IDENTIFICATION OF THE DYNAMIC COEFFICIENTS OF ACTIVE MAGNETIC BEARING

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ABSTRACT
It is important to identify the dynamic coefficients of rotor-active magnetic bearings (RAMBs) system. In most papers published, only force stiffness coefficients are identified, but moment stiffness coefficients caused by coupling effects between thrust bearing and journal bearings are not considered. But the coupling effects should not be ignored if the structure of AMB system is non-symmetrical. In this paper, identification of force and moment stiffness coefficients of a non-symmetrical 5 degree of freedom (5DOF) RAMBs system is presented, which uses least-square method under multi-frequency current excitations. The validity of this method is verified by experiments on a test rig RAMBs system. Furthermore, both simulation and experiment results show that the coupling between thrust bearing and journal bearings affect system stability greatly, therefore should be considered in design of magnetic bearings.

NOMENCALTURE

A_r, (A_t)	effective gap area of journal (thrust) bearing
C_{0r}, (C_{0t})	air gap of journal (thrust) bearing
e_x, ey	eccentric distance in x, y direction
$F_x, F_y F_{zc}, M_x, M_y M_{xc}, M_{yc}$	components of magnetic force and moment in x, y, z direction
i_x, i_y, i_z (i_{x0} i_{y0}, i_{z0})	dynamic (static) control currents
J_{0x}, J_{0y}	equatorial moments of inertia
J_{0xz}, J_{0yz}	moments of inertia product
J_{0z}	polar moment of inertia
k_{ls} ($l=x, y, s=x, y, \varphi, \psi, i_x, i_y$), k_{zt} ($t=z, \varphi, \psi, i_z$)	force stiffness coefficients
k_{mls} ($l=x, y, s=x, y, \varphi, \psi, i_x, i_y$), k_{mlt} ($t=z, \varphi, \psi, i_z$)	moment stiffness coefficients
l_a, l_b	distance between left (right) journal bearing and centroid of rotor
l_{sa}, l_{sb}	distance between left (right) journal sensor and centroid of rotor
m	mass of rotor
N_r, (N_t)	numbers of turns of winding of journal (thrust) bearing
p	magnetic force of journal bearing
R_1, R_2, R_3, R_4	radial of the annuli of thrust bearing
x, y, z, (x_0, y_0, z_0)	dynamic (static) displacements
ϕ	cylindrical coordinate
φ, ψ, (φ_0, ψ_0)	dynamic (static) tilting angles of rotor
Ω	angular velocity of rotor
$\delta_1 \ldots \delta_5$	current excitations
μ	permeability of air

INTRODUCTION
Magnetic bearings have some virtues, such as noncontact, elimination of lubrication and low power losses etc., compared with conventional bearings [1,2]. This leads to applications mainly in three areas: vacuum or clean room environments, machine tools and turbomachinery and centrifuges. In applications of AMBs, the predicted dynamic behaviors disagree with the measured ones sometimes because of inaccuracy of the model. So it is important to identify the dynamic parameters of RAMBs systems.

Most papers published about the identification of RAMBs system are mainly concerned with dynamic analysis and parameter identification algorithms. Michael Baloh [3] has used adaptive estimation to identify unknown parameters and disturbances for a simple one-dimensional magnetic bearing system.

Shun-Chang Chang [4] studied the technique of identifying a non-linear electromagnetic system experimentally based on the principle of harmonic balance and the curve fitting method. Srinivasan [5] has proposed two approaches to obtain accurate AMB plant models for the purpose of control design: physical modeling and system identification. Chong-Won Lee [6] has used the frequency response function for system parameter identification. Seung-Jong Kim [7] has used a modified least-mean-squares algorithm to identify the current and position stiffnesses of AMBs equipped with four pairs of built-in radial magnetic force transducers.

