DYNAMICS OF A RIGID ROTOR - MAGNETIC BEARING SYSTEM EQUIPPED WITH A THRUST MAGNETIC BEARING

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ABSTRACT
This paper is concerned with the dynamics of a rigid rotor supported by journal and thrust active magnetic bearings. For small perturbation, the action of the thrust bearing can be described by a series of stiffness coefficients. The system equations are formulated by combining the equations of motion of the rotor and the equations of the decentralized PID controllers. The eigenvalues of system are calculated and examined. The results indicate that the thrust magnetic bearing has great influence on the eigenvalues corresponding to low-frequency conical whirls, and causes the system stability to degrade.

INTRODUCTION
Magnetic bearings have found wide applications in fields such as aerospace, petrochemical and power generation industries due to its advantages over traditional rolling or sliding bearings. As the system is working on closed-loop, the performance of an active magnetic bearing-rotor system relies heavily on the dynamic characteristics of the entire system composed of rotor, journal and thrust magnetic bearings including coils, sensors, controllers and amplifiers, and other parameters of the rotor system. The unmodelled dynamics related to these factors may lead to system failures, and thus need to be investigated. In this paper, the effect of a thrust magnetic bearing on the dynamics of a rigid rotor supported by magnetic bearings is studied.
Thrust magnetic bearings are used as supports to balance axial loads and therefore their effects are often neglected when performing tuning of the controllers or dynamic analysis of systems in lateral directions. Studies on hydrodynamic bearings have revealed that hydrodynamic thrust bearings may have great effects upon the lateral static and dynamic behavior of rotor systems due to the moments produced by them[1,2,3]. For a rotor system equipped with a thrust magnetic bearing, such effects may also exhibit. The moment stiffness is negative, and the moments force the runner towards the thrust magnetic bearing, and affect the system stability.
The dynamic behavior of a thrust magnetic bearing can be described by a series of moment and force stiffness coefficients[4]. In this paper, they are used in formulating the equations of motion of a rigid rotor supported by journal and thrust magnetic bearings considering the effect of thrust bearing. Applying the equations of a decentralized PID control system, system equations in state space are obtained. The eigenvalues of the system are calculated, and the effect of thrust magnetic bearing on system stability is examined thereafter.

FORMULATIONS
Equations of motion of the rotor
A five-degree-of-freedom rigid rotor-active magnetic bearing system is depicted in **FIGURE 1**. The equations of motion of the rotor can be formulated as

$$M_B \ddot{q}_B + G_B \dot{q}_B = Q\,, \qquad (1)$$

where M_B and G_B are respectively the mass and damping matrices of the rotor, Q is force exerted on the rotor, q_B is displacement vector of the rotor,

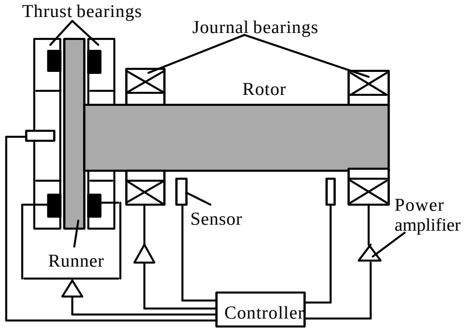

FIGURE 1: Structure of a Rotor-Active Magnetic Bearing System

$$M_B=\begin{bmatrix} \frac{l_b}{l}m & 0 & \frac{l_a}{l}m & 0 & 0 \\ 0 & \frac{l_b}{l}m & 0 & \frac{l_a}{l}m & 0 \\ -\frac{1}{l}J_{oy} & 0 & \frac{1}{l}J_{oy} & 0 & 0 \\ 0 & -\frac{1}{l}J_{ox} & 0 & \frac{1}{l}J_{ox} & 0 \\ 0 & 0 & 0 & 0 & m \end{bmatrix},$$

$$G_B=\frac{J_{oz}w}{l}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$Q=\begin{Bmatrix} -\Delta F_{xa}-F_{xb} \\ \Delta F_{ya}-\Delta F_{yb} \\ \Delta F_{xa}l_a-\Delta F_{xb}l_b-\Delta M_{yc} \\ \Delta F_{ya}l_a-\Delta F_{yb}l_b-\Delta M_{xc} \\ -\Delta F_{zc} \end{Bmatrix},\quad q_B=\begin{Bmatrix} x_a \\ y_a \\ x_b \\ y_b \\ z \end{Bmatrix},$$

m is mass, J_{ox} and J_{oy} are equatorial moments of inertia, J_{oz} is polar moment of inertia, w is angular velocity, l_a and l_b are the distances of journal bearings to the center of mass O (**FIGURE 2**), $l=l_a+l_b$, ΔF_{xj} and ΔF_{yj} (j=a,b) are dynamic forces of journal bearings in the x and y directions, ΔM_x and ΔM_y are dynamic moments of thrust bearing in the x and y directions, and ΔF_z is dynamic force of thrust bearing in the axial direction.

