CONTROL OF BALANCE BEAM WITH MAGNETIC BEARING: A LINEAR DESIGN APPROACH

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ABSTRACT

In the control of magnetic bearing systems, it is known that there is a conflict between performance enhancement and power-loss reduction, which is directly related to the biasing level. In this paper, we first propose a general framework to coordinate the transient performance and the steady state performance. Then we apply this framework to handle the conflict between fast convergence rate and low biasing level. Our study is based on a linear model which is obtained by using appropriate nonlinear current allocation strategy. Our theoretical results are verified on an experimental system, a beam balancing test rig, which consists of a beam free to rotate on a pivot at its center of mass, and stabilized by electromagnets located at both ends of the beam.
Keywords: Magnetic bearings, stabilization, current biasing, control performances

1 INTRODUCTION

This work is intended to develop a systematic design approach to the control of magnetic bearing systems by using a simple experimental setup at the University of Virginia. Active magnetic bearings (AMB) have several appealing advantages over traditional bearings, such as very low power-loss, very long life, elimination of oil supply, low weight, reduction of fire hazard, vibration control and diagnostic capability[1]. They have been untilized in a variety of rotating machines ranging from artificial heart pumps, compressors, high speed milling spindles to flywheel energy storage systems.

The experimental system to be studied in this paper is a beam balancing test rig (see Fig. 1). It consists of a beam free to rotate on a pivot at its center of mass, and stabilized by electromagnets located at both ends of the beam. This experiment mimics the dynamics of a single axis AMB system yet is quite simple from a mechanical viewpoint. It has attracted significant interest. Two invited sessions were organized at 2000 ACC[8, 9]. Participants of these sessions examined various aspects of this benchmark system. More recently, in [5], we attempted to characterize the relationship between several performances and the biasing level through numerical optimization method.

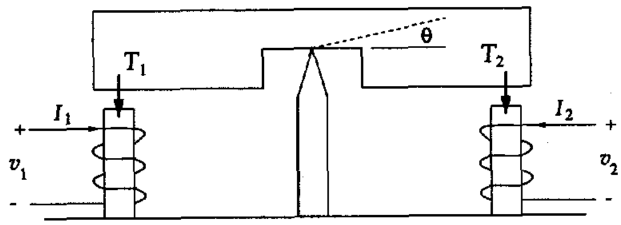

Figure 1: The beam balancing test rig.

The dynamics of the beam can be modeled by the following differential equation (see, e.g., [3]):

$$J\ddot{\theta} = -D\dot{\theta} + T_2 - T_1 + d, \qquad (1)$$

where the variable θ is the angle between the beam and the horizontal direction. T_1 and T_2 are the torques generated by the two electromagnets and d is disturbance due to unbalance or other unknown sources. The total torque provided by the electromagnets is $T := T_2 - T_1$. The system parameters are: J - the moment of mass, and D - system damping due to air and pivot friction.

The torques are determined by the currents I_1 and I_2 in the two circuit systems through the following relation:

$$T_1 = c_{t1}\left(\frac{g_0 I_1}{g_0+\theta}\right)^2, \quad T_2 = c_{t2}\left(\frac{g_0 I_2}{g_0-\theta}\right)^2,$$

where g_0 is the maximal angle which is reached when one end of the beam touches an electromagnet. c_{t1} and c_{t2} are constant parameters. In this work, we adopt the current mode, i.e., the control inputs are I_1 and I_2. For simplicity, we assume that $c_{t1} = c_{t2} = c_t$.

Different magnetic bearing systems can be modeled similarly to the above balance beam system. For example, [10] considered a rotor whose one dimensional position is controlled by a pair of electromagnets. The model in [10] is mathematically the same as the one in this paper. If we have several pairs of electromagnets, then the whole system can be broken into small components similar to the above model, possibly with some couplings.

