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ON THE USE OF SCHROEDER PHASED HARMONIC SEQUENCES IN MULTI-FREQUENCY VIBRATION CONTROL OF FLEXIBLE ROTOR/MAGNETIC BEARING SYSTEMS

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ABSTRACT
Control strategies based on synchronous vibration cancellation have been successfully applied to rotor-bearing systems incorporating magnetic bearings. In this paper, such a control strategy is extended to provide attenuation of sub and super-harmonic vibration components resulting from multi-frequency excitation of the system. System identification is achieved using test signals applied through the magnetic bearings that consist of multiple frequency components. These components are superimposed with appropriate selection of the relative phasing so that the possibility of the rotor displacement exceeding clearances or the bearing force reaching saturation is minimised. The selection of phases is based on a Schroeder Phased Harmonic Sequence, which minimises peak signal values. The scheme for multi-frequency identification and vibration control is applied to an experimental flexible rotor supported by two magnetic bearings. An electromagnetic shaker connected to the foundation provides externally generated multi-frequency disturbances. The results demonstrate the effectiveness of the controller in suppressing multi-frequency vibrations.

INTRODUCTION
Magnetic bearings have a limited force capacity, and cannot match the high stiffness properties of oil-film bearings when film thickness becomes small. It is common practice to incorporate auxiliary bearings or bushes to prevent damage to the rotor and stator laminations when large rotor orbits occur. However, when the rotor makes contact with the auxiliary bearing, the resulting non-linear dynamic response can be destructive, and the system may need to be shut down to prevent damage. Furthermore, the auxiliary bearings often have to be replaced after only a few rotor contact events. Therefore, it is important to utilise fully the active control capability of magnetic bearings to prevent rotor contact with auxiliary bearings, particularly when operating conditions change or faults develop.

Various control methods have been developed that utilise real-time analysis of measured vibration signals to quantify synchronous vibration components. Optimally chosen control inputs are then used to cancel these vibration components through an adaptive or closed loop algorithm [1-7]. With this type of controller, the frequency for control update or adaptation is usually an integer multiple of the synchronous frequency.

This type of strategy was first considered in open loop/adaptive form for synchronous vibration control of flexible rotor-bearing systems incorporating magnetic bearings [1,2]. The approach is simple to implement and does not affect rotor stability. It requires no prior knowledge of system dynamics, as system response behaviour is automatically identified prior to control optimisation. The technique is often implemented with additional feedback control of the magnetic bearings to provide stable rotor support, shaft centring and alignment. Various safety features for fault detection and fault tolerance have been introduced recently [8,9] to improve the robustness to faulty signals or degradation of operational capabilities within the magnetic bearings.

In many environments, rotor systems may be subject to non-synchronous excitation. Typical examples can be found in rotating machine systems in ships, aircraft and sea-platforms, where non-synchronous vibrations arise from external sources and are transmitted through machine foundations. This paper introduces an extension to the synchronous control technique that will allow optimal attenuation of measured vibration in safety-critical rotating systems subject to quasi-stationary disturbance signals with multi-frequency components. The control scheme is devised to include additional on-line identification, monitoring and adaptation features.

In synchronous vibration control applications a sinusoidal synchronous test signal can be applied through the magnetic bearings in order to identify the influence coefficients that relate measured synchronous vibration components to control inputs. To achieve multi-frequency vibration control for a particular running speed, the influence coefficients must be identified separately for all the excitation frequencies. If this is done in a sequential manner, i.e. by applying a test signal at each frequency in turn, long test times may result in an unacceptably slow controller response: by the time the controller has adapted it may be too late to prevent the rotor contacting the auxiliary bearings. However, if test signals with different frequency components are superimposed then the resulting signal may cause magnetic bearing force saturation or unacceptably large rotor vibration. Therefore, an appropriate selection of the relative phasing of each component of the test signal is required to obtain the lowest possible peak value signal. The Schroeder Phased Harmonic Sequence (SPHS) can achieve this, and is a very simple signal to construct.

