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PERFORMANCE OPTIMIZATION OF A BEARINGLESS PUMP USING FEEDBACK LINEARIZATION TECHNIQUES

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ABSTRACT

The reliability as well as the lifetime of rotating fluid pumps which have to fulfill very high requirements on chemical resistance and purity may be improved considerably when all rotating parts are suspended magnetically with a bearingless motor. A very simple and hence cost-efficient mechanical design can be accomplished when a bearingless slice motor with PM excitation featuring only four concentrated coils is employed. In this paper a nonlinear controller design is presented which improves the performance of the bearingless pump substantially. By means of a nonlinear state transformation and a static state feedback control law the nonlinear mathematical model of the bearingless motor can be transformed into a linear and time-invariant one. Moreover, the resulting linear model may then be split into independent systems for force and torque generation. Accordingly, independent controllers for the rotor position as well as rotational speed can be implemented.

INTRODUCTION

Until today only pneumatically driven bellow pumps or diaphragm pumps have been used when very high requirements on chemical resistance, purity and reliability are demanded. The disadvantages of these pumps are relatively short lifetimes, low energy efficiency and mechanical wear. Considerable improvements with respect to lifetime and reliability can be accomplished when the impeller and the shaft of a centrifugal pump is levitated without any mechanical contact by means of electromagnetic forces. As a result the pump housing can be sealed hermetically and hence absolutely no mechanical wear occurs. For contact-free levitation and drive of a shaft different configurations with magnetic bearings and motors or bearingless motors have been published [3], [5], [6].

When the process only demands moderate pressure (a few bars) a very compact pump design with an integrated bearingless slice motor becomes feasible [1], [4]. A characteristic feature of a bearingless slice motor is that only three spatial degrees of freedom are stabilized actively, the other three degrees of freedom - the axial position of the rotor and the tilting angles - are stabilized passively.

MATHEMATICAL MODEL

Owing to the simplicity of the mechanical design a bearingless slice motor with concentrated windings is well suited for low-cost pump applications. Especially, when the required mechanical power is in the lower power range, a motor design featuring four concentrated coils - as shown in Figure 1 - can be employed to bring about further reduction in the overall system costs. However, the major drawback of this mechanical design is that the mathematical model becomes relatively complex. Thus, a sophisticated controller is required to achieve good operation characteristics.

Before the mathematical model can be formulated it is necessary to have a closer look at the whole drive including the driver electronics. The structure of the model varies whether the bearingless motor is fed from a current or a voltage source inverter. Typically, a PWM voltage source driver is used when the required power is in the range of several hundred watts. Therefore, in this case it is sufficient to deal with a mathematical model for the operation of

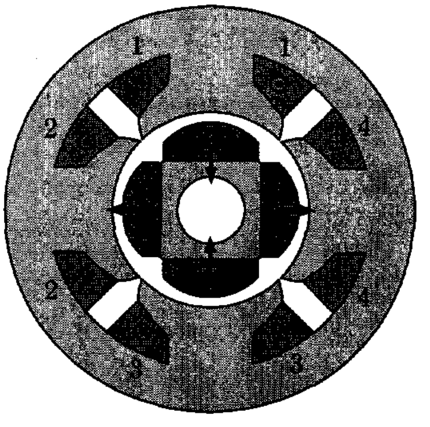

FIGURE 1: Bearingless single-phase motor with four concentrated coils

the bearingless motor in conjunction with a voltage source inverter.

A characteristic feature of the bearingless motor shown in Figure 1 is that each coil produces radial forces and torque at the same time. For the operation the task is to feed the coils with a set of appropriate currents to generate desired forces and torque. However, the bearingless motor under study has four concentrated coils to stabilize only three degrees of freedom actively. This means that there are many different sets of phase currents that produce exactly the same required radial forces and torque. But only a few sets of currents are of practical relevance, like those minimizing the resistive power losses. In this paper it is assumed that the motor is operated with the highest possible efficiency.