Traditionally, a bias current I_b is introduced in the current mode such that $I_1 = I_b - I$ and $I_2 = I_b + I$, where I is used as a control input to produce a net torque. Using bias current makes the system easier to control but has the drawback of increasing power-loss. Much effort has been made toward reducing the bias or the power consumption (see, e.g., [2, 4, 7, 14, 15]). Recently, we made some effort in [5, 6] to reduce the power-loss by characterizing the relationship between the bias current and several performances. It was found out that decreasing bias current would reduce the achievable performances and slow down the convergence rate. In particular, it was shown in [6] that decreasing bias current would reduce the size of the largest possible stability region. These results were verified experimentally. For example, it is very easy to stabilize the beam with $I_b = 0.5A$, it is harder to stabilize it with $I_b = 0.2A$ and we never succeeded to do the stabilization with $I_b = 0.1A$.

In this paper, we propose a new design and control approach that will coordinate system performances and low power-loss (low biasing level). We first address a general problem of coordinating transient and steady state performances in Section 2. In Section 3, we apply the results in Section 2 to handle the conflict between fast convergence rate and low biasing level. We will replace the conventional linear relation $I_1 = I_b - I$ and $I_2 = I_b + I$ with a nonlinear one

$$I_1 = (I_b + I)\frac{g_0 + \theta}{g_0}, \quad I_2 = (I_b - I)\frac{g_0 - \theta}{g_0},$$

which results in an exact linear dynamical relation between θ and I and simplifies the design significantly. Experimental results show that this design approach is very effective. In particular, we successfully stabilized the beam with $I_b = 0.1A$. In fact, the beam will reach the balance position no matter what the initial condition is.

Notation: For a vector u, the infinity norm of u is $\|u\|_\infty = \max_i |u_i|$ and the 2−norm is $\|u\| = (\sum_i u_i^2)^{\frac{1}{2}}$. We use sat : $\mathbf{R}^m \to \mathbf{R}^m$ to denote the standard saturation function of appropriate dimensions. For $u \in \mathbf{R}^m$, the ith component of sat(u) is $\text{sign}(u_i)\min\{1, |u_i|\}$.

2 COORDINATION OF TRANSIENT AND STEADY STATE PERFORMANCES

As we will see later in this paper, the problem considered in this section is motivated by the control of magnetic bearing systems. It may also arise from other control systems. Here we formulate the problem into a more general framework.

2.1 Parameter-dependent Linear Systems

Consider a parameter-dependent linear system

$$\dot{x} = (A_0 + pA_1)x + (B_0 + pB_1)u, \ x \in \mathbf{R}^n, \ u \in \mathbf{R}^m, \tag{2}$$

subject to input saturation and state constraint:

$$\|u\|_\infty \le 1, \ \ x \in X_c,$$

where p is a scalar that can be adjusted on line. (The results in this section can be easily generalized to the case where $A = A_0 + p_1A_1 + p_2A_2 + \cdots$ and $B = B_0 + p_1B_1 + p_2B_2 + \cdots$. We only consider the scalar case for simplicity). The state constraint set X_c is generally some polytope determined by a matrix $G \in \mathbf{R}^{q\times n}$ as follows,

$$X_c = \{x \in \mathbf{R}^m : \ \|Gx\|_\infty \le 1\}.$$

When applied to the magnetic bearing system in this paper, p is the bias current I_b or the bias flux ϕ_b. It is known that large I_b is good for fast transient response and small I_b is necessary for low power loss. To achieve fast transient response as well as low power loss, we may try to change I_b, or the parameter p for a general system (2), according to the location of the state x. A straightforward solution seems to be using a p and a corresponding controller that is good for transient response when $\|x\| > c$ for some positive number c, and use another p and its corresponding controller that is good for steady state performance when $\|x\| \le c$. However, we must answer two questions: How to guarantee closed-loop stability and how to make the transition smooth? In this section, we will present a design scheme that would coordinate the transient and steady state performances.