In this experimental study a number of parallel control loops are used to attenuate measured rotor vibration components of a flexible rotor supported by two radial magnetic bearings. On-line identification of the speed-dependant multi-frequency influence coefficients is achieved using a test signal derived from a Schroeder Phased Harmonic Sequence. An electromagnetic shaker connected to the foundation provides externally generated multi-frequency disturbances.

CONTROL STRATEGY

Consider the linearised dynamics of a flexible rotor – bearing system, represented through a frequency response model appropriate to periodic multi-frequency disturbances:

$$\mathbf{Q}(\Omega, j\omega) = \mathbf{Q}_0(\Omega, j\omega) + \mathbf{R}(\Omega, j\omega)\mathbf{U}(j\omega) \quad (1)$$

The vectors $\mathbf{Q}_0$ and $\mathbf{Q}$ are the spectral amplitudes of the measured rotor displacements at a rotational speed Ω, with and without the application of the control forces respectively. The control force vector, $\mathbf{u}(t)$, has a corresponding spectral amplitude vector $\mathbf{U}(j\omega)$. $\mathbf{R}$ is the rotational speed dependant partial receptance matrix. If the control force vector is synthesised from a number of discrete frequency sinusoidal components

$$\mathbf{u}(t) = \text{Re}\left\{\sum_{n=1}^{N} \mathbf{U}_n e^{j\omega_n t}\right\} \quad (2)$$

then

$$\mathbf{Q}_n = \mathbf{R}_n\mathbf{U}_n + \mathbf{Q}_0(\Omega, j\omega_n) \quad (3)$$

where $\mathbf{Q}_n = \mathbf{Q}(\Omega, j\omega_n)$ and $\mathbf{R}_n = \mathbf{R}(\Omega, j\omega_n)$. If the control input is chosen to minimise the sum of squares of the components $\mathbf{Q}_n$, the optimum control input vectors $\hat{\mathbf{U}}_n$ can be calculated from the following expression [1].

$$\hat{\mathbf{U}}_n = -(\mathbf{R}_n^T\mathbf{R})\mathbf{R}_n^T\mathbf{Q}_0(\Omega, j\omega_n) = \mathbf{H}_n\mathbf{Q}_0(\Omega, j\omega_n), \quad n = 1 \ldots N \quad (4)$$

Optimal control forces can be synthesised on-line through a closed loop algorithm, which will respond continuously to changes in disturbance amplitudes [11]:

$$\mathbf{U}_n(t+T) = \mathbf{U}_n(t) + g\mathbf{H}_n\mathbf{Q}_n(t) \quad (5)$$

Here, T is the synchronous period and $\mathbf{U}_n$ and $\mathbf{Q}_n$ must be updated each synchronous cycle. The scalar g is a gain that can be selected online to give optimal settling times. This type of algorithm can compensate for small mismatch errors in the control and disturbance frequencies as amplitudes and phases are continually optimised. Moreover, it can generally produce faster response times than adaptive vibration control algorithms.

Estimation of the matrix $\mathbf{R}$ is achieved by injecting a trial force at each control input in turn and measuring the corresponding change in rotor response $\Delta\mathbf{Q}$. Each column of the $\mathbf{R}$ matrix is obtained from

$$\mathbf{R}_k(\Omega, j\omega) = \frac{\Delta\mathbf{Q}(\Omega, j\omega)}{S_k(j\omega)}, \; k = 1..M \quad (6)$$

The subscript k denotes the k^{th} column of the $\mathbf{R}$ matrix and M is the number of control inputs. S_k is the complex trial force component applied at the k^{th} input channel. In the case of synchronous vibration control, the tests are carried out at the synchronous frequency only i.e. $\omega = \Omega$. However, for multi-frequency