By means of an optimization problem [8] a simple transformation for the phase currents $\mathbf{i}_1$ of the form

$$\mathbf{i}_1 = \mathbf{V}\,\bar{\mathbf{i}}_1 \qquad (1)$$

with

$$\mathbf{i}_1 = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}, \qquad \bar{\mathbf{i}}_1 = \begin{bmatrix} i_a \\ i_b \\ i_t \end{bmatrix}$$

$$\mathbf{V} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ -1 & -1 & 1 \\ -1 & 1 & -1 \end{bmatrix}$$

can be found which guarantees minimization of the resistive power losses for an arbitrary set of currents $\bar{\mathbf{i}}_1$. More specifically, transformation (1) maps the currents i_a, i_b and i_t of a fictitious auxiliary winding system on the phase currents i_1 to i_4 of the bearingless motor. Instead of formulating the mathematical model for the original bearingless motor with four concentrated windings it is more convenient to transform all electrical values into the fictitious auxiliary winding system with three phases. When the transformation is supposed to be invariant with respect to the power, the phase voltages are transformed as

$$\bar{\mathbf{u}}_1 = \mathbf{V}^T \mathbf{u}_1,$$

where $\mathbf{u}_1$ and $\bar{\mathbf{u}}_1$ denote the vectors of the phase voltages in the original system and the auxiliary system, respectively. Transformation of the resistance and inductance matrices into the auxiliary system yields

$$\bar{\mathbf{R}}_1 = \mathbf{V}^T \mathbf{R}_1 \mathbf{V}, \qquad \bar{\mathbf{L}}_1 = \mathbf{V}^T \mathbf{L}_1 \mathbf{V}.$$

On condition that the motor design is symmetrical the resistance and inductance matrices of the auxiliary three phase system become

$$\bar{\mathbf{R}}_1 = \begin{bmatrix} 4R_1 & 0 & 0 \\ 0 & 4R_1 & 0 \\ 0 & 0 & 4R_1 \end{bmatrix}$$

and

$$\bar{\mathbf{L}}_1 = \begin{bmatrix} \bar{L}_{11} & 0 & 0 \\ 0 & \bar{L}_{22} & 0 \\ 0 & 0 & \bar{L}_{33} \end{bmatrix}$$

$$\begin{aligned} \bar{L}_{11} &= 4L_{11} - 4L_{13} \\ \bar{L}_{22} &= 4L_{11} - 4L_{13} \\ \bar{L}_{33} &= 4L_{11} - 8L_{12} + 4L_{13}, \end{aligned}$$

where R_1 denotes the phase resistance and L_{11}, L_{12} and L_{13} are the self and coupling inductances of the original system.

When only the dominant nonlinearities are taken into account and the rotor is assumed to be in the centered position the current to force relationship[1] results in

$$\begin{bmatrix} F_x \\ F_y \\ T \end{bmatrix} = \bar{\mathbf{T}}_m(\varphi) \begin{bmatrix} i_a \\ i_b \\ i_t \end{bmatrix} \qquad (2)$$

with

$$\bar{\mathbf{T}}_m(\varphi) = \begin{bmatrix} T_{11}(\varphi) & T_{12}(\varphi) & 0 \\ -T_{12}(\varphi) & T_{11}(\varphi) & 0 \\ 0 & 0 & T_{33}(\varphi) \end{bmatrix},$$

where F_x and F_y denote the force acting on the rotor, T is the torque and $T_{xx}(\varphi)$ are nonlinear functions of the rotor angle φ. By defining a state vector as

$$\mathbf{x} = \begin{bmatrix} i_a & i_b & i_t & x_r & y_r & \varphi & v_{rx} & v_{ry} & \omega \end{bmatrix}^T,$$

[1] A more accurate model for the current to force relationship can be found in [9].

where x_r and y_r denote the position of the rotor in radial directions, v_{rx}, v_{ry} and ω are the velocities of the rotor in radial directions and the angular velocity, respectively, a nonlinear state space representation for the bearingless motor with concentrated coils can be found

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{0} & \mathbf{A}_{13} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \\ \mathbf{A}_{31} & \mathbf{A}_{32} & \mathbf{0} \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} \overline{\mathbf{L}}_1^{-1} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \overline{\mathbf{u}}_1(t)$$
$$\mathbf{y}(t) = \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} \end{bmatrix} \mathbf{x}(t). \tag{3}$$

