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APPLICATION OF ZERO-POWER MAGNETIC SUSPENSION TO VIBRATION ISOLATION SYSTEM

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ABSTRACT

An active vibration isolation system using zero-power magnetic suspension is studied both analytically and experimentally. Since a zero-power system behaves as if it has a negative stiffness, connecting it with a normal spring in series can generate infinite stiffness against disturbances on the isolation table. The fundamental characteristics of the system are clarified with a basic model. A single-axis apparatus is manufactured for experimental study. The experimental results support the analytical predictions well.

INTRODUCTION

For microvibration isolation, high performance in suppressing the effects of direct disturbance is required in addition to sufficient isolation from ground vibration. However, a trade-off between them is inevitable in passive-type vibration isolation systems because higher stiffness of suspension is better for the former while lower stiffness is better for the latter.

They can be made compatible by applying active control technology [1]. However, conventional active-type vibration isolation systems need high-performance sensors such as servo-type accelerometers so that they tend to be costly, roughly speaking, nearly ten times as expensive as passive ones. This restricts the application fields of active vibration isolation system.

Mizuno has proposed a unique approach to breaking through the trade-off [2, 3]. The approach utilizes the serial connection of two springs, one of which has negative stiffness (negative spring) and the other has positive stiffness (normal spring). When the absolute value of negative stiffness equals the latter, the total stiffness becomes infinite. This research applies this idea to vibration isolation systems, and uses zero-power magnetic suspension for realizing negative stiffness. The fundamental characteristics of the developed system are studied both analytically and experimentally.

VIBRATION ISOLATION SYSTEM

Serial Connection of Two Springs

It will be shown that infinite stiffness can be realized by connecting a normal spring with a spring that has negative stiffness in series. When two springs with spring constants of k_1 and k_2 are connected in series as shown by Fig.1, the total stiffness k_c is given by

$$k_c = \frac{k_1 k_2}{k_1 + k_2}. \tag{1}$$

This equation shows that the total stiffness becomes lower than that of each spring when normal springs are connected. However, if one of the springs has negative

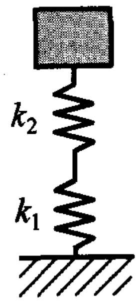


FIGURE 1: Series springs

stiffness that satisfies

$$k_1 = -k_2, \tag{2}$$

the resultant stiffness becomes infinite, that is

$$|k_c| = \infty. \tag{3}$$

This research applies this principle of generating high stiffness against direct disturbance to vibration isolation systems.

Zero-power Magnetic Suspension

Systems containing passive elements with negative stiffness are generally unstable because they generate force in the direction opposite to restoring. Active control is, therefore, necessary to realize negative stiffness without instability. Zero-power control is used in this research.

Model. Figure 2 shows a single-degree-of-freedom-of-motion model for describing zero-power magnetic suspension [4]. The suspended object with mass of m is assumed to move only in the vertical direction translationally. The equation of motion is given by

$$m\ddot{x}(t) = k_s x(t) + k_i i(t) + f_d(t), \tag{4}$$

where

x : displacement of the suspended object,
k_s : gap-force coefficient of the magnet,
k_i : current-force coefficient of the magnet,
i : control current,
f_d : disturbance acting on the suspended object.

Zero-Power Controller. The zero-power control operates to accomplish

$$\lim_{t\to\infty} i(t) = 0 \quad \textit{for stepwise disturbances.} \tag{5}$$

The controller achieving the control objective (5) is generally represented by [4]

$$I(s) = -\frac{s\tilde{h}(s)}{g(s)} X(s), \tag{6}$$

where $g(s)$ and $h(s)$ are coprime polynomials in s and selected for the closed-loop system to be stable.

The minimal-order compensator achieving zero-power control and assigning the closed-loop poles arbitrarily can be represented as [4]

$$I(s) = -\frac{s(\tilde{h}_2 s + \tilde{h}_1)}{s^2 + g_1 s + g_0 s} X(s), \tag{7}$$

The poles to be assigned being given, the coefficients g_i's and h_i's are determined uniquely.