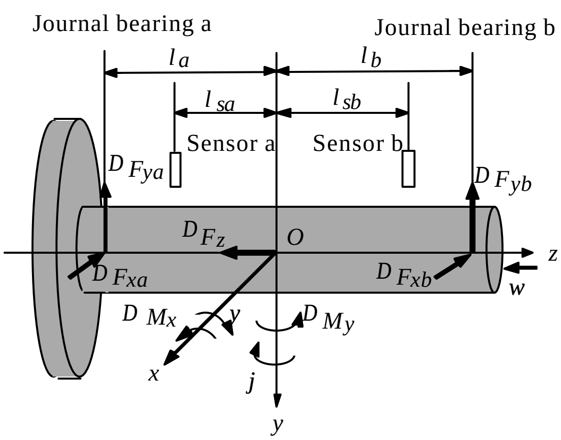

FIGURE 2: A Rigid Rotor in Equilibrium

In the case of small perturbation, the dynamic actions of magnetic bearings can be described by a series of displacement and current stiffness coefficients. Therefore the dynamic forces produced by the journal bearings are

$$\begin{Bmatrix} \Delta F_{xj} \\ \Delta F_{yj} \end{Bmatrix}=\begin{bmatrix} k_{xx}^j & 0 & k_{xi}^j & 0 \\ 0 & k_{yy}^j & 0 & k_{yi}^j \end{bmatrix}\begin{Bmatrix} x_j \\ y_j \\ i_{xj} \\ i_{yj} \end{Bmatrix}\ (j=a,b)\,, \qquad (2)$$

where k_{xx}^j and k_{yy}^j are force-displacement stiffness coefficients of bearing j, k_{xi}^j and k_{yi}^j are force-current stiffness coefficients of bearing j, x_j and y_j are displacements of journal j in the x and y directions respectively, i_{xj} and i_{yj} are control currents of journal bearing j in the x and y directions respectively.

The dynamic forces and moments produced by the thrust bearing are [4]

$$\begin{Bmatrix} \Delta F_{zc} \\ \Delta M_{xc} \\ \Delta M_{yc} \end{Bmatrix}=\begin{bmatrix} k_{zz} & k_{zj} & k_{zy} & k_{zi} \\ k_{xz}^m & k_{xj}^m & k_{xy}^m & k_{xi}^m \\ k_{yz}^m & k_{yj}^m & k_{yy}^m & k_{yi}^m \end{bmatrix}\begin{Bmatrix} z \\ j \\ y \\ i_z \end{Bmatrix}, \qquad (3)$$

where $k_{zn}\,(n=z,y,j \text{ and } i_z)$ are force stiffness coefficients in the axial direction, k_{xn}^m and k_{yn}^m are moment stiffness coefficients in the x and y directions respectively, z is axial displacement of rotor, j and y are tilting angles of runner, i_z is control current.

The tilt of a rigid rotor can be written as

$$\begin{cases} j=(x_b-x_a)/l \\ y=(y_b-y_a)/l \end{cases}. \qquad (4)$$

Substitution of Eqs.(2),(3) and (4) into Eq. (1) gives

$$Q=-K_Bq_B-K_{IB}I_B\,, \qquad (5)$$

where

$$K_B=\begin{bmatrix} k_{xx}^a & 0 & k_{xx}^b & 0 & 0 \\ 0 & k_{yy}^a & 0 & k_{yy}^b & 0 \\ -k_{xx}^a l_a-\frac{k_{yj}^m}{l} & -\frac{k_{yy}^m}{l} & k_{xx}^b l_b+\frac{k_{yj}^m}{l} & \frac{k_{yy}^m}{l} & k_{yz}^m \\ -\frac{k_{xj}^m}{l} & -k_{yy}^a l_a-\frac{k_{xy}^m}{l} & \frac{k_{xj}}{l} & k_{yy}^b l_b+\frac{k_{xy}^m}{l} & k_{xz}^m \\ -\frac{k_{zj}}{l} & -\frac{k_{zy}}{l} & \frac{k_{zj}}{l} & \frac{k_{zy}}{l} & k_{zz} \end{bmatrix},$$