In above papers, only force stiffness coefficients including force-current and force-position stiffness coefficients are identified. Moment stiffness coefficients caused by coupling between thrust AMB and journal AMBs are not considered. Usually, moment stiffness coefficients are small and may be ignored if the structure of AMB system is symmetrical. However, sufficient attentions should be paid to them when the structure is non-symmetrical.

Identification of dynamic coefficients, including force and moment stiffness coefficients, of a 5DOF non-symmetrical RAMBs system is presented in this paper, which uses least-square method [10] under multi-frequency current excitations [8,9]. First, dynamic coefficients are defined to describe the dynamic behavior of AMB system. Then differential equations are established and coefficients are identified by the least square method. The theoretical results agree well with the experimental results.

DEFINITION OF DYNAMIC COEFFICIENTS

Diagram of a 5DOF non-symmetrical RAMBs system is shown in Figure 1. The position of the thrust disc mounted on the shaft can be described by displacements in the x, y and z directions, and the tilting angles φ and ψ.

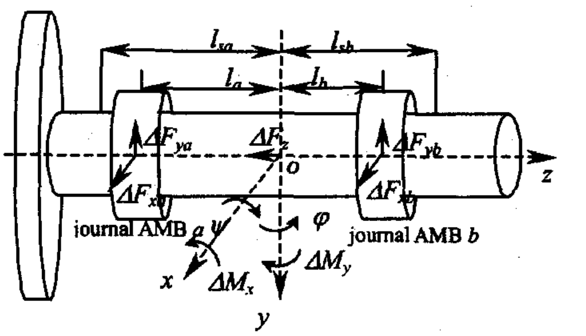
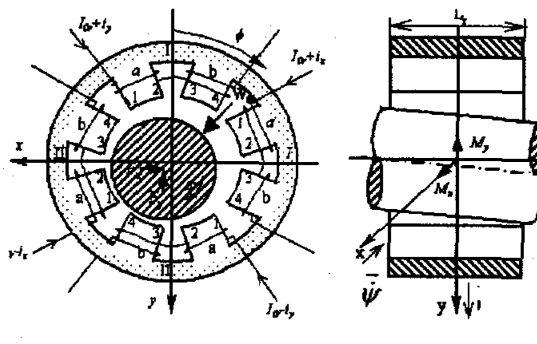


FIGURE 1: Diagram of RAMBs system



FIGRUE 2: Structure Diagram of journal bearing

Dynamic Coefficients Of Journal Bearing

φ and ψ can be written as

$$\begin{cases} \varphi = \dfrac{x_b - x_a}{l_a + l_b} \\ \psi = \dfrac{y_b - y_a}{l_a + l_b} \end{cases} \tag{1}$$

Assuming that the static working point of the rotor is $s_0(x_0, y_0, z_0, \varphi_0, \psi_0, i_{x0}, i_{y0}, i_{z0})$, the magnetic force and moment components of journal bearing in x and y directions are defined as

$$\begin{cases} F_{x0} = \iint p\big|_{s_0} \sin\phi r d\phi dz \\ F_{y0} = \iint p\big|_{s_0} \cos\phi r d\phi dz \\ M_{x0} = \iint p\big|_{s_0} z\cos\phi r d\phi dz \\ M_{y0} = \iint p\big|_{s_0} z\sin\phi r d\phi dz \end{cases} \tag{2}$$

respectively.

The force and moment stiffness coefficients are:

$$\begin{cases} k_{xs} = \dfrac{\partial F_x}{\partial s}\Big|_{s_0} = \iint \dfrac{\partial p}{\partial s}\Big|_{s_0} \sin\phi r d\phi dz \\ k_{ys} = \dfrac{\partial F_y}{\partial s}\Big|_{s_0} = \iint \dfrac{\partial p}{\partial s}\Big|_{s_0} \cos\phi r d\phi dz \\ k_{mxs} = \dfrac{\partial M_x}{\partial s}\Big|_{s_0} = \iint \dfrac{\partial p}{\partial s}\Big|_{s_0} z\cos\phi r d\phi dz \\ k_{mys} = \dfrac{\partial M_y}{\partial s}\Big|_{s_0} = \iint \dfrac{\partial p}{\partial s}\Big|_{s_0} z\sin\phi r d\phi dz \end{cases}$$

$$(s = x, y, \varphi, \psi, i_x, i_y) \tag{3}$$

k_{xs} and k_{ys} are force stiffness coefficients in x and y directions.

