8th International Symposium on Magnetic Bearing
August 26-28, 2002, Mito, Japan

FORCES AND MOMENTS IN AXIALLY POLARIZED RADIAL PERMANENT MAGNET BEARINGS

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ABSTRACT

Permanent magnet ring bearings can be used in many different applications where they must be designed to have certain force and moment properties. In this paper, the magnetic vector potential is employed to obtain a three dimensional analytical solution for axially polarized radial bearings for analysis and design purposes. A more general solution for the radial and axial forces plus the moments is obtained than previous published works. It requires only a simple numerical evaluation and is relatively easily carried out with currently available mathematical programs. This analysis extends the method to the operation of the rings over a large operating range of clearance unlike other authors who evaluate the forces only at small perturbations from a centered location. The analysis of forces and moments are employed to evaluate the complete 5x5 stiffness matrix in a companion paper to "Stiffness Analysis of Axially Polarized Radial Permanent Magnet Bearings," by Jiang, Allaire, Baloh, and Wood.

INTRODUCTION

Permanent magnet (PM) radial bearings are becoming more used in magnetic suspension in industrial devices. Generally, PM bearings are axially polarized due to the difficulty of constructing and magnetizing radially polarized bearings although some advantages are obtained with radial polarization. It is important to be able to evaluate all three force components - two transverse and one axial - as well as two moments - one about each transverse coordinate axis for multiple ring configurations. The forces and moments are evaluated for rotor operation over a large range of clearance unlike other authors who carry out analysis only for small perturbations about a centered operating point. This paper is the first of a series of two papers and is followed by "Stiffness Analysis of Axially Polarized Radial Permanent Magnet Bearings" [Jiang, Allaire, Baloh and Wood, 2002] published separately.

Several previous works have developed an analysis of PM bearings. Many previous works have used a two-dimensional approximation for the three-dimensional rings. Yonnet [1981a] developed an analytical solution for a wide range of PM bearings and couplings based upon a dipole distribution in a bar shaped magnet for an unrolled configuration where the airgap dimension is very small compared to the magnetic cross sectional dimensions. Yonnet [1981b] also presented another analytical solution for a distribution of magnetic poles where the ring curvature is large compared to any of the ring cross-section dimensions. The magnetic forces and stiffnesses were obtained between two rings with either axial or radial polarization. Okuda et al [1984] developed an analysis of ring permanent magnets and eddy current damping configurations using equivalent surface currents. Delamare et al [1994] developed a five axis magnetic suspension using two axially polarized permanent magnet rings for radial and moment control of a mass and one active magnetic bearing for axial control. Delamare et al [1995] considered several different PM configurations different from those normally found in the literature.

Marinescu and Marinescu [1994] employed an axial-symmetric magnetic vector potential analysis of residual magnetic flux densities with current sheets to obtain the radial and axial force as well as radial stiffness between two PM rings forming a magnetic bearing configuration. They found that only one component of the magnetic vector potential, the circumferential component, is needed for the analysis. However, they did not consider moments on the bearing.

The purpose of this work is to obtain equations for all of the forces and moments components for axially polarized PM bearings which overcomes many of the limitations of previous analyses. A magnetic vector potential is used to evaluate the magnetic field due to an equivalent current sheet placed in ring form. The self-energy and mutual-energy of the fields are determined and then employed to evaluate the force and moment between PM ring magnets which are near each other. There are five generalized displacements or degrees of freedom x, y, z, α, β (not counting the rotational degree) and five generalized forces $F_x, F_y, F_z, M_\alpha, M_\beta$ (not counting the rotational moment). A fully three dimensional solution is obtained for ring magnets which is easy to evaluate using existing mathematical programs such as *Mathematica.* This solution is more general than the previous analyses available in the literature.

NOMENCLATURE

Main Variable/Function

a, b	=Stationary and Moving Ring Radius
A	=Magnetic Vector Potential Function
B	=Magnetic Flux Density
C	=Moment Reference Center
F	=Force
h	=Ring Height
Hc	=Axial Length of Moving Ring Center and Moment Reference Center
I	=Useful Integral Function
J	=Current Density
r, z	=Radial and Axial Coordinates
M	=Moment
s	=Polarization Sign (±1)
V	=Volume of Ring
W	=Magnetic Energy Function
ϕ	=Rotational Angular Coordinate (Around z Axis)
μ_0	=Free Space Magnetic Permeability
x, y, z	=Transitional Coordinate
α, β	=Moment Coordinate
θ	=Angle between Vector Potential A_a and Current J_b