$$K_{IB}=\begin{bmatrix} k_{xi}^a & 0 & k_{xi}^b & 0 & 0 \\ 0 & k_{yi}^a & 0 & k_{yi}^b & 0 \\ -k_{xi}^a l_a & 0 & k_{xi}^b l_b & 0 & k_{yi}^m \\ 0 & -k_{yi}^a l_a & 0 & k_{yi}^b l_b & k_{xi}^m \\ 0 & 0 & 0 & 0 & k_{zi} \end{bmatrix},\ I_B=\begin{Bmatrix} i_{xa} \\ i_{ya} \\ i_{xb} \\ i_{yb} \\ i_z \end{Bmatrix}.$$

Therefore the equations of motion of the rotor are

$$M_B\ddot{q}_B+G_B\dot{q}_B+K_Bq_B+K_{IB}I_B=0\,. \qquad (6)$$

Equations of sensors, amplifiers and PID controllers

The signal flow in a rotor-magnetic bearing system is shown in **FIGURE 3**. In the dynamic analysis of such a system, the dynamics of the sensors, power amplifiers and controllers should be considered.

The power amplifier can be modeled as a first-order system, whose transfer function is

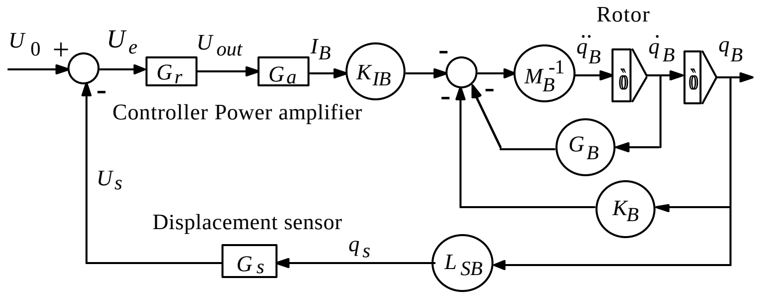

FIGURE 3: Signal Flow in a Rotor-Active Magnetic Bearing System

$$G_a(s) = \frac{A_a}{1+T_a s}, \tag{7}$$

where A_a is gain, T_a is time constant, and the corresponding differential equation is

$$T_a \dot{I}_B + I_B + A_a U_{out} = 0, \tag{8}$$

where I_B is control current, U_{out} is output voltage of controller or input voltage of amplifier.

The displacement sensor is again represented by a first-order model, i.e.

$$G_s(s) = \frac{A_s}{1+T_s s}, \tag{9}$$

where A_s is gain, T_s is time constant, and the corresponding differential equation is

$$T_s \dot{U}_s + U_s - A_s L_{SB} q_B = 0, \tag{10}$$

where U_s is output voltage of sensor, L_{SB} is the coupling matrix to account for the effect of noncollcation between sensor and actuator,

$$L_{SB} = \frac{1}{l}\begin{bmatrix} l_b + l_{sa} & 0 & l_a - l_{sa} & 0 & 0 \\ 0 & l_b + l_{sa} & 0 & l_a - l_{sa} & 0 \\ l_b - l_{sb} & 0 & l_a + l_{sb} & 0 & 0 \\ 0 & l_b - l_{sb} & 0 & l_a + l_{sb} & 0 \\ 0 & 0 & 0 & 0 & l \end{bmatrix},$$

l_{sa} and l_{sb} are the distances of sensors in the lateral directions to the center of mass O (**FIGURE 2**).

The transfer function of the PID controller is

$$G_r(s) = K_{pr} + \frac{K_{ir}}{s} + \frac{K_{dr} s}{1+T_{dr} s}, \tag{11}$$

where K_{pr} is proportional gain, K_{ir} is integral gain, K_{dr} is derivative gain and T_{dr} is time constant, the corresponding differential equation is

$$(K_{dr} + K_{pr} T_{dr})\ddot{U}_e + (K_{pr} + K_{ir} T_{dr})\dot{U}_e$$
$$+ K_{ir} U_e - \dot{U}_{out} - T_{dr} \ddot{U}_{out} = 0, \tag{12}$$

where U_e is error voltage.