Under the condition of small perturbation, the increments of F_x, F_y, M_x and M_y can be expressed with force stiffness coefficients as

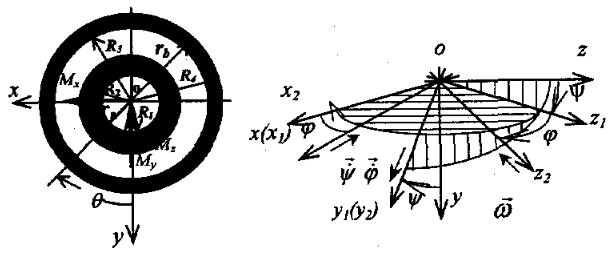
$$\begin{Bmatrix} \Delta F_x \\ \Delta F_y \\ \Delta M_x \\ \Delta M_y \end{Bmatrix} = \begin{bmatrix} k_{xx} & k_{xy} & k_{x\varphi} & k_{x\psi} & k_{xi_x} & k_{xi_y} \\ k_{yx} & k_{yy} & k_{y\varphi} & k_{y\psi} & k_{yi_x} & k_{yi_y} \\ k_{mxx} & k_{mxy} & k_{mx\varphi} & k_{mx\psi} & k_{mxi_x} & k_{mxi_y} \\ k_{myx} & k_{myy} & k_{my\varphi} & k_{my\psi} & k_{myi_x} & k_{myi_y} \end{bmatrix} \begin{Bmatrix} \Delta x \\ \Delta y \\ \Delta\varphi \\ \Delta\psi \\ \Delta i_x \\ \Delta i_y \end{Bmatrix} \tag{4}$$

where dynamic coefficients k_{yx}, k_{xy}, $k_{y\varphi}$, $k_{x\psi}$, k_{yix}, k_{xiy}, k_{mxx},

k_{myy}, $k_{mx\varphi}$, $k_{my\psi}$, k_{mxix}, k_{myiy} are high-order terms compared with k_{xx}, k_{yy}, $k_{x\varphi}$, $k_{y\psi}$, k_{xix}, k_{yiy}, k_{mxx}, k_{myx}, $k_{my\varphi}$, $k_{mx\psi}$, k_{myix} and k_{mxiy}. Moments caused by coupling effect between two journal AMBs are smaller than those caused by coupling between thrust AMB and journal AMBs. So generally only six stiffness coefficients (k_{xx}, k_{yy}, k_{xix}, k_{yiy}, $k_{my\varphi}$, $k_{mx\psi}$) of journal AMBs are considered when the rotor is not tilting.

Dynamic Coefficients Of Thrust bearing

The configuration of the thrust bearing is shown in Figure 3. Three reference systems are used to describe the motion of the thrust disc. (x, y, z) is fixed on the thrust bearing. Rotating reference (x_1, y_1, z_1) is defined relative to fixed reference (x, y, z) by a single rotation ψ about the x-axis. Reference (x_2, y_2, z_2) is defined relative to (x_1, y_1, z_1) by a single rotation φ about the y_1-axis. Its origin is at the center of the thrust disc and the z-axis coincides with the rotor's centerline.