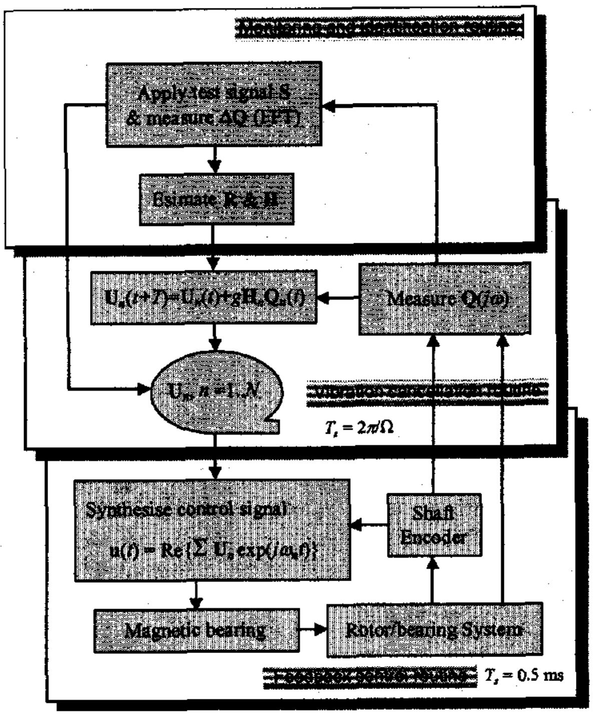

FIGURE 1: Flowchart of the hierarchical control scheme for multi-frequency vibration attenuation

vibration control, the trial force and the measurement of the rotor response must include all the frequencies of interest, i.e. $\omega = i\omega_0$, $i=1...N$, where N is the number of harmonics. The fundamental frequency ω_0 is chosen to cover the frequencies of interest and be consistent with the data analysis hardware and software. Although it is not a requirement, it is preferable to choose ω_0 to be an integer factor of the rotational frequency in order to cover the synchronous vibration as well. The proposed control strategy is summarised in Fig. 1, where the details regarding fault detection and decision-making processes are omitted but can be found in [8].

It is important to note that the speed dependent **R** matrix, and hence the **H** matrix, is not a function of the external disturbances, and is not expected to change unless there is a change in the rotor or bearing dynamics, for example, due to changing speed. However, the in-situ identification will be necessary for those cases and may also be used as a part of a condition monitoring and/or fault detection procedure [8]. Moreover, when new frequencies of excitation occur during operation this will necessitate identification of the **R** matrix for the appropriate disturbance frequencies.

As shown in Fig. 1, the changes in **Q** due to external excitation are immediately acted upon according to Eq. 5 without waiting for the re-identification and re-evaluation of the **H** matrix. However, re-identification of the **R** matrix is still performed automatically to check whether the recorded changes in **Q** are attributable to changes in system dynamics or changes in disturbance.

MULTI-FREQUENCY TEST SIGNAL

In order to identify the **R** matrix for all the frequency components at a given rotational speed Ω, a periodic test signal that excites these frequencies is required. Using sinusoidal signals will require $N \times M$ tests, each involving a few periods of steady state response with averaging to reduce noise. If all the frequency components are added together with arbitrary phase then the resulting system response may produce unacceptable peak rotor displacements or cause force saturating of the magnetic bearing. Decreasing the excitation amplitudes of all the frequencies is not a good solution, because this will result in weak perturbations in the frequency domain and will consequently increase the estimation errors.

However, it is possible to generate a low-peak periodic signal $s(t)$ having any user specified power spectrum P_i, $i = 1,..,N$ where P_i is the ratio of the power at $\omega = i\omega_0$ to the total power. i.e.

$$\sum_{i=1}^{N} P_i = 1 \tag{7}$$

Selecting the phase of each frequency component in accordance with the Schroeder Phased Harmonic Sequence (SPHS):

$$\phi_i = \phi_{i-1} - 2\pi \sum_{m=1}^{i-1} P_m \,, i = 1..N \tag{8}$$

the signal can then be constructed as

$$s(t) = \sum_{i=1}^{N} A_i . Cos(i\omega_0 t + \phi_i) \tag{9}$$

where A_i is the amplitude of the i^{th} harmonic and is given by

$$A_i = \sqrt{\frac{P_i}{2}} \tag{10}$$

The amplitude of each frequency component can be selected freely to obtain a balanced response.