Differentiating Eq.(8), we have

$$T_a \ddot{I}_B + \dot{I}_B + A_a \dot{U}_{out} = 0, \tag{13}$$

Considering the summing point, $U_e = U_0 - U_s$, and for ideal PID controllers, reference input voltage $U_0 = 0$, we have

$$T_s \ddot{U}_e + \dot{U}_e + A_s L_{SB} \dot{q}_B = 0, \tag{14}$$

and

$$-T_{dr} \ddot{U}_{out} - \dot{U}_{out} + (K_{pi} - K_{dp} T_s^{-1} - K_{ir} T_s)\dot{U}_e$$
$$- K_{dp} T_s^{-1} A_s L_{SB} \dot{q}_B - K_{ir} A_s L_{SB} q_B = 0, \tag{15}$$

where

$K_{dp} = K_{dr} + K_{pr} T_{dr}$,
$K_{pi} = K_{pr} + K_{ir} T_{dr}$.

System equations

Combining the equations for the rotor, the sensors, the amplifiers and the PID controllers, the system equations are obtained

$$\begin{cases} M_B \ddot{q}_B + G_B \dot{q}_B + K_B q_B + K_{IB} I_B = 0 \\ T_a \ddot{I}_B + \dot{I}_B + A_a \dot{U}_{out} = 0 \\ T_s \ddot{U}_e + \dot{U}_e + A_s L_{SB} \dot{q}_B = 0 \\ -T_{dr} \ddot{U}_{out} - \dot{U}_{out} + (K_{pi} - K_{dp} T_s^{-1} - K_{ir} T_s)\dot{U}_e \\ -K_{dp} T_s^{-1} A_s L_{SB} \dot{q}_B - K_{ir} A_s L_{SB} q_B = 0 \end{cases}. \tag{16}$$

When state variables $X_m = (q_B, I_B, \dot{q}_B, \dot{I}_B, \dot{U}_e, \dot{U}_{out})^T$ are introduced, the corresponding system equations in state space are

$$\dot{X}_m = A_m X_m, \tag{17}$$

where

$$A_m = \begin{bmatrix} 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 \\ A_{m13} & A_{m23} & A_{m33} & 0 & 0 & 0 \\ 0 & 0 & 0 & -T_a^{-1} w^{-1} & 0 & -T_a^{-1} w^{-1} A_a \\ 0 & 0 & A_{m35} & 0 & -T_s^{-1} w^{-1} & 0 \\ A_{m16} & 0 & A_{m36} & 0 & A_{m56} & -T_{dr}^{-1} w^{-1} \end{bmatrix},$$

with $A_{m13} = -M_B^{-1} w^{-2} K_B$, $A_{m16} = -T_{dr}^{-1} w^{-2} K_{ir} A_s L_{SB}$,

$A_{m23} = -M_B^{-1} w^{-2} K_{IB}$, $A_{m33} = -M_B^{-1} w^{-1} G_B$,
$A_{m35} = -T_s^{-1} w^{-1} A_s L_{SB}$, $A_{m36} = -T_{dr}^{-1} w^{-1} K_{dp} T_s^{-1} A_s L_{SB}$,

$A_{m56} = T_{dr}^{-1} w^{-1}(K_{pi} - K_{dp} T_s^{-1} - K_{ir} T_s)$.

This generalized eigenvalue problem can be solved by using the QR algorithm.

NUMERICAL RESULTS AND DISCUSSIONS

The rotor depicted by **FIGURE 2** is analyzed with parameters given in **TABLE 1**. The parameters of the controllers, power amplifiers and sensors are given in **TABLE 2**. The parameters of journal bearings are: radial clearance 0.0004m, bias current is 4A, winding number is 57, width is 0.044m, diameter is 0.06m. The parameters of thrust bearing are: clearance is 0.0005m, bias current is 4A, winding number is 145, diameter is 0.15m, axial load is 1500N. The rotating speed is 30,000r/min.

The dimensionless eigenvalues of the rotor system are given in **TABLE 3**. The corresponding dimensionless eigenmodes in the lateral directions are depicted in **FIGURE 4**.