a Configuration of thrust Bearing b Tilt coefficient of thrust plate

FIGURE 3: Configuration of thrust bearing

Force and moment stiffness coefficients of thrust bearing are defined as [11],

$$\begin{cases} k^c_{zt} = \dfrac{\partial F_{zc}}{\partial t} \\ k^c_{mxt} = \dfrac{\partial M_{xc}}{\partial t} \\ k^c_{myt} = \dfrac{\partial M_{yc}}{\partial t} \end{cases} \qquad (t=z,\ \varphi,\ \psi,\ i_z) \quad (5)$$

where

$$\begin{cases} F_{zc0} = \iint\limits_{\Omega} (p_1 - p_2)|_{s_0}\ r dr d\theta \\ M_{xc0} = -\iint\limits_{\Omega} (p_1 - p_2)|_{s_0}\ r^2 \cos\theta dr d\theta \\ M_{yc0} = -\iint\limits_{\Omega} (p_1 - p_2)|_{s_0}\ r^2 \sin\theta dr d\theta \end{cases}$$

The integral area Ω is the shadow areas shown in Figure 3. p_1 and p_2 are magnetic forces of inner and outer rings respectively.

The magnetic force and moment components of thrust bearing can be written as

$$\begin{Bmatrix} \Delta F_{zc} \\ \Delta M_{xc} \\ \Delta M_{yc} \end{Bmatrix} = \begin{bmatrix} k^c_{zz} & k^c_{z\varphi} & k^c_{z\psi} & k^c_{ziz} \\ k^c_{mxz} & k^c_{mx\varphi} & k^c_{mx\psi} & k^c_{mxiz} \\ k^c_{myz} & k^c_{my\varphi} & k^c_{my\psi} & k^c_{myiz} \end{bmatrix} \begin{Bmatrix} \Delta z \\ \Delta\varphi \\ \Delta\psi \\ \Delta i_z \end{Bmatrix} \qquad (6)$$

Four stiffness coefficients (k^c_{zz}, k^c_{ziz}, $k^c_{mx\psi}$, $k^c_{my\varphi}$) should be considered if the rotor is not titling. So the stiffness coefficient vector to be identified will be

$$k = \left[k^a_{xx}, k^a_{yy}, k^b_{xx}, k^b_{yy}, k^c_{zz}, k^a_{xix}, k^a_{yiy}, k^b_{xix}, k^b_{yiy}, k^c_{ziz}, k^c_{my\varphi}, k^c_{mx\psi}\right]^T$$

THE IDENTIFICATION METHOD

Displacements of the rotor at the positions of magnetic bearings and sensors are

$$\begin{cases} q_B = (x_a,\ y_a,\ x_b,\ y_b,\ z_c)^T \\ q_s = (x_{sa},\ y_{sa},\ x_{sb},\ y_{sb},\ z_{sc})^T \end{cases} \qquad (7)$$

where subscript a, b and c denote displacements at bearing a, b and c, and sa, sb and sc denote displacements at sensor a, b and c. The relationship between q_B and q_s is

$$L_B q_B = L_s q_s \qquad (8)$$

where

$$l = l_a + l_b,\quad l_s = l_{sa} + l_{sb}$$

$$L_B = \frac{1}{l}\begin{bmatrix} l_b & 0 & l_a & 0 & 0 \\ 0 & l_b & 0 & l_a & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & l \end{bmatrix},\ L_s = \frac{1}{l_s}\begin{bmatrix} l_{sb} & 0 & l_{sa} & 0 & 0 \\ 0 & l_{sb} & 0 & l_{sa} & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & l_s \end{bmatrix}$$