Let p_t be the parameter p that is good for transient response and let p_s be the parameter p that is good for steady state. The main idea is to let p vary continuously between p_t and p_s, such that under the control of $u = F(p)x$, the trajectories starting from a given set of initial conditions will converge to the origin with satisfactory transient responses.

Suppose that all the possible initial conditions of the system (2) is in a bounded polytope, described by the convex hull of a set of points $x_{10}, x_{20}, \cdots x_{\ell 0} \in \mathbf{R}^n$. To state the stability and performance requirements for systems subject to state and input constraints, we need some notation.

For a general system $\dot{x} = f(x,t)$, the convergence rate can be defined in terms of a quadratic function $V(x) = x^{\mathrm{T}}Px$ for a positive definite matrix P ($P > 0$). Let $x(t), t \ge 0$ be a trajectory. The convergence rate along the trajectory is

$$\inf\left\{-\frac{\dot{V}(x(t))}{V(x(t))} : \ t \ge 0\right\}.$$

Consider a closed-loop linear system $\dot{x} = (A+BF)x$. The convergence rate is the largest positive number α such that

$$(A+BF)^{\mathrm{T}}P + P(A+BF) \leq -\alpha P. \quad (3)$$

For $P > 0$, denote

$$\mathcal{E}(P) := \{x \in \mathbf{R}^n : \ x^{\mathrm{T}}Px \leq 1\}.$$

For a feedback matrx $F \in \mathbf{R}^{m\times n}$, denote

$$\mathcal{L}(F) := \{x \in \mathbf{R}^n : \ \|Fx\|_\infty \leq 1\}.$$

2.2 On Maximizing the Convergence Rate

Consider a linear system

$$\dot{x} = Ax + Bu, \ x \in \mathbf{R}^n, \ u \in \mathbf{R}^m, \quad (4)$$

subject to input saturation and state constraint

$$\|u\|_\infty \leq 1, \quad \|Gx\|_\infty \leq 1.$$

We would like to maximize the convergence rate by using a linear state feedback $u = Fx$ that satisfies the control bound. To ensure that the control bound $\|Fx\|_\infty \leq 1$ and the state bound $\|Gx\|_\infty \leq 1$ are satisfied and the maximal convergence rate is achieved for the whole trajectory, we need to construct some invariant set inside the region $\mathcal{L}(F)\cap\mathcal{L}(G)$. If P satisfies (3) and $\mathcal{E}(P) \subset \mathcal{L}(F)$, then $\mathcal{E}(P)$ is an invariant set, which means that all the trajectories starting from inside it will stay inside. At this point, we need to ensure that the ellipsoid $\mathcal{E}(P)$ include all the initial conditions $x_{01}, x_{02}, x_{03}, \cdots$, which are the vertices of the given set of initial conditions. In summary, we need to find P and F such that $x_{01}, x_{02}, x_{03}, \cdots \in \mathcal{E}(P) \subset \mathcal{L}(F)\cap\mathcal{L}(G)$ and the convergence rate α is maximized. This can be formulated as the following optimization problem:

$$\begin{aligned} &\sup_{P>0,F} \ \alpha && (5)\\ \text{s.t.} \ &a) \ (A+BF)^{\mathrm{T}}P + P(A+BF) \leq -\alpha P\\ &b) \ \mathcal{E}(P) \subset \mathcal{L}(F)\cap\mathcal{L}(G)\\ &c) \ x_{01}, x_{02}, x_{03}, \cdots \in \mathcal{E}(P). \end{aligned}$$

The above optimization problem can be exactly transformed into an LMI problem and be solved efficiently.