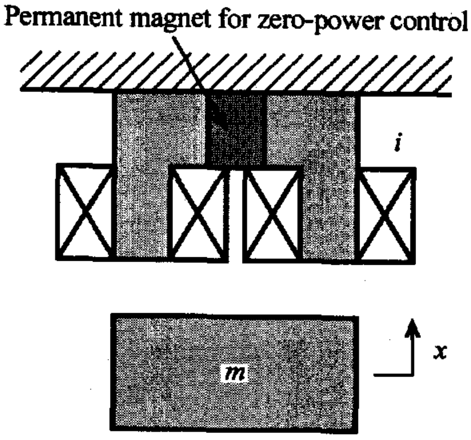


FIGURE 2: Basic model of a zero-power magnetic suspension system

Negative Stiffness. When a constant force F_0 is applied to the suspended object, the suspended object is maintained at a position satisfying

$$0 = k_s x(\infty) + k_i i(\infty) + F_0, \tag{8}$$

in the steady states. In the zero-power control system, the coil current converges to zero, that is,

$$i(\infty) = 0. \tag{9}$$

Therefore,

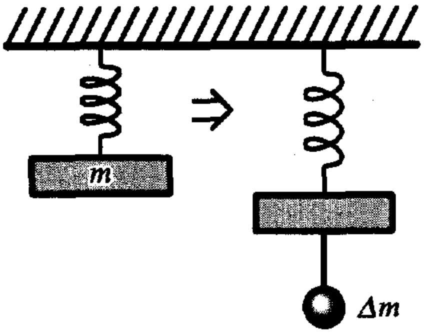
$$x(\infty) = -\frac{F_0}{k_s}. \tag{10}$$

The negative sign appearing in the right-hand side verifies that the new equilibrium position is in the direction opposite to the applied force. It indicates that the zero-power control system behaves as if it has negative stiffness. When an external force is applied to the mass in common mass-spring systems, the mass moves to the direction of the applied force as shown in Fig.3a. In the zero-power controlled system, the suspended object moves in the opposite direction as shown in Fig.3b.

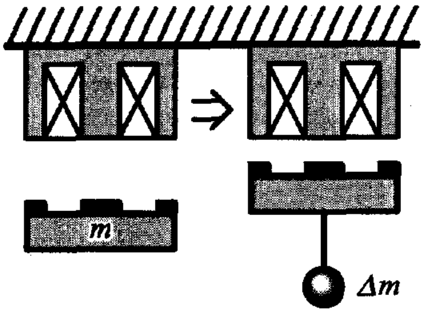
Configuration of Vibration Isolation System

A schematic drawing of the proposed vibration isolation system is shown in Fig.4 [2, 3]. A middle table m_1 is connected to the base through a spring k_1 and a damper c_1 that work as a conventional vibration isolator. An electromagnet for zero-power magnetic suspension is fixed to the middle table. The part of an isolation table m_2 facing the electromagnet is made of soft iron material.

This system can reduce vibration transmitted from ground by setting k_1 small and at the same time have



(a) Normal spring



(b) Zero-power magnetic suspension system

FIGURE 3: Comparison between normal spring and zero-power magnetic suspension system

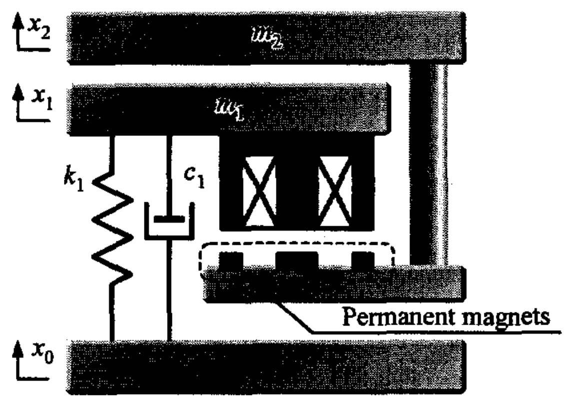


FIGURE 4: Schematic drawing of the proposed vibration isolation system using zero-power magnetic suspension

infinite stiffness against direct disturbance by setting the amplitude of negative stiffness equal to k_1.