Subscript

x, y, z	=Transitional Coordinate
α, β	=Moment Coordinate
a, b	=Source Origination (Nominal Position)
b'	=Source Origination (Perturbation Position)

Superscript

$'$	= Perturbation of Coordinate
z^T	=Temp Integral Variable at Height of Ring b
z^{TT}	=Temp Integral Variable at Height of Ring a
$\bar{}$	=Normalized Variable/Function (Bar on Top)

MAGNETIC VECTOR POTENTIAL

Let the three-dimensional magnetic vector potential A of one PM ring exerted upon the other be defined as $B = \nabla \times A$ where B is the three-dimensional magnetic flux density. Faraday's equation gives $\nabla \times B = \mu J$ where J is the three-dimensional current density in the current sheet [Jackson, 1999]. The Coulomb gage bold $\nabla \times A = 0$ has been employed in this formulation. We also have the conservation of magnetic flux density bold $\nabla B = 0$. Faraday's law is really three simultaneous three-dimensional differential equations to be solved. The differential axial-symmetric magnetic vector potential at point in space b due to current density J_a in ring (sheet) located at a radius a, as shown in Fig. 1, is

$$dA_a(b, z^{TT}) = \frac{\mu_0 J_a dz^{TT}}{4\pi} aI(z^{TT}) \qquad (1)$$

where

$$I(z^{TT}) = \int_0^{2\pi} \frac{\cos(\phi_a)}{\left[a^2 + b^2 - 2ab\cos(\phi_a) + (z^{TT})^2\right]^{\frac{1}{2}}} d\phi_a \qquad (2)$$

in cylindrical coordinates [Krause, 1953]. Here μ_0 is the magnetic permeability of free space. The magnetic vector potential expression is valid for any type of magnetization, radial or axial. This is an elliptic integral and is rather difficult to integrate in general but specific values can be obtained using available mathematical codes such as *Mathematica.*

The self energy and mutual energy of the static magnetic field is given by

$$W_{self} = \frac{1}{2}\int A_a J_a dV, W_{mutual} = \frac{1}{2}\int A_a J_b dV \qquad (3)$$

These terms are used to obtain the force and moment in the following analysis.

AXIAL POLARIZATION

Fig. 2 shows the geometry of the two axisymmetric PM rings, one of radius a, height h_a, current density J_a and the other of radius b, height h_b, current density J_b. In the case of an axially polarized bearing, the full magnetic vector potential expression is obtained by integration of Equation (1) with the result

$$A_a(b, z^T) = \frac{\mu_0 J_a}{4\pi}\int_{z^T - z_a}^{z^T - z_a - h_a} [aI(z^{TT})]dz^{TT} \qquad (4)$$

where z^T is the axial coordinate of ring b, as shown in Fig. 2, and the definite integral $I(z^{TT})$ is defined as Equation (2).

The nominal position of the ring b is z=0. The mutual energy is easily found as the following equation for the PM magnetic field.

$$W_{ab}(b,z) = \int_{z_b+z}^{z_b+z+h_b} \left[2\pi J_b b A_a(b,z^T)\right] dz^T$$
$$= \frac{\mu_0 J_a J_b}{2} \int_{z_b+z}^{z_b+z+h_b} b \left[\int_{z^T-z_a}^{z^T-(z_a+h_a)} aI(z^{TT}) dz^{TT} \right] dz^T \quad (5)$$

AXIAL FORCE

The axial force between the two current sheets is

$$F(z) = -\frac{\partial W_{ab}(b,z)}{\partial z}$$
$$= -\frac{\mu_0 J_a J_b}{2} [f(z_b + z + h_b) - f(z_b + z)] \quad (6)$$

where the function f is given by

$$f(z) = b \int_{z-z_a}^{z-(z_a+h_a)} aI(z^{TT}) dz^{TT} \quad (7)$$

The function $f(z)$ is evaluated to determine the force at any axial position z in the PM bearing, as shown in Fig. 2. This function is expressed in the form

$$F_z(s_a, a, z_a, h_a, s_b, b, z_b, h_b, z) = -\frac{\mu_0 J_a J_b}{2} s_a s_b \times$$
$$[f(a, z_a, h_a, b, z_b + z + h_b) - f(a, z_a, h_a, b, z_b + z)] \quad (8)$$

where $f(a,z_a,h_a,b,z)$ is a function evaluated with *Mathematica* or similar code. Also $s_a=\pm1$ and $s_b=\pm1$ to indicate the current direction (indicating the PM axial polarization).