If it is required to minimise the peak displacement of the rotor at the p^{th} measurement site during the identification then the test signal must be constructed taking account of the appropriate influence coefficients:

$$s_k(t) = \mathrm{Re}\left\{ \sum_{i=1}^{N} R_{pk}^{-1}(ji\omega_0) A_i e^{j(i\omega_0 t + \phi_i)} \right\} \tag{11}$$

This will then compensate for the change in phase and amplitude produced by the rotor response dynamics. For example, if any of the discrete frequencies coincides with a natural frequency, then the relative

FIGURE 2: Flexible rotor with magnetic bearings

magnitude of the test signal component will be reduced. The peak response of the rotor will be minimal for the selected vibration component amplitudes A_i. This method does require an initial estimate of **R** to use in eq. 11. In many cases this estimate will be available from similar previous operating conditions.

For frequencies at which no significant vibration is detected, the corresponding amplitudes can be set to zero. For each new set of test signal requirements, the phases can easily be calculated from Eq. 8.

EXPERIMENTAL SETUP

The test rig (Fig. 2) comprises a 2 m long rotor with a mass of 100 kg supported by two radial magnetic bearings. The maximum rotational frequency is around 100 Hz, while the first and second rotor flexural modes have natural frequencies around 27 and 70 Hz respectively. Pairs of eddy current displacement sensors are located in four planes along the length of the rotor. The signals from all these sensors are used in the control optimisation. The two magnetic bearings each have two opposing coil pairs, oriented at 45° to the vertical, giving a total of four control force axes. Eight current controlled Pulse Width Modulated (PWM) amplifiers power the magnet coils with a bias current of 4.3 A, giving a bearing negative stiffness of $2\text{x}10^6$ N/m, a current gain of 487 N/A and a peak control force of approximately 1500 N. The bearing stator-rotor radial air gap is 1.2 mm and adjacent retainer bearings have a radial clearance of 0.7 mm.

The control scheme is implemented on a PC-based digital signal processor (DSP). The real-time control algorithm is comprised of three main sub-routines, as shown in Fig. 1. The lowest level routine calculates the control signals that drive the magnetic bearing amplifiers and operates at the highest sample frequency of 2000 Hz. The control signals are calculated each iteration by superimposing the PID displacement feedback signal with the multi-frequency vibration cancellation signal. The vibration cancellation routine is executed once per rotor revolution and must calculate the rotor vibration components $\mathbf{Q}_n$ for the previous revolution and then use them to calculate the control force components $\mathbf{U}_n$ for the next revolution. The vibration components may be obtained using an FFT-based algorithm, although a direct integration method [11] was used here to improve execution time. The identification and monitoring routine is executed as a background task on the PC, and will trigger re-identification of the partial receptance matrix **R** when significant changes in the measured vibration occur. This routine also regulates the controller gain g and will, for example, set g to zero during identification of **R** in order to hold constant the control components $\mathbf{U}_n$.

Vibration of the rig foundation was achieved using an electromagnetic shaker connected to the base, which was mounted on elastomeric isolators. The shaker was positioned to excite the base in a horizontal direction using a DSP generated driving signal with selected vibration components. The natural frequencies of the translatory and yaw rigid body modes of the whole rig were 7 and 4 Hz respectively.