The behavior of this system for direct disturbance acting on the isolation table is explained as follows. Assuming that the table is subject to a downward force, the gap between the electromagnet and the table becomes smaller due to zero-power control; in other words, the table would move upwards if the middle table were fixed. Meanwhile, the middle table moves downwards because of the increase of the electromagnetic force. When the decrease of the gap is cancelled by the downward displacement of the middle table, the isolation table is maintained at the same position as before the downward force is active.

In the following, this property will be shown first analytically and then experimentally.

ANALYSIS

Basic Equations

The translation motions in the vertical direction will be treated here. The equations of motion are

$$m_1\ddot{x}_1 = -k_1(x_1 - x_0) - c_1(\dot{x}_1 - \dot{x}_0) - f_e, \tag{11}$$

$$m_2\ddot{x}_2 = f_e + f_d, \tag{12}$$

where

x_0 : displacement of the floor,
x_1 : displacement of the middle table,
x_2 : displacement of the isolation table,
f_e : control force produced by the electromagnet,
f_d : direct disturbance acting on the isolation table.

The control force is approximately represented by

$$f_e = k_s(x_2 - x_1) + k_i i. \tag{13}$$

According to the discussion in the section on Zero-Power Controller, the control current achieving the zero-power control is generally represented by

$$I(s) = -c_2(s)s(X_2(s) - X_1(s)), \tag{14}$$

where

$$c_2(s) = \frac{\tilde{h}(s)}{g(s)}. \tag{15}$$

Response to Direct Disturbance

It assumed for simplicity that the initial values are zero. From (11) to (15), we get

$$X_1(s) = \frac{(c_1 s + k_1)t_2(s)}{t_c(s)} X_0(s) + \frac{k_i c_2(s)s - k_s}{t_c(s)} F_d(s). \tag{16}$$

$$X_2(s) = \frac{(c_1 s + k_1)(k_i c_2(s)s - k_s)}{t_c(s)} X_0(s) + \frac{t_1(s) + k_i c_2(s)s - k_s}{t_c(s)} F_d(s). \tag{17}$$

where

$$t_1(s) = m_1 s^2 + c_1 s + k_1, \tag{18}$$

$$t_2(s) = m_2 s^2 + k_i c_2(s) s - k_s, \tag{19}$$

$$t_c(s) = t_1(s) t_2(s) + m_2 s^2 (k_i c_2(s) s - k_s). \tag{20}$$

To estimate the stiffness for direct disturbance, the direct disturbance f_d is assumed to be stepwise, that is,

$$F_d = \frac{F_0}{s} \quad (F_0 : \text{const}). \tag{21}$$

When the vibration of the floor is neglected, the steady-state displacement of the table is obtained as

$$\frac{x_2(\infty)}{F_0} = \lim_{s \to 0} \frac{t_1(s) + k_i c_2(s) s - k_s}{t_c(s)}$$

$$= \frac{1}{k_1} - \frac{1}{k_s}. \tag{22}$$

Therefore, when

$$k_1 = k_s, \tag{23}$$

the following equation is obtained

$$\frac{x_2(\infty)}{F_0} = 0. \tag{24}$$

Equation (24) shows that the suspension system between the isolation table and the floor has infinite stiffness because there is no steady-state deflection even in the presence of stepwise disturbances acting on the table.

The steady-state displacement of the middle table is given by

$$\frac{x_1(\infty)}{F_0} = \frac{1}{k_1}, \tag{25}$$

Equation (25) verifies that the middle table moves downward when a downward force acts on the isolation table ($F_0 < 0$). The gap between the electromagnet and the isolation table is given by

$$\frac{x_1(\infty) - x_2(\infty)}{F_0} = \frac{1}{k_s}. \tag{26}$$

Equation (26) indicates that the gap decreases when $F_0 < 0$. These results support well the predictions on the behavior of the proposed vibration isolation system described in the previous chapter.

EXPERIMENT

Single-Axis Experimental Setup

Figures 5 and 6 are a schematic diagram and a photo of the developed apparatus for experimental study. The middle and isolation tables are guided to be in translation in the vertical direction by linear bearings. An electromagnet is fixed to the middle table corresponding to m_1 in Fig.4. Permanent magnets providing bias flux are made of NdFeB materials. They are built in the target iron of the isolation table.