2.3 Coordination of Transient Performance and Steady State Performance

Our controller design is based on the following result:

Proposition 1 *Consider the closed-loop system*

$$\dot{x} = (A_0 + pA_1)x + (B_0 + pB_1)Fx. \quad (6)$$

Suppose that p is a time varying parameter that takes value between p_t and p_s. Let P be a positive-definite matrix satisfying

$$\begin{aligned} &(A_0+B_0F)^{\mathrm{T}}P+P(A_0+B_0F)\\ &\quad +p_t(A_1+B_1F)^{\mathrm{T}}P+p_tP(A_1+B_1F) \leq -\alpha_t P \quad (7)\\ &(A_0+B_0F)^{\mathrm{T}}P+P(A_0+B_0F)\\ &\quad +p_s(A_1+B_1F)^{\mathrm{T}}P+p_sP(A_1+B_1F) \leq -\alpha_s P \quad (8) \end{aligned}$$

for some $\alpha_t, \alpha_s > 0$, then all the trajectories will converge to the origin with a convergence rate α between α_s and α_t.

If F and P satisfy the conditions in Proposition 1, then the stability of the closed-loop system (6) is ensured no matter how we change p between p_t and p_s. For good steady state performance, we should have $p \to p_s$ as $\|x\| \to 0$. For fast convergence rate, we need to set p close to p_t when x is away from the origin. For smooth transition, we need p to change continuously. In view of these points, we propose the following function of p

$$p = (1-\lambda)p_s + \lambda p_t, \quad \lambda = \mathrm{sat}(k\|x\|),$$

where k is used to adjust the transient response. For those x where $k\|x\| \geq 1$, we have $\lambda = 1$ and $p = p_t$. As $x \to 0$, $\lambda \to 0$ and $p \to p_s$. We note that k should not be too large due to the possible dynamics in p.

One design approach based on Proposition 1 is to fix α_s and maximize α_t. Of course, we also need to ensure that $x_{01}, x_{02}, x_{03}, \cdots \in \mathcal{E}(P) \subset \mathcal{L}(F)\cap\mathcal{L}(G)$. The optimization problem can be stated as follows:

$$\begin{aligned} &\sup_{P>0,F} \ \alpha_t && (9)\\ \text{s.t.} \ &a) \ (7), \ (8)\\ &b) \ \mathcal{E}(P) \subset \mathcal{L}(F)\cap\mathcal{L}(G)\\ &c) \ x_{01}, x_{02}, x_{03}, \cdots \in \mathcal{E}(P) \end{aligned}$$

In a special situation where $B_1 = 0$, we can design a parameter dependent feedback law for further improvement of the transient response.

Proposition 2 *Consider the open-loop system*

$$\dot{x} = (A_0 + pA_1)x + B_0u. \quad (10)$$

Suppose that p is a time varying parameter that takes value between p_s and p_t. Let P be a positive definite matrix and $F_t, F_s \in \mathbf{R}^{m\times n}$ be two feedback matrices such that

$$\begin{aligned} &(A_0 + B_0F_t)^{\mathrm{T}}P + P(A_0 + B_0F_t)\\ &\qquad +p_tA_1^{\mathrm{T}}P + p_tPA_1 \leq -\alpha_t P \quad (11)\\ &(A_0 + B_0F_s)^{\mathrm{T}}P + P(A_0 + B_0F_s)\\ &\qquad +p_sA_1^{\mathrm{T}}P + p_sPA_1 \leq -\alpha_s P \quad (12) \end{aligned}$$

for some $\alpha_t, \alpha_s > 0$. Denote $H_t = F_t P^{-1}, H_s = F_s P^{-1}$ and $F(p) = (\frac{p-p_s}{p_t-p_s} H_t + \frac{p_t-p}{p_t-p_s} H_s)P$. Then for the closed-loop system

$$\dot{x} = (A_0 + pA_1)x + B_0 F(p)x, \qquad (13)$$

all the trajectories will converge to the origin with a convergence rate α between α_s and α_t.