The sub-matrices of the nonlinear dynamic matrix have the form

$$\mathbf{A}_{11} = -\overline{\mathbf{L}}_1^{-1}\overline{\mathbf{R}}_1$$
$$\mathbf{A}_{13} = -\overline{\mathbf{L}}_1^{-1}\overline{\mathbf{T}}_m^T(\varphi)$$
$$\mathbf{A}_{31} = \begin{bmatrix} \frac{1}{m_r} & 0 & 0 \\ 0 & \frac{1}{m_r} & 0 \\ 0 & 0 & \frac{1}{I_z} \end{bmatrix} \overline{\mathbf{T}}_m(\varphi)$$
$$\mathbf{A}_{32} = \begin{bmatrix} \frac{k_x}{m_r} & 0 & 0 \\ 0 & \frac{k_x}{m_r} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

with m_r the mass of the rotor, I_z the moment of inertia, $\mathbf{I}$ a 3×3 unit matrix and k_x the linearized displacement stiffness.

FEEDBACK LINEARIZATION

Nonlinear state transformation

Since derivation of the nonlinear state transformation for the bearingless motor with four concentrated coils is beyond the scope of this paper, only the basic ideas and the final results for a nonlinear change of the coordinates and a static state feedback for the motor under study are presented.

When a general plant modeled by nonlinear differential equations of the form

$$\dot{\mathbf{x}} = \mathbf{a}(\mathbf{x}) + \mathbf{b}(\mathbf{x})\,\mathbf{u}$$
$$\mathbf{y} = \mathbf{c}(\mathbf{x}) \tag{4}$$

is considered, simplification in the description and the behavior may be feasible by means of a nonlinear change of the coordinates with a transformation

$$\mathbf{z} = \mathbf{T}(\mathbf{x}) \tag{5}$$

where $\mathbf{z}$ is the new state vector and $\mathbf{T}(\mathbf{x})$ represents a function of n variables

$$\mathbf{T}(\mathbf{x}) = \begin{bmatrix} T_1(x_1, x_2, \cdots, x_n) \\ T_2(x_1, x_2, \cdots, x_n) \\ \cdots \\ T_n(x_1, x_2, \cdots, x_n) \end{bmatrix}.$$

If $\mathbf{T}(\mathbf{x})$ is invertible the original state vector can be expressed as

$$\mathbf{x} = \mathbf{T}^{-1}(\mathbf{z}).$$

By differentiating both sides of Equation (5) with respect to time we obtain

$$\dot{\mathbf{z}} = \frac{d\mathbf{z}}{dt}$$
$$\dot{\mathbf{z}} = \frac{\partial \mathbf{T}}{\partial \mathbf{x}} \frac{d\mathbf{x}}{dt}$$
$$\dot{\mathbf{z}} = \frac{\partial \mathbf{T}}{\partial \mathbf{x}} \left(\mathbf{a}(\mathbf{x}) + \mathbf{b}(\mathbf{x})\mathbf{u}\right).$$

Accordingly, the original description of the system of Equation (4) can be replaced by a new description when expressing $\mathbf{x}$ as $\mathbf{T}^{-1}(\mathbf{z})$

$$\dot{\mathbf{z}} = \widehat{\mathbf{a}}(\mathbf{z}) + \widehat{\mathbf{b}}(\mathbf{z})\,\mathbf{u}$$
$$\mathbf{y} = \widehat{\mathbf{c}}(\mathbf{z}),$$

with

$$\widehat{\mathbf{a}} = \left.\frac{\partial \mathbf{T}}{\partial \mathbf{x}} \mathbf{a}(\mathbf{x})\right|_{\mathbf{x}=\mathbf{T}^{-1}(\mathbf{z})}$$
$$\widehat{\mathbf{b}} = \left.\frac{\partial \mathbf{T}}{\partial \mathbf{x}} \mathbf{b}(\mathbf{x})\right|_{\mathbf{x}=\mathbf{T}^{-1}(\mathbf{z})}$$
$$\widehat{\mathbf{c}} = \left.\mathbf{c}(\mathbf{x})\right|_{\mathbf{x}=\mathbf{T}^{-1}(\mathbf{z})}.$$

For the bearingless motor with concentrated coils a nonlinear transformation of the state variables of the form

$$\begin{aligned} z_1 &= T_{11}(\varphi)\, i_a + T_{12}(\varphi)\, i_b \\ z_2 &= -T_{12}(\varphi)\, i_a + T_{11}(\varphi)\, i_b \\ z_3 &= T_{33}(\varphi)\, i_t \\ z_4 &= x_r \\ z_5 &= y_r \\ z_6 &= \varphi \\ z_7 &= v_{rx} \\ z_8 &= v_{ry} \\ z_9 &= \omega \end{aligned} \tag{6}$$

can be found to simplify the description of the model (3) substantially. With this nonlinear transformation only the three currents of the auxiliary winding system are mapped on new state variables z_1 to z_3 and all the other state variables remain unchanged. Moreover, if the current to force relationship of Equation (2) is considered it can be seen that the state variables z_1, z_2 and z_3 are forces and torque rather than currents.