Assumed that the rotor is rigid and not tilting, and the system operates at steady state. Then the differential linearized equation of motion of the rotor can be expressed as

$$M_B \ddot{q}_B + G_B \dot{q}_B = Q_o + E_o C_s \qquad (9)$$

where

$$M_B = \begin{bmatrix} \frac{l_b}{l}m & 0 & \frac{l_a}{l}m & 0 & 0 \\ 0 & \frac{l_b}{l}m & 0 & \frac{l_a}{l}m & 0 \\ -\frac{1}{l}J_{oy} & 0 & \frac{1}{l}J_{oy} & 0 & 0 \\ 0 & -\frac{1}{l}J_{ox} & 0 & \frac{1}{l}J_{ox} & 0 \\ 0 & 0 & 0 & 0 & m \end{bmatrix}$$

$$G_B = \frac{J_{oz}\Omega}{l}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},\quad E_o = \Omega^2 \begin{bmatrix} me_x & me_y \\ me_y & -me_x \\ J_{oxz} & J_{oyz} \\ J_{oyz} & -J_{oxz} \\ 0 & 0 \end{bmatrix}$$

$$Q_o = -\begin{bmatrix} \Delta F_{xa} + \Delta F_{xb} \\ \Delta F_{ya} + \Delta F_{yb} \\ -\Delta F_{xa} l_a + \Delta F_{xb} l_b + \Delta M_{yc} \\ -\Delta F_{ya} l_a + \Delta F_{yb} l_b + \Delta M_{xc} \\ \Delta F_{zc} \end{bmatrix}, \quad C_s = \begin{Bmatrix} \cos\Omega t \\ \sin\Omega t \end{Bmatrix}$$

M_B is the mass matrix and G_B is the damping matrix. Q_O is force exerted on the rotor, and $E_O C_S$ is gyroscopic force. Eq. (10) and (11) can be derived from Eq.(4) and (6) respectively.

$$\begin{bmatrix} \Delta F_{xn} \\ \Delta F_{yn} \end{bmatrix} = \begin{bmatrix} k_{xx}^n & 0 & k_{xix}^n & 0 \\ 0 & k_{yy}^n & 0 & k_{yiy}^n \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta i_{xn} \\ \Delta i_{yn} \end{bmatrix} \quad (n\text{=a, b}) \tag{10}$$

$$\begin{bmatrix} \Delta F_{zc} \\ \Delta M_{xc} \\ \Delta M_{yc} \end{bmatrix} = \begin{bmatrix} k_{zz}^c & 0 & 0 & k_{ziz}^c \\ 0 & k_{mx\varphi}^c & 0 & 0 \\ 0 & 0 & k_{my\psi}^c & 0 \end{bmatrix} \begin{bmatrix} \Delta z \\ \Delta\varphi \\ \Delta\psi \\ \Delta i_z \end{bmatrix} \tag{11}$$

Substituting Eq. (1), (10) and (11) into Eq. (9) yields

$$M_B \ddot{q}_B + G_B \dot{q}_B + K_B q_B + K_{IB} I_B = F \tag{12}$$

where

$$K_B = \begin{bmatrix} k_{xx}^a & 0 & k_{xx}^b & 0 & 0 \\ 0 & k_{yy}^a & 0 & k_{yy}^b & 0 \\ -k_{xx}^a l_a - \dfrac{k_{my\varphi}^c}{l} & 0 & k_{xx}^b l_b + \dfrac{k_{my\varphi}^c}{l} & 0 & 0 \\ 0 & -k_{yy}^a l_a - \dfrac{k_{mx\psi}^c}{l} & 0 & k_{yy}^b l_b + \dfrac{k_{mx\psi}^c}{l} & 0 \\ 0 & 0 & 0 & 0 & k_{zz}^c \end{bmatrix}$$

Extra equations should be added to solve out stiffness coefficients in Eq. (17) because there are twelve unknowns in five equations. Ten extra equations are added in this paper.