The relative displacement of the isolation table to the middle table is detected with an eddy-current gap sensor with a resolution of 1μm. The output signal is inputted to a DSP-based digital controller. A second order compensator given by eq.(7) is implemented for realizing zero-power control. The control period is 100μs.

The relative displacement of the middle table to the base is also detected with another gap sensor for monitoring.

Experimental Results

First, the static characteristic of the implemented zero-power control system is measured. The middle table is fixed during measurement. Figure 7 shows the results. The upward displacement of the isolation table is plotted to the mass of weights added to the isolation table. The mass of the table m is set to be

(a)700[g], (b)1310[g], (c)1700[g].

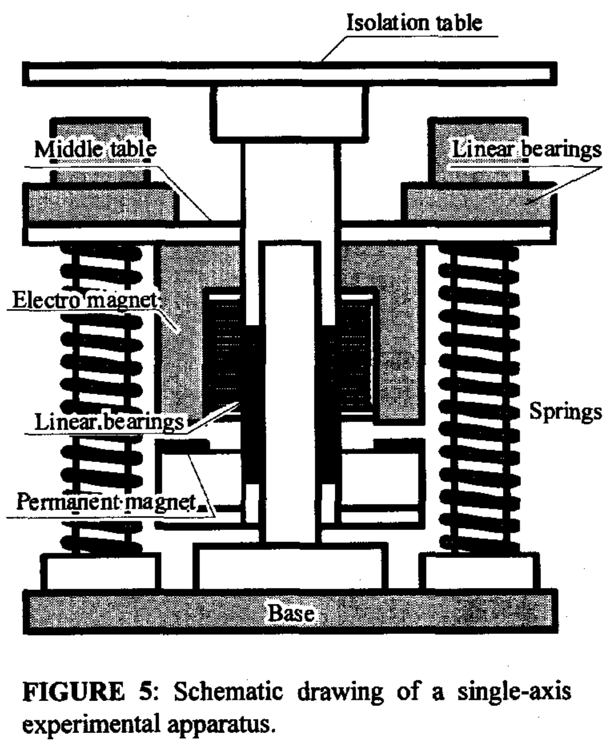


FIGURE 5: Schematic drawing of a single-axis experimental apparatus.

The force produced by adding weights is *downward* while the displacement of the table is *upwards*. It verifies that the static stiffness of the magnetic suspension system is *negative*. The relation between displacement and external force is linear when added weights are small. It is observed that the magnitude of stiffness becomes larger as the mass of the table increases. It is due to the decrease of gap between the electromagnet and the suspended object.

Next, the static response of the isolation table to direct disturbance is measured. The middle table, suspended by springs, is released to move freely, and weights are added to the isolation table. Figure 8 shows the relative displacement of the isolation table to the base. The stiffness of suspension of the middle table k_1 and the magnitude of the negative stiffness of magnetic suspension k_s are also plotted. It is found that the displacement of the isolation table $|x_2|$ is very small when $k_1 \cong k_s$ ($m \cong 1380[g]$). The estimated stiffness between the isolation table and the base is shown in Fig.9. The maximum value is 490[kN/m] and about 70 times the values of k_1 and k_s ($\cong 6.8$ [kN/m]).

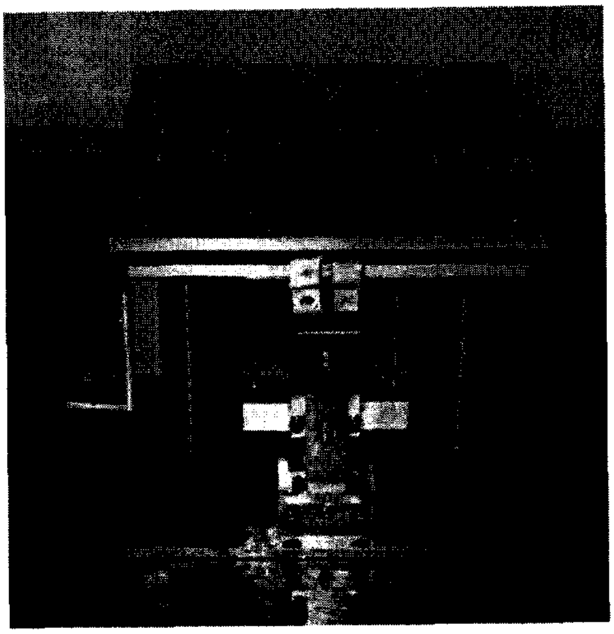


FIGURE 6: Photograph of the single-axis experimental apparatus.