Exact linearization via feedback

The nonlinear change of the coordinates presented in the previous section yields a state space representation that is still nonlinear. However, by means of a static state feedback control law of the form

$$\mathbf{u} = \boldsymbol{\alpha}(\mathbf{z}) + \boldsymbol{\beta}(\mathbf{z})\,\mathbf{v} \qquad (7)$$

the nonlinear system can be transformed into a linear and controllable one. In Equation (7) the variables $\mathbf{u}$ and $\mathbf{v}$ represent the control input of the original and the new system, respectively, and $\boldsymbol{\beta}(\mathbf{z})$ is assumed to be nonzero for all $\mathbf{z}$. When the static state feedback of Equation (7) is applied to the model (4) we obtain

$$\begin{aligned}\dot{\mathbf{z}} &= \widehat{\mathbf{a}}(\mathbf{z}) + \widehat{\mathbf{b}}(\mathbf{z})\boldsymbol{\alpha}(\mathbf{z}) + \widehat{\mathbf{b}}(\mathbf{z})\boldsymbol{\beta}(\mathbf{z})\mathbf{v} \\ \mathbf{y} &= \widehat{\mathbf{c}}(\mathbf{z}),\end{aligned}$$

where $\boldsymbol{\alpha}(\mathbf{z})$ and $\boldsymbol{\beta}(\mathbf{z})$ should be chosen in such a way as to get a linear system.

For the bearingless motor that is considered in this paper we choose

$$\boldsymbol{\alpha}(\mathbf{z}) = \overline{\mathbf{L}}_1 \frac{\partial \overline{\mathbf{T}}_m^{-1}(\varphi)}{\partial \varphi}\,\omega\,\mathbf{z}_1 + \overline{\mathbf{T}}_m^T(\varphi)\,\mathbf{z}_3$$

and

$$\boldsymbol{\beta}(\mathbf{z}) = \overline{\mathbf{T}}_m^{-1}(\varphi)$$

with

$$\begin{aligned}\mathbf{z}_1 &= [z_1, z_2, z_3]^T \\ \mathbf{z}_2 &= [x_r, y_r, \varphi]^T \\ \mathbf{z}_3 &= [v_{rx}, v_{ry}, \omega]^T .\end{aligned}$$

Thus the resulting closed-loop system is governed by the equations

$$\begin{aligned}\dot{\mathbf{z}} &= \begin{bmatrix} \hat{\mathbf{A}}_{11} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \\ \hat{\mathbf{A}}_{31} & \hat{\mathbf{A}}_{32} & \mathbf{0} \end{bmatrix} \dot{\mathbf{z}} + \begin{bmatrix} \overline{\mathbf{L}}_1^{-1} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \mathbf{v} \\ \mathbf{y} &= \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} \end{bmatrix}\end{aligned} \qquad (8)$$

consisting of the sub-matrices

$$\begin{aligned}\hat{\mathbf{A}}_{11} &= -\overline{\mathbf{L}}_1^{-1}\overline{\mathbf{R}}_1 \\ \hat{\mathbf{A}}_{31} &= \begin{bmatrix} \frac{1}{m_r} & 0 & 0 \\ 0 & \frac{1}{m_r} & 0 \\ 0 & 0 & \frac{1}{J_z} \end{bmatrix} \\ \hat{\mathbf{A}}_{32} &= \begin{bmatrix} \frac{k_x}{m_r} & 0 & 0 \\ 0 & \frac{k_x}{m_r} & 0 \\ 0 & 0 & 0 \end{bmatrix}.\end{aligned}$$