The responses of displacements and currents are got when multi-frequency current excitations are imposed So the dynamic coefficients of RAMBs may be recognized via the least square method.

$$K_{IB} = \begin{bmatrix} k_{xix}^a & 0 & k_{xix}^b & 0 & 0 \\ 0 & k_{yiy}^a & 0 & k_{yiy}^b & 0 \\ -k_{xix}^a l_a & 0 & k_{xix}^b l_b & 0 & 0 \\ 0 & -k_{yiy}^a l_a & 0 & k_{yiy}^b l_b & 0 \\ 0 & 0 & 0 & 0 & k_{ziz}^c \end{bmatrix}$$

$$F = E_O C_S, \quad I_B = \left[i_{xa}, i_{ya}, i_{xb}, i_{yb}, i_{zc} \right]^T$$

Values of M_B and G_B and F can be calculated if the rotor speed is given. q_B and I_B could be are measured by sensors. So the unknown quantities in Eq. (12) are stiffness coefficients in K_B and K_{IB}. The least square method is used to identify these coefficients, which is explained further in the following part.

The input-output relation of a n dimensional linear system can be expressed as

$$Y = X\theta \tag{13}$$

$$Y = \begin{bmatrix} y(1) \\ y(2) \\ \vdots \\ y(m) \end{bmatrix}, X = \begin{bmatrix} x_1(1) & \cdots & x_n(1) \\ x_1(2) & & x_n(2) \\ \vdots & & \vdots \\ x_1(m) & \cdots & x_n(m) \end{bmatrix}, \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_3 \end{bmatrix}$$

where Y is output, X is a $n \times m$ matrix (m>n), and θ is a $n \times 1$ parameter vector.

Assumed that $\hat{\theta}$ is the optimum estimation, it satisfies the following equation

$$\frac{\partial (Y - X\theta)^T (Y - X\theta)}{\partial \theta} \Big|_{\theta = \hat{\theta}} = 0 \tag{14}$$

therefore,

$$\hat{\theta} = (X^T X)^{-1} X^T Y \tag{15}$$

This result is defined as least square estimate of θ

Performing the Fourier transformation of Eq. (12), we get

$$\begin{aligned} &-\omega^2 M_B q_B(j\omega) + j\omega G_B q_B(j\omega) + \\ &K_B q_B(j\omega) + K_{IB} I_B(j\omega) = F(j\omega) \end{aligned} \tag{16}$$

And rearranging above equation, we have:

$$Ck = F(j\omega) - (-\omega^2 M_B + j\omega G_B) q_B(j\omega) \tag{17}$$

where

$$C = \begin{bmatrix} x_a & 0 & x_b & 0 & 0 & i_{xa} & 0 & i_{xb} & 0 & 0 & 0 & 0 \\ 0 & y_a & 0 & y_b & 0 & 0 & i_{ya} & 0 & i_{yb} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & z_c & 0 & 0 & 0 & 0 & i_{zc} & 0 & 0 \\ -x_a l_a & 0 & x_b l_b & 0 & 0 & -l_a i_{xa} & 0 & l_b i_{xb} & 0 & 0 & \dfrac{x_b - x_a}{l} & 0 \\ 0 & -y_a l_a & 0 & y_b l_b & 0 & 0 & -l_a i_{ya} & 0 & l_b i_{yb} & 0 & 0 & \dfrac{y_b - y_a}{l} \end{bmatrix}$$

$$k = (\bar{C}^T \bar{C})^{-1} \bar{C}^T \bar{Y} \tag{18}$$

where

$$\bar{C} = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix}, \bar{Y} = \begin{bmatrix} F(j\omega_1) - (-\omega_1^2 M_B + j\omega_1 G_B) q_B(j\omega_1) \\ F(j\omega_2) - (-\omega_2^2 M_B + j\omega_2 G_B) q_B(j\omega_2) \\ F(j\omega_3) - (-\omega_3^2 M_B + j\omega_3 G_B) q_B(j\omega_3) \end{bmatrix}$$