One design approach based on Proposition 2 is also to fix α_s and maximize α_t. Since we use different feedback matrices F_s and F_t for p_s and p_t, it is expected that the maximal α_t will be greater than the case of using a constant F. The optimization problem corresponding to Proposition 2 can be stated as follows:

$$\sup_{P>0,F} \alpha_t \qquad (14)$$
$$\text{s.t.} \quad a) \ (11), \ (12)$$
$$b) \ \mathcal{E}(P) \subset \mathcal{L}(F) \cap \mathcal{L}(G)$$
$$c) \ x_{01}, x_{02}, x_{03}, \cdots \in \mathcal{E}(P)$$

3 THE LINEAR DESIGN APPROACH

The dynamics of the beam balancing test rig under the current mode can be modeled by the following differential equation,

$$J\ddot{\theta} = -D\dot{\theta} + c_t \left(\left(\frac{g_0 I_2}{g_0 - \theta} \right)^2 - \left(\frac{g_0 I_1}{g_0 + \theta} \right)^2 \right), \qquad (15)$$

where g_0 is the maximal angular displacement which is reached when one end of the beam touchs the electromagnets. So we have $|\theta| \leq g_0$. In the current mode, we assume that I_1 and I_2 are the control inputs that can be exactly generated.

3.1 A Conventional Current Biasing Strategy

In (15), the currents appear in the form of I_1^2 and I_2^2, which are highly nonlinear for a control system. A conventional way to reduce this nonlinearity is to introduce a bias current I_b and let I_1 and I_2 operate symmetrically around I_b, i.e.,

$$I_1 = I_b + I, \quad I_2 = I_b - I, \qquad (16)$$

where I is used as a control input that produces a net torque on the beam. With I_1 and I_2 determined from (16), the dynamical relation between the input I and the output θ is,

$$J\ddot{\theta} = -D\dot{\theta} + c_t \left(\left(\frac{g_0(I_b - I)}{g_0 - \theta} \right)^2 - \left(\frac{g_0(I_b + I)}{g_0 + \theta} \right)^2 \right). \qquad (17)$$

Linearizing the system at $(\theta, \dot{\theta}) = (0, 0)$, we obtain

$$\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{4c_t I_b^2}{J g_0} & -\frac{D}{J} \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{4c_t I_b}{J} \end{bmatrix} I. \qquad (18)$$

Denote $x = [\ \theta \quad \dot{\theta}\ ]^T$ and

$$A_L = \begin{bmatrix} 0 & 1 \\ \frac{4c_t I_b^2}{J g_0} & -\frac{D}{J} \end{bmatrix}, B_L = \begin{bmatrix} 0 \\ \frac{-4c_t I_b}{J} \end{bmatrix},$$

then $\dot{x} = A_L x + B_L I$. This linearized model approximates the nonlinear system (17) very well when θ is close to zero. When θ is close to g_0, the nonlinearity gets stronger, which usually cause the beam to stick to one of the electromagnets. We notice that the linearized system (18) has an unstable pole since $\det(A_L) < 0$. As we have discussed in [6], under the input constraint $|I| \leq I_b$ and the state constraint $|\theta| \leq g_0$, the largest possible stability region that can be achieved for system (18) is a strip (see Fig. 2 for the largest possible stability regions under different bias currents $I_b = 0.1, 0.2$ and $0.5A$). The feedback laws that actually achieve the largest stability region have the following form:

$$I = I_b \text{sat}(\gamma F_0 x), \quad \gamma > 1,$$

where

$$F_0 = \frac{J}{4c_t I_b^2} [\ \lambda_1 \lambda_2 \quad -\lambda_1\ ] \qquad (19)$$

and $\lambda_1 > 0, \lambda_2 < 0$ are the eigenvalues of A_L.