By means of the two transformations - change of the coordinates and static state feedback - the state representation for the bearingless motor becomes linear. Moreover, the ninth order system may be split into three independent systems of the form

$$\begin{aligned}\begin{bmatrix} \dot{z}_1 \\ \dot{x}_r \\ \dot{v}_{rx} \end{bmatrix} &= \mathbf{A}_{S1} \begin{bmatrix} z_1 \\ x_r \\ v_{rx} \end{bmatrix} + \mathbf{b}_{S1} v_1 \\ y_1 &= x_r,\end{aligned} \qquad (9)$$

$$\begin{aligned}\begin{bmatrix} \dot{z}_2 \\ \dot{y}_r \\ \dot{v}_{ry} \end{bmatrix} &= \mathbf{A}_{S1} \begin{bmatrix} z_2 \\ y_r \\ v_{ry} \end{bmatrix} + \mathbf{b}_{S1} v_2 \\ y_2 &= y_r,\end{aligned} \qquad (10)$$

$$\begin{aligned}\begin{bmatrix} \dot{z}_3 \\ \dot{\varphi} \\ \dot{\omega} \end{bmatrix} &= \mathbf{A}_{S3} \begin{bmatrix} z_3 \\ \varphi \\ \omega \end{bmatrix} + \mathbf{b}_{S3} v_3 \\ y_3 &= \varphi.\end{aligned} \qquad (11)$$

with

$$\mathbf{A}_{S1} = \begin{bmatrix} -\frac{\overline{R}_1}{\overline{L}_{11}} & 0 & 0 \\ 0 & 0 & 1 \\ \frac{1}{m_r} & \frac{k_x}{m_r} & 0 \end{bmatrix}, \qquad \mathbf{b}_{S1} = \begin{bmatrix} \frac{1}{\overline{L}_{11}} \\ 0 \\ 0 \end{bmatrix}$$

and

$$\mathbf{A}_{S3} = \begin{bmatrix} -\frac{\overline{R}_3}{\overline{L}_{33}} & 0 & 0 \\ 0 & 0 & 1 \\ \frac{1}{I_z} & 0 & 0 \end{bmatrix}, \qquad \mathbf{b}_{S3} = \begin{bmatrix} \frac{1}{\overline{L}_{33}} \\ 0 \\ 0 \end{bmatrix}.$$

The structures of the first two sub-systems (9) and (10) are equal and, moreover, their form corresponds

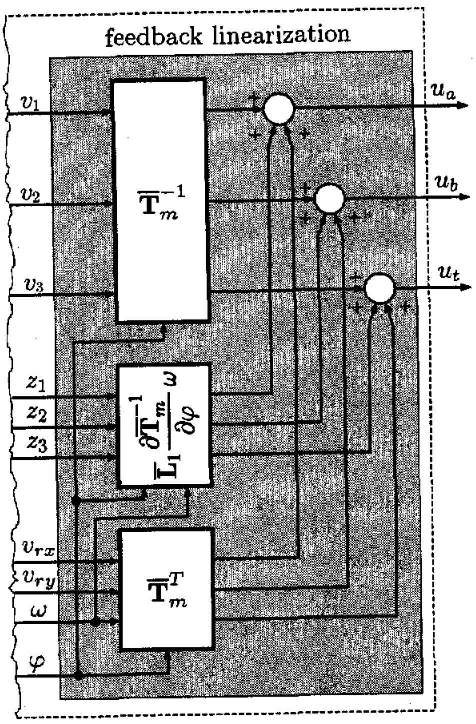

FIGURE 2: Block diagram of the proposed feedback linearization

to the model of a one degree of freedom voltage controlled active magnetic bearing system [7]. Furthermore, the structure of Equation (11) corresponds to a model of a conventional DC motor without emf.

CONTROLLER IMPLEMENTATION

The proposed feedback linearization is shown in form of a block diagram in Figure 2. A characteristic property of the linearized model is that any reference signal on the new control input v_1 only influences the radial position of the rotor in x-direction. Accordingly, with the control input v_2 the rotor position in y-direction may be altered and finally v_3 allows the rotational speed to be modified. As a result of this property, the design of a position controller may apply any design method for SISO LTI systems. To implement the control scheme the state variables have to be known precisely. Accordingly, the state variables must be measured or, in the case of cost efficient drives be calculated by means of a state observer.