$$C_j = \begin{bmatrix} x_{aj} & 0 & x_{bj} & 0 & 0 & i_{xaj}+\delta_{1j} & 0 & i_{xbj}+\delta_{2j} & 0 & 0 & 0 & 0 \\ 0 & y_{aj} & 0 & y_b & 0 & 0 & i_{yaj}+\delta_{3j} & 0 & i_{ybj}+\delta_{4j} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & z_c & 0 & 0 & 0 & 0 & i_{zcj}+\delta_{5j} & 0 & 0 \\ -x_{aj}l_a & 0 & x_{bj}l_b & 0 & 0 & -l_a(i_{xaj}+\delta_{1j}) & 0 & l_b(i_{xbj}+\delta_2) & 0 & 0 & \frac{x_{bj}-x_{aj}}{l} & 0 \\ 0 & -y_{aj}l_a & 0 & y_{bj}l_b & 0 & 0 & -l_a(i_{yaj}+\delta_{3j}) & 0 & l_b(i_{ybj}+\delta_{4j}) & 0 & 0 & \frac{y_{bj}-y_{aj}}{l} \end{bmatrix}, (j=1,2,3)$$

In fact, the gyroscopic effect has little effect on the identification, and the dynamic coefficients can be thoroughly recognized when the rotor is at rest. This is convenient for implementation. So Eq. (18) can be simplified when Ω =0.

Since the sensors and magnetic bearings are mounted at different positions, the displacements at the sensor and bearing are not the same. The displacements at the bearing can be obtained form Eq.(8). Substituting Eq. (8) into Eq. (18), we get

$$k = (\bar{C}^T\bar{C})^{-1}\bar{C}^T \begin{bmatrix} \omega_1^2 M_B L_B^{-1} L_S q_B(j\omega_1) \\ \omega_2^2 M_B L_B^{-1} L_S q_B(j\omega_2) \\ \omega_3^2 M_B L_B^{-1} L_S q_B(j\omega_3) \end{bmatrix} \quad (19)$$

EXPERIMENTAL RESULTS

In order to identify the dynamic coefficients of AMBs, we have established a simple measurement system to measure the dynamic currents and displacements of AMBs. In this system, analog controller, amplifier and electromagnet are used as vibration exciter, eddy current sensor and hall sensor as pickup and a self-developed measurement and analysis system as measurement and analysis instrument.

The rotor is standstill in experiments. Multi-frequency current excitations are added in electromagnet winding. Different excitation modes are tried to obtain better measured data. Table 1 shows amplitudes and phases of coil currents and displacements of journal AMBs when both journal AMBs are excited in x direction. Table 2 shows partial measurement data when both journal AMBs are excited in x and y directions. Parameters of the test rig are in Table 3 .

TABLE 1: Amplitudes and phases of displacements and currents when journal AMBs are excited in x direction.

f (Hz)	17.10		22.89		30.96		37.06	
	Amplitude	Phase	Amplitude	Phase	Amplitude	Phase	Amplitude	Phase
δ_1, δ_2 mA	486.67	141.00	488.38	-63.40	488.36	43.60	488.50	-150.80
$i_{xa}+\delta_1$ mA	921.5	-28.00	974.59	122.80	1098.55	-136.60	1259.97	27.62
$i_{xb}+\delta_2$ mA	960.52	-31.70	950.34	121.00	944.25	-140.50	949.22	24.11
i_{ya} mA	179.25	-26.37	219.62	126.70	320.90	-138.70	460.45	20.80
i_{yb} mA	118.50	113.20	108.90	-97.50	110.61	2.39	115.52	164.10
x_a um	60.62	154.00	63.02	-57.30	68.13	42.00	74.66	-156.50
x_b um	60.98	153.00	61.23	-59.50	61.07	39.80	60.94	-158.80
y_a um	7.61	163.18	9.39	-56.05	13.76	35.03	19.86	-169.50
y_b um	4.78	-57.21	4.50	79.79	4.57	173.77	4.99	-25.37