If there are several PM rings, the properties of each ring can be expressed in terms of their respective current rings. Then, the forces and moments are taken as the superposition of the magnetic vector potential solution for all of the PM current rings. The numerical values of the forces and moments are evaluated by the use of a simple numerical integration over the single argument, which is numerically stable.

RADIAL FORCE

When the ring b is displaced radially by a distance r as shown in Fig. 3, the displaced co-energy is

$$W_{ab'}(r,z) = \int_{z_b+z}^{z_b+z+h_b} J_b b \left[\int_0^{2\pi} A_a(b', z^T) \cos(\theta) d\phi_b \right] dz^T \quad (9)$$

where the length b' and $\cos(\theta)$ are

$$b'(r,\phi_b) = \sqrt{b^2 - 2br\cos(\phi_b) + r^2}$$
$$\cos(\theta) = \sqrt{1 - \left(\frac{r}{b'(r,\phi_b)} \sin(\phi_b) \right)^2} \quad (10)$$

Define the dimensionless variable as below

$$\bar{r} \equiv \frac{r}{b} \quad \bar{z} \equiv \frac{z}{b} \quad \bar{a} \equiv \frac{a}{b} \quad \bar{b}' \equiv \frac{b'}{b} \quad \bar{h}_a \equiv \frac{h_a}{b} \quad \bar{h}_b \equiv \frac{h_b}{b}$$
$$F(\bar{r}) = -\frac{1}{b}\frac{\partial \bar{W}_{ab'}(\bar{r},\bar{z})}{\partial \bar{r}} \quad K(\bar{r}) = -\frac{1}{b}\frac{\partial F(\bar{r})}{\partial \bar{r}} \quad (11)$$

The co-energy function becomes

$$W_{ab'}(\bar{r},\bar{z}) = \int_{\bar{z}_b+\bar{z}}^{\bar{z}_b+\bar{z}+\bar{h}_b} J_b b^2 \left[\int_0^{2\pi} A_a(\bar{b}', \bar{z}^T) \cos(\theta) d\phi_b \right] d\bar{z}^T \quad (12)$$

From equation (11) the radial force can be found as

$$F_r(\bar{r},\bar{z}) = \int_{\bar{z}_b+\bar{z}}^{\bar{z}_b+\bar{z}+\bar{h}_b} J_b b \left[\int_0^{2\pi} -\frac{\partial [A_a(\bar{b}', \bar{z}^T)\cos(\theta)]}{\partial \bar{r}} d\phi_b \right] d\bar{z}^T \quad (13)$$

Use Taylor expansion for (13) with considerable algebra, the radial force is given by

$$F_r(\bar{r}) = o(\bar{r}^3) +$$
$$\int_{\bar{z}_b+\bar{z}}^{\bar{z}_b+\bar{z}+\bar{h}_b} J_b b\, \pi \bar{r} [A_a(1,\bar{z}^T) - \frac{\partial A_a(1,\bar{z}^T)}{\partial \bar{b}'} - \frac{\partial^2 A_a(1,\bar{z}^T)}{(\partial \bar{b}')^2}] d\bar{z}^T \quad (14)$$

All non-zero terms are retained up to $o(\bar{r}^3)$. The actual terms are evaluated in *Mathematica* or a similar code using numerical integration.

ANGULAR DISPLACEMENT AND MOMENT

Moments were not evaluated in previously published works but they are very important for full evaluation of PM bearing performance. Consider the small angular displacement α of the b PM ring as shown in Fig. 4 relative to the a PM ring. Then the dimensionless axial position of the ring is given by

$$\bar{z}'(\alpha) = \bar{z}^{TT} + \sin(\alpha)\sin(\phi_b)$$
$$= \bar{z}^{TT} + \alpha \sin(\phi_b) + o(\alpha^3) \quad (15)$$

The dimensionless radially displaced distance is determined as

$$\bar{b}'(\alpha) = \sqrt{1 - (\bar{z}'(\alpha) - \bar{z}^{TT})^2}$$
$$= \left[1 - \frac{\alpha^2}{2}\sin^2(\phi_b) \right] + o(\alpha^4) \quad (16)$$

The cosine term is

$$\cos(\theta) = \sqrt{1 - (\sin(\alpha)\cos(\phi_b))^2}$$
$$= 1 - \frac{\alpha^2}{2}\cos^2(\phi_b) + o(\alpha^4) \quad (17)$$

