect of control of unbalancing vibration is better. It has been verified that the proposed control algorithm is effective and robust.

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