TABLE 2 Amplitudes and phases of displacements and currents when journal AMBs are excited in x, y directions

f (Hz)	17.08		22.90		31.08		36.89	
	Amplitude	Phase	Amplitude	Phase	Amplitude	Phase	Amplitude	Phase
$\delta_1 \ldots \delta_4$ mA	481.74	-54.10	482.61	131.51	483.74	171.42	483.61	-114.45
$i_{xa}+\delta_1$ mA	696.95	133.68	719.24	-43.13	787.21	-3.25	870.58	71.77
$i_{xb}+\delta_2$ mA	942.68	130.45	935.13	-50.23	945.67	-14.10	967.45	56.43
$i_{ya}+\delta_3$ mA	1366.53	140.60	1472.50	-40.07	1687.61	-5.17	1941.68	65.33
$i_{yb}+\delta_4$ mA	877.31	134.15	886.67	-46.53	876.30	-13.26	865.95	57.88
x_a um	50.80	-41.59	52.21	138.21	55.27	173.17	58.76	-114.93
x_b um	60.05	-43.91	60.46	133.47	60.70	165.82	61.03	-124.19
y_a um	80.39	-35.42	86.11	140.07	95.25	170.54	101.78	-121.65
y_b um	58.23	-40.66	59.10	135.61	58.82	167.04	58.16	-123.60

TABLE 3: Paremeters of RAMBs

Parameters	Values	Parameters	Values
M	8.7 kg	N_r	80
l_a	0.065 m.	N_t	70
l_b	0.080 m	R_1	0.032 m
l_{sa}	0.116 m	R_2	0.036 m
l_{sb}	0.130 m	R_3	0.052 m
C_{0r}	0.25 mm	R_4	0.055 m
C_{0t}	0.2 mm	I_{0r}	4 A
A_r	3417.6mm^2	I_{0t}	2 A
A_t	3144.7mm^2		

The theoretical values of stiffness coefficients can be obtained from Eq. (3) and (5):

$$\begin{cases} k_{xx}^a = k_{yy}^a = k_{xx}^b = k_{yy}^b = -2.8\times10^7 N/M \\ k_{zz}^c = -9.0\times10^6 N/M \\ k_{xix}^a = k_{yiy}^a = k_{xix}^b = k_{yiy}^b = 1.8\times10^3 N/A \\ k_{ziz}^c = 9.0\times10^2 N/A \\ k_{my\varphi}^c = k_{mx\psi}^c = -1.1\times10^4 N\cdot M \end{cases}$$

The estimated values can be derivded from Eq. (8).

$$\begin{cases} k_{xx}^a = k_{yy}^a = k_{xx}^b = k_{yy}^b = -3.5\times10^7 N/M \\ k_{zz}^c = -8.2\times10^6 N/M \\ k_{xix}^a = k_{yiy}^a = k_{xix}^b = k_{yiy}^b = 1.2\times10^3 N/A \\ k_{ziz}^c = 8.4\times10^2 N/A \\ k_{my\varphi}^c = k_{mx\psi}^c = -1.3\times10^4 N\cdot M \end{cases}$$

The relative errors of stiffness coefficients between theoretical calculation and experimental results is less than 30%. Moreover errors of journal stiffness coefficients are greater than that of thrust stiffness coefficients. Errors may come from two aspects. One is that we only consider the coupling effects of the thrust bearing and the different positions of sensors and bearings, but other trivial coupling effects such as the coupling between two journal bearings are ignored. However they may also affect measured data in actual rotor-AMB system. The other is disturbance from the measurement system. For example, hall sensor and eddy current sensor are disturbed by electromagnetic fields.

CONCLUSION

(1) Equations of motion for a 5DOF RAMBs system is established. The effect of thrust bearing on the lateral vibration and the different mounting positions of sensors and bearings are considered.

(2) The identification method is simple and valid, and the theoretical results agree well with the experiment ones

(3) Moment stiffness coefficients of thrust bearing are negative. It will deteriorate system stability. The values of moment stiffness coefficients show that coupling between radial and thrust bearings should be considered in analysis and design of the RAMBs system.

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