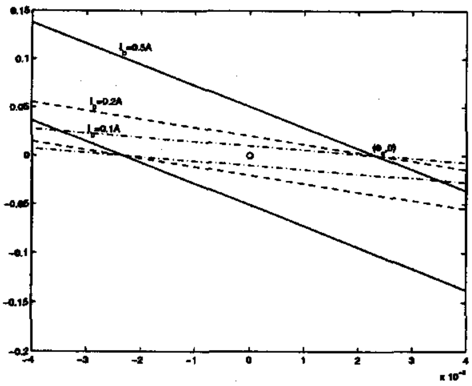

Figure 2: The largest possible stability region under different bias currents

As we can see from Fig. 2, the stability regions grow larger when we increase the bias current. With $I_b = 0.1A$, the stability region is a narrow strip. This makes it very difficult to stabilize the beam at the balance position since it is hard to set an initial state exactly inside the narrow strip. Besides, a small disturbance will drive the state outside of the strip even if it is already at the balance position. Actually, we have never been successful in balancing the beam with $I_b = 0.1A$. In what follows, we will present a new control approach which will overcome the difficulty in stabilization by using small bias current.

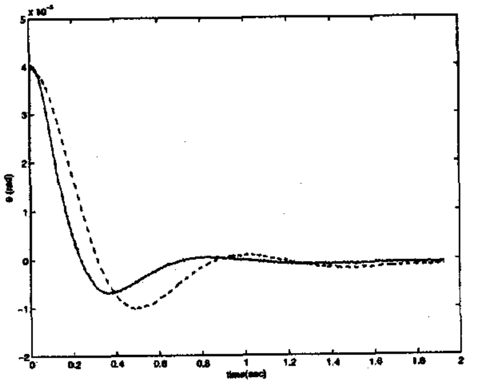

Figure 4: The time resposes of θ under constant I_b and varying I_b

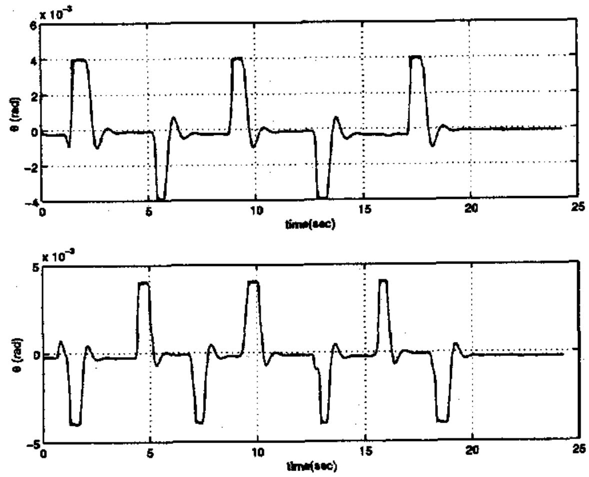

Figure 5: The time resposes of θ

4 CONCLUSIONS

We proposed a general framework to coordinate the transient performances and steady state performances. By applying this framework to the control of magnetic bearing systems, we developed a systematic design approach to handle the conflict between the convergence rate and low bias current. Experimental results confirmed the effectiveness of our proposed design approach.

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3.2 An Exact Linear Model

The simple linear relation between I_1, I_2 and I in (16) results in a nonlinear dynamical relation (17) between I and θ. This nonlinearity becomes severe when θ is close to $\pm g_0$. When θ is very close to $\pm g_0$, if the currents are nonzero, then the attractive force between the beam and the electromagnet at one end will be very large, causing this end to stick to the electromagnet. To overcome this situation, we should make I_1 and I_2 proportional to the air gap, or proportional to $g_0 \pm \theta$. Specifically, we propose the following relation between I_1, I_2 and I,

$$I_1 = (I_b + I)\frac{g_0 + \theta}{g_0}, \quad I_2 = (I_b - I)\frac{g_0 - \theta}{g_0}. \quad (20)$$

In this way, the dynamical relation between I and θ is simply

$$J\ddot{\theta} = -D\dot{\theta} - 4c_t I_b I. \quad (21)$$

or,

$$\begin{aligned} \begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ 0 & -\frac{D}{J} \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + I_b \begin{bmatrix} 0 \\ -\frac{4c_t}{J} \end{bmatrix} I \\ &=: A_0 x + I_b B_1 I, \quad (22) \end{aligned}$$

This system is not only linear, but also maginally stable with one open-loop pole at 0 and another one at $-4c_t I_b/J$. We note that there is no problem in generating the currents given by (20) since the values of $\frac{g_0 \pm \theta}{g_0}$ are between 0 and 2 ($|\theta| \le g_0$).