EXPERIMENTAL RESULTS AND DISCUSSIONS

The experiments were carried out at 24 Hz running speed. Figure 3 shows the uncontrolled and controlled rotor orbit and maximum amplitude of the vibration components, with no base excitation, at the non-driven end disk. As expected, significant vibration reduction is achieved with the synchronous controller. In order to extend the controller to cover non-synchronous frequencies, the identification of the **R** matrix at these frequencies is necessary. A multi-frequency test signal of 8 harmonics with a fundamental frequency of 12 Hz (half of the rotational speed) is selected with the following amplitudes in Newton:

$$A_i = 40 \text{ x } [1, 2, 0, 3, 0, 3, 0, 3] \qquad (12)$$

The half-synchronous frequency component is weighted less as 12 Hz is very close the first critical speed of the system. Higher weightings of the second, third and fourth harmonics of the synchronous speed reflect the lower sensitivity of the response at these frequencies. In other words, previous knowledge of the system dynamics is utilised in selecting the relative amplitudes of each frequency component in

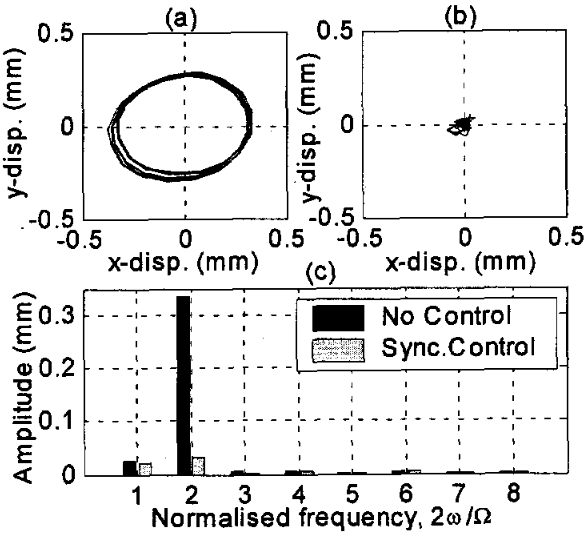

FIGURE 3: Rotor vibration at non-driven end.
(a) without control, (b) with synchronous control
(c) component amplitudes

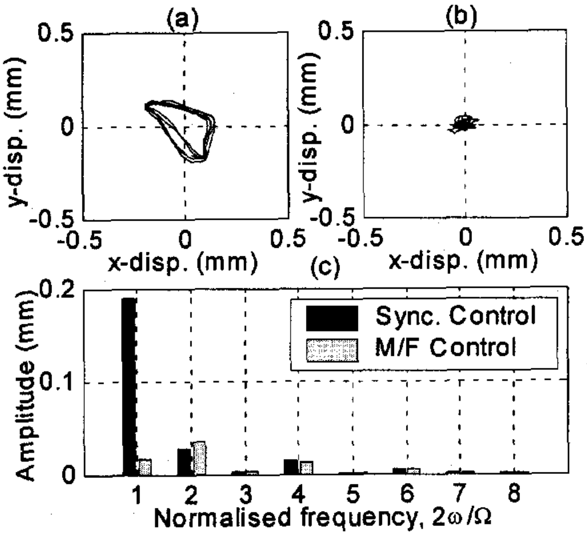

FIGURE 5: Rotor vibration at non-driven end.
(a) with synchronous control, (b) with multi-frequency controller, (c) Component amplitudes