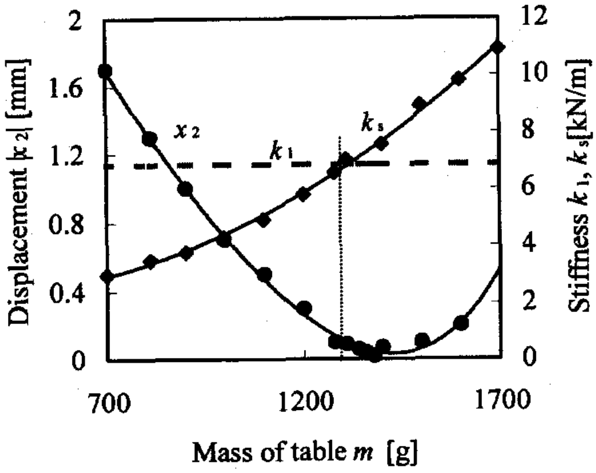


FIGURE 8: Displacement of the isolation table, the stiffness of the mechanical spring and the negative stiffness.

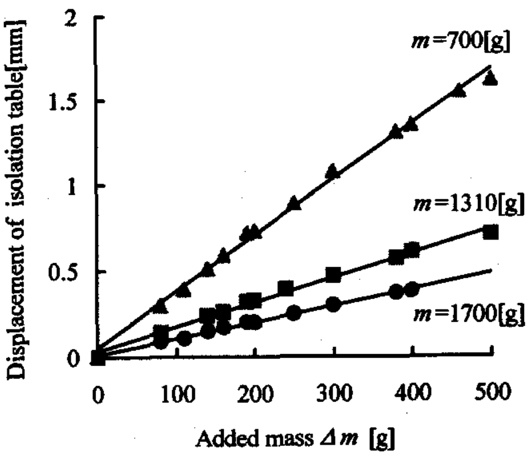


FIGURE 7: Measurement of the negative stiffness of the zero-power magnetic suspension system.

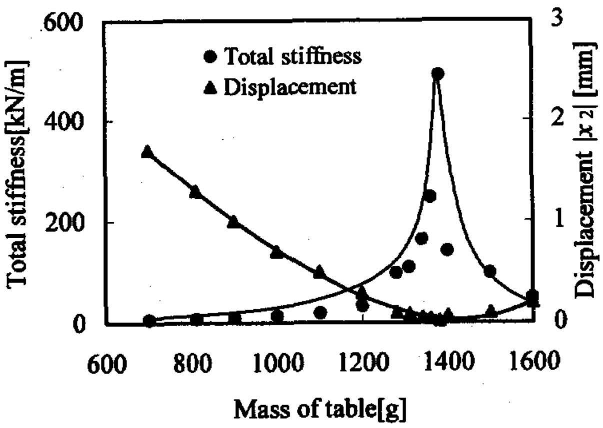


FIGURE 9: Stiffness of the vibration isolation system to static direct disturbance.

CONCLUSIONS

The vibration isolation system using zero-power magnetic suspension was studied both theoretically and experimentally. The serial connection of a normal spring with a zero-power magnetic suspension system enables an isolation system to have infinite stiffness against direct disturbance acting on the isolation table theoretically. This characteristic was confirmed experimentally with the manufactured single-axis experimental apparatus.

This paper focused on the static characteristics of the proposed vibration isolation system. A three-axis apparatus is developed for experimental study on dynamic direct disturbance [5]. It actively controls three degrees of freedom of motions in the vertical direction of the isolation table by the proposed method. The control method of realizing infinite stiffness is generalized to be applicable to systems in which a hybrid magnet in suspension is replaced by a linear actuator [6].

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