The magnetic vector potential becomes

$$A_a[\bar{b}'(\alpha), \bar{z}'(\alpha)] = \frac{\mu_0 J_a}{4\pi} \int_{\bar{z}^T-\bar{z}_a}^{\bar{z}^T-\bar{z}_a-\bar{h}_a} b\bar{a} I[\bar{b}'(\alpha), \bar{z}'(\alpha)] d\bar{z}^{TT} \quad (18)$$

The co-energy is

$$W_{ab'}(\alpha,\bar{z})$$
$$= b \int_{\bar{z}_b+\bar{z}}^{\bar{z}_b+\bar{z}+\bar{h}_b} J_b b \left[\int_0^{2\pi} A_a[\bar{b}'(\alpha), \bar{z}'(\alpha)] \cos(\theta) d\phi_b \right] d\bar{z}^T \quad (19)$$

The moment induced by this angular tilt is

$$M_a(\alpha) = -\frac{\partial W_{ab'}(\alpha,\bar{z})}{\partial \alpha} \quad (20)$$

Expanding this out in a Taylor's series form gives

$$M_\alpha(\alpha) = o(\alpha^3) + b\int_{\bar{z}_b+\bar{z}}^{\bar{z}_b+\bar{z}+\bar{h}_b} J_b b\ \pi\alpha\left[A_a + \frac{\partial A_a}{\partial \bar{b}'} - \frac{\partial^2 A_a}{(\partial \bar{z}')^2}\right]_{\alpha=0} d\bar{z}^T \quad (21)$$

where all non-zero terms are retained up to $o(\alpha^3)$. The actual terms are evaluated in *Mathematica* or a similar code using numerical integration.

FORCES AND MOMENTS

The final results for the forces and moments have now been obtained. Each of these terms on the right has already been evaluated. These terms are then used to evaluate the PM bearing 5x5 stiffness matrix.

CONCLUSIONS

The general analysis presented in this paper develops a three-dimensional magnetic vector potential analysis of axially polarized ring permanent magnets. This analysis is applied to axial and radial translational displacements as well as angular displacements to evaluate the bearing forces and moments as a general function of axial position z and radial position r. Ring symmetry properties have been included where applicable. The formulas obtained in this paper can be evaluated using standard commonly available mathematical packages. This paper provides the basis for the extension to full stiffness matrices for PM rings as given in the companion paper Jiang et al [2002]. There is good agreement between an example radial bearing as presented in the companion paper and the results by Marinescu [1994]. Two additional example cases are presented in the companion paper.

REFERENCES

Delamare, J., Yonnet, J. P., Rulliere, E., 1994, "A Compact Magnetic Suspension With Only One Axis Control," IEEE Trans. on Magnetics, Vol. 30, No. 6, November, pp. 4746-4748.

Delamare, J., Rulliere, E., and Yonnet, J. P., 1995, "Classification and Synthesis of Permanent Magnet Bearing Configurations," IEEE Trans. on Magnetics, Vol. 31, No. 6, November, pp. 4190-4192.

Jackson, J. D., 1999, Electrodynamics, Third Edition, Wiley, New York.

Jiang, W., Allaire, P. E., and Baloh, M. J., "Stiffness Analysis of Axially Polarized Radial Permanent Magnet Bearings", 2002.

Krause, J. D., 1953, Electromagnetics, McGraw-Hill, New York.

Marinescu, M., and Marinescu, N., "A New Improved Method for Computation of Radial Stiffness of Permanent Magnet Bearings," IEEE Transactions on Magnetic Bearings," Vol. 30, No. 5, Sept 1994, pp. 3491-3494.

Okuda, H., Abukawa, T., Arai, K., and Ito, M., "Characteristics of Ring Permanent Magnet Bearing," IEEE Transactions on Magnetics, Vol. MAG-20, No. 5, September, 1984, pp. 1693-1695.

Panofsky, W. K. H., and Phillips, 1955, Classical Electricity and Magnetism, Addision-Wesley, Reading, Mass.

Yonnet, J. P., 1981a, "Permanent Magnet Bearings and Couplings," IEEE Trans. on Magnetics, Vol. Mag-17, No. 1, January, pp. 1169-1173.

Yonnet, J. P., 1981b, "Analytical Calculation of Magnetic Bearings," Paper No. V-3, Fifth International Workshop on Rare Earth-Cobalt Permanent Magnets and Their Applications," Roanoke, VA, USA, pp. 199-215.