In what follows, we will present some controller design schemes following the framework in Section 2.

3.3 Controller Design

Ideally, we would like that the fluxes of the electromagnets are below saturation level. This can be guaranteed by restrict the magnitudes of the currents I_1 and I_2 to be less than some number I_M: $|I_1|, |I_2| \le I_M$. Since $|(g_0 \pm \theta)/g_0| \le 2$, we need to restrict $|I| \le I_M/2 - I_b$. Consider a linear controller of the form $I = (I_M/2 - I_b)Fx$. The closed-loop system is

$$\dot{x} = A_0 x + B_1 I_b (I_M/2 - I_b) F x.$$

To avoid flux saturation, we need to restrict $|Fx| \le 1$. Let $p = I_b(I_M/2 - I_b)$. The maximal p is obtained at $I_b = I_M/4$ with the value of $p_t = I_M^2/16$. Since large I_b will produce large power loss, we would like to choose $I_b \le I_M/4$.

Using the method in Section 2, it is easy to design F to maximize the convergence rate for a fixed I_b. We can also design a feedback matrix F which produces a fast convergence rate for large I_b and guarantees certain convergence rate for small I_b.

3.4 Experimental Results

We consider the balance beam test rig at University of Virginia. The parameters of the experimental system are

$$J = 0.0948kgm, \quad g_0 = 0.004\text{rad}$$

$$c_t = 0.1384kgm/A^2, \quad I_M = 2A.$$

In the steady state, we choose $I_b = 0.1A$. Following Section 2's method, we designed the following feedback law

$$I = (1 - I_b)Fx, \quad F = [171.1263 \ 21.8343], \quad (23)$$

and I_b is chosen as a function of the state x:

$$I_b = 0.1 + 0.4\text{sat}(10(\theta^2 + 0.001\dot{\theta}^2)/0.004^2). \quad (24)$$

The maximal I_b is $0.5A$.

We compared the transient responses under the control of (23) by using a varying bias current (24) and a constant bias current $I_b = 0.1$ with both simulation and experiment. In simulation, the initial state is taken as $x = \begin{bmatrix} 0.0039 \\ 0 \end{bmatrix}$. The simulation results are shown in Fig. 3, where the solid curve is the time response under a varying bias and the dashed curve is the time response under a constant bias $I_b = 0.1$. Fig. 3 shows clearly that the transient response by using a varying I_b is much better than that by using a constant I_b.

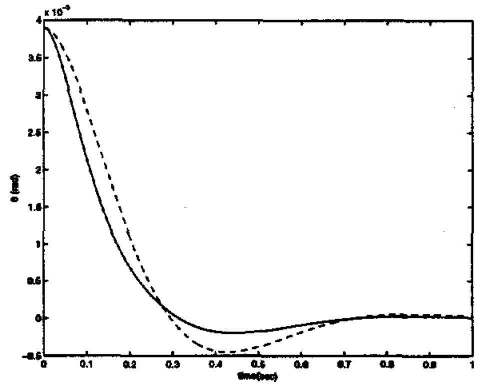

Figure 3: The time resposes of θ under constant I_b and varying I_b

The experimental results are shown in Fig. 4, where the solid curve is the time response under a varying I_b and the dashed curve is that under a constant I_b. The initial conditions are both chosen as $x = \begin{bmatrix} 0.004 \\ 0 \end{bmatrix}$. The two time responses in Fig. 5 are obtained by pushing the beam to touch one of the electromagnets several times after the balance position had been reached. We see that the beam always go back to the balance position. This was impossible to do with a bias current $I_b = 0.1A$ for the linearized system (18).