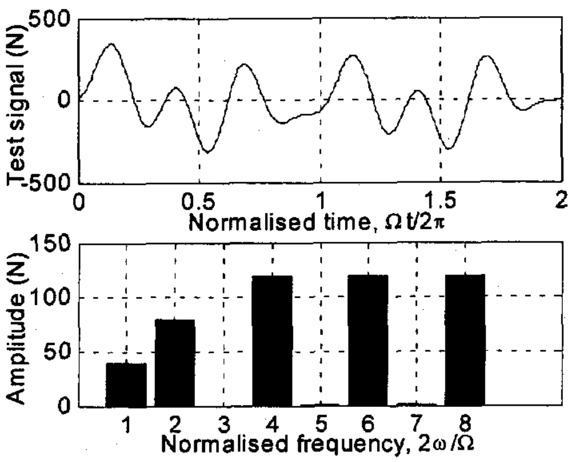

FIGURE 4: SPHS Test signal and component amplitudes

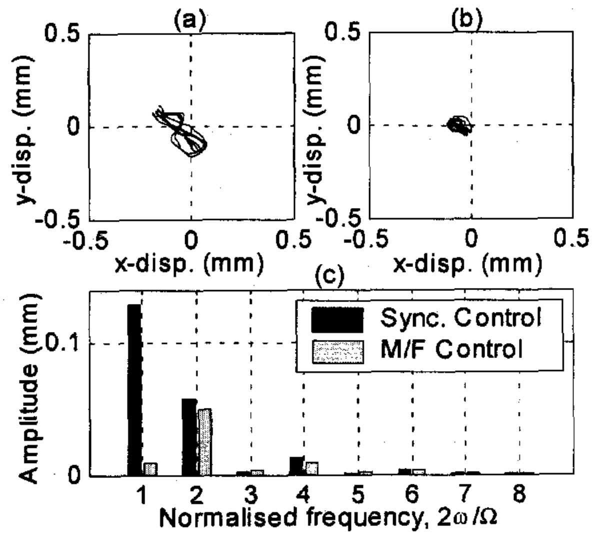

FIGURE 6: Rotor vibration at driven end.
(a) with synchronous control, (b) with multi-frequency controller, (c) Component amplitudes

the test signal. Having selected the desired amplitude response of the signal, the phases (radian) of each component can be calculated from Eq. 8 as:

$$\Phi_i=[0,-0.1963,0,-2.1598,0,-1.3744,0,-4.1233] \quad (13)$$

This gives a low-peak factor test signal as shown in Fig. 4(a), and the FFT of the signal, shown in Fig. 4(b), confirms that the signal has the required frequency components as specified in Eq. (12). This will cover frequencies up to the forth harmonic of the rotational speed. It is advantageous to include the frequencies of all expected vibration components. In this way the controller can quickly react when vibrations arise anywhere within this frequency range without the need to first identify the **R** (and **H**) matrix. In any case the identification is carried out by the monitoring system if and when necessary to detect any changes in system internal characteristics, such as bearing or rotor properties [8].

While in the synchronous control mode, half and twice-synchronous base excitations were introduced through the shaker. The multi-frequency controller was then activated. The orbit and maximum amplitude of the vibration components at the non-driven end of the rotor with and without the multi-frequency controller are shown in Fig. 5. Without the multi-frequency controller, i.e. with synchronous controller only, significant vibrations at the half-synchronous frequency were evident within the rotor: the controller is only effective for the synchronous vibrations. Twice-synchronous vibrations were not very significant because of the force limitations of the shaker. However, it is apparent from Fig. 5 that the multi-frequency controller has significantly reduced the half-synchronous vibrations and the orbit size. Similar controller performance can be observed at the driven end disk as shown in Fig. 6 with significant vibration reductions at the half-synchronous frequency, and some improvements at the twice-synchronous frequency despite the low level of excitation.

CONCLUSION

A closed loop harmonic vibration control algorithm has been developed to attenuate multi-frequency rotor vibration. The control algorithm requires the identification of the partial receptance matrix relating the measurement sites and the control inputs for all frequencies of interest. This is achieved by applying a multi-frequency test signal at each of the control inputs in turn. The amplitude of each component is chosen using initial estimations of the system dynamics. The phasing of each component is optimised using Schroeder Phased Harmonic Sequences (SPHS), which ensure a low peak value signal in the time domain while providing specified amplitude characteristics in the frequency domain. This choice of test signal allows much faster identification by eliminating the need to apply a separate test signal for each frequency. The experimental results showed that the multi-frequency identification of the system dynamics could be carried out effectively using a SPHS-based test signal. The subsequent calculation of the optimum closed loop controller gain allows significant reduction in the vibration of various non-synchronous components as well as maintaining effective synchronous vibration control.

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