FIGURES

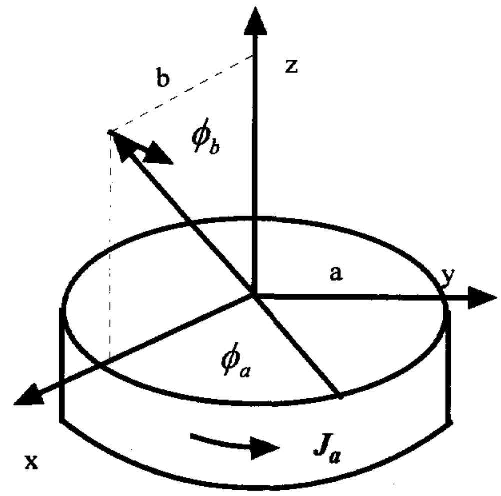
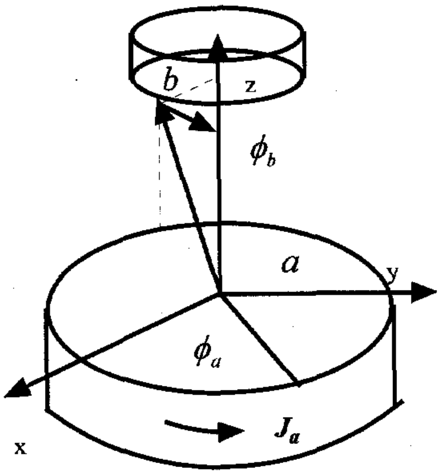


FIG. 1: Vector Potential for a Current Sheet



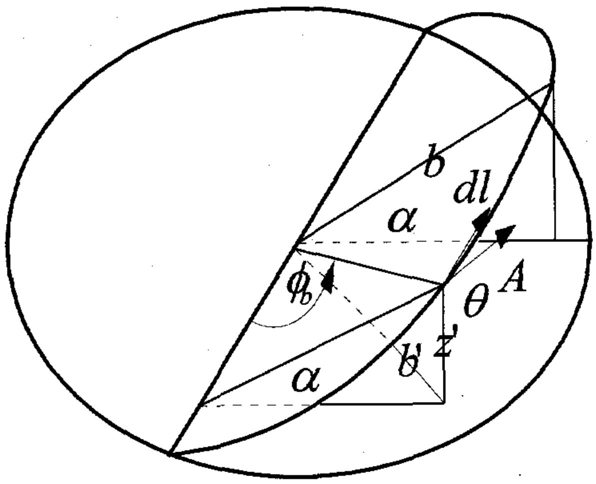


FIG. 4: Angular Displacement of PM Rings

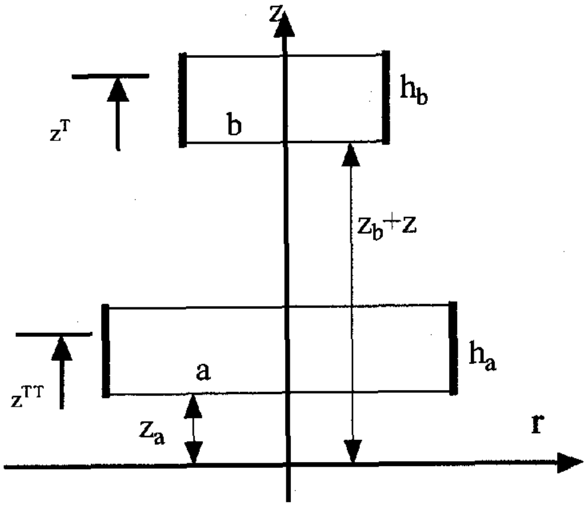


FIG. 2: Geometry of Two PM Rings Acting Upon Each Other

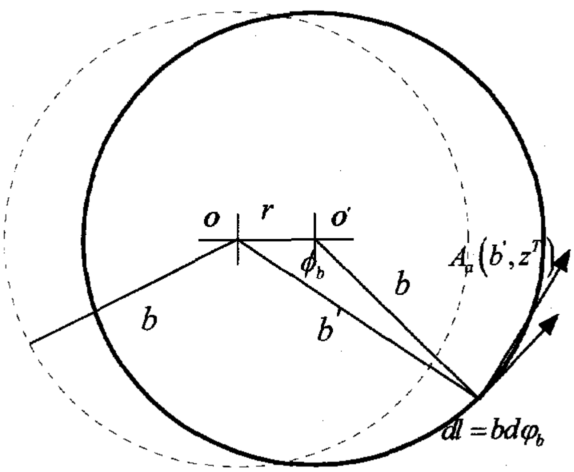


FIG.3: Radial Displacement of PM rings