Finally, the control scheme requires an additional transformation that maps the voltages of the auxiliary winding system on the physical winding configuration. This transformation can be found as

$$\mathbf{u}_1 = \overline{\mathbf{V}}\,\overline{\mathbf{u}}_1$$

FIGURE 3: Bearingless pump with four concentrated coils

with

$$\overline{\mathbf{V}} = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ -1 & -1 & 1 \\ -1 & 1 & -1 \end{bmatrix}.$$

EXPERIMENTAL RESULTS

The bearingless pump with a rated electrical power of 200 W is shown in Figure 3. To confirm improvements in the performance, measurements have been carried out. Figure 4 shows the two different orbits of the rotor both for a rotational speed of 3500 rpm. The solid curve represents the orbit when the proposed feedback linearization technique was implemented and the dashed curve shows the orbit without the proposed control law. Displacement of the rotor is almost five times larger without the nonlinear state feedback control law because of the non-sinusoidal relationship between the phase currents and force generation. Consequently, the phase currents become much higher, as depicted in Figure 5. This figure shows the instantaneous values of the phase current of phase one when the control law is disabled. Since the delivered flow rate and pressure of the pump have not been changed the mechanical power of the bearingless drive remains unchanged, too, and thus the resistive power loss increases considerably.

This means that the efficiency of the bearingless motor could have been increased substantially by the implementation of the feedback linearization technique. Even the attainable flow rate as well as pressure could have been increased to 45 liters per minute and 2.5 bars, respectively, because the rotational speed could have been increased from 3800 rpm

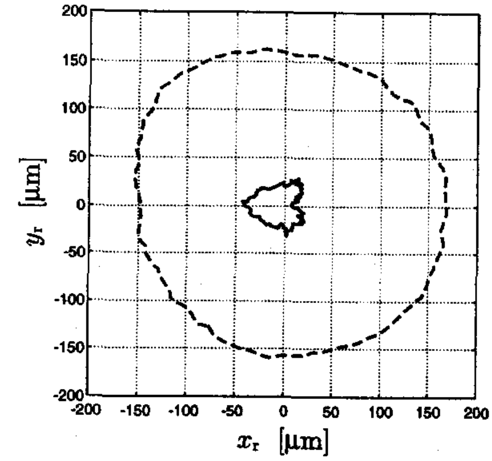

FIGURE 4: Orbit of the rotor when the pump operates at a rotational speed of 3500 rpm with (solid line) and without (dashed line) the proposed linearization technique

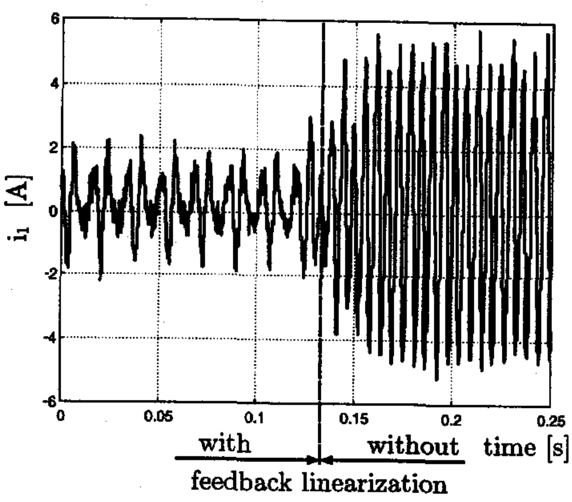

FIGURE 5: Instantaneous value of the phase current when the state feedback control law is disabled

to 8300 rpm.

CONCLUSIONS

In this paper a novel nonlinear control scheme for bearingless PM motor has been introduced. Especially, for a cost efficient bearingless slice motor with concentrated coils the non-sinusoidal relationship between the phase currents and the bearing forces demands a sophisticated controller to achieve sufficient high motor performance. A very elegant way to meet the requirements is to apply a feedback linearization technique.

The basic idea behind feedback linearization is to modify the nonlinear description and the behavior of a mathematical model to achieve some prescribed behavior. More specifically, by means of two nonlinear transformations - nonlinear change of the state variables and nonlinear state feedback - the model of the bearingless motor is transformed into a linear and time-invariant one. Consequently, linear position and speed controllers can be designed.

With the proposed control law it has been possible to considerably improve the efficiency of the motor as well as the pump characteristics.

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