TABLE 1: Parameters of Rotor

Parameter	Value
l_a (mm)	76.3
l_b (mm)	103.7
l_{sa} (mm)	46.3
l_{sb} (mm)	73.7
m (kg)	9.78
J_{ox} (kg/m^2)	0.143
J_{oy} (kg/m^2)	0.143
J_{oz} (kg/m^2)	0.0102

TABLE 2: Parameters of Controllers, Sensors and Power Amplifiers

Bearing type	Parameter	Value
Journal bearings	A_a (1/Ω)	1
	A_s (V/m)	7800
	T_a (s)	3.2e-5
	T_s (s)	3.2e-5
	T_{dr} (s)	6.0e-4
	k_{pr}	3.8e-4
	k_{ir} (1/s)	200
	k_{dr} (s)	0.01
Thrust bearing	A_a (1/Ω)	1
	A_s (V/m)	7800
	T_a (s)	3.2e-5
	T_s (s)	3.2e-5
	T_{dr} (s)	1.0e-4
	k_{pr}	3.8e-4
	k_{ir} (1/s)	200
	k_{dr} (s)	0.01

(1)The modes mainly fall into four categories, namely cylindrical whirls, conical whirls, their combinations and axial translation.

(2)As the effective stiffness of magnetic bearings is much lower than that of oil-lubricated bearings, the mechanically-dominant vibrations(modes 2-5) lie in the low-frequency zone, while the electrically-dominant vibrations in the high-frequency zone(modes 6-13).

(3)In the low-frequency zone, the first three modes (modes 1-3) correspond to the axial translation and cylindrical whirls. Therefore the thrust bearing show little effect on them. The physical meaning is obvious: the dynamic moments provided by the thrust bearing are only related to the dynamic tilt of rotor, therefore the effect on these two kinds of motions is insignificant. The fourth- and fifth-order modes are typically conical ones. As a result, the effect of thrust bearing on them is significant. The whirling frequencies are raised by 32.2% and 20.4% respectively, while the absolute values of the real parts of eigenvalues are reduced by 26.8% and 36.5% respectively. The dimensionless damping, which is the absolute value of the ratio of the real part to the imaginary part, is thereby reduced, and the system stability degrades.

(4)In the modes corresponding to the lateral whirls(modes 2-13), components related to the axial motion are zeros, while in those corresponding to the axial motion(modes 1,14,15), the components related to the lateral whirls are zeros. This indicates that the lateral motion shows little effect upon the axial translation.

(5)The variation of the fourth- and fifth- order eigenvalues with the bias current of thrust bearing is shown in **FIGURE 5**. The axial load is 0N. The action of the thrust bearing becomes more significant when the current increases, which is due to the increase of the negative moment stiffness of thrust bearing.

TABLE 3: Dimensionless Eigenvalues l_i / w

Mode	Not consider TAMB	Consider TAMB
1(AM)	-0.8674 ± **i**0.2570	-0.8674 ± **i**0.2570
2(LM)	-0.7826 ± **i**0.3495	-0.7826 ± **i**0.3495
3(LM)	-0.7826 ± **i**0.3495	-0.7826 ± **i**0.3495
4(LM)	-0.6327 ± **i**0.5608	-0.4633 ± **i**0.7411
5(LM)	-0.6393 ± **i**0.6974	-0.4057 ± **i**0.8395
6(LE)	-4.1909 ± **i**10.209	-4.4090 ± **i**10.040
7(LE)	-3.4585 ± **i**11.715	-3.6395 ± **i**11.510
8(LE)	-200.55 ± **i**12.989	-200.55 ± **i**12.990
9(LE)	-200.47 ± **i**13.007	-200.47 ± **i**13.008
10(LE)	-2.2094 ± **i**19.981	-2.2107 ± **i**19.978
11(LE)	-2.1954 ± **i**20.024	-2.1976 ± **i**20.020
12(LE)	-201.99 ± **i**21.491	201.99 ± **i**21.491
13(LE)	-201.99 ± **i**21.492	-201.99 ± **i**21.492
14(AE)	-8.0686 ± **i**58.272	-8.0686 ± **i**58.272
15(AE)	-222.66 ± **i**65.531	-222.66 ± **i**65.531

*A: axial translation; L: lateral whirl;
M: mechanically dominant; E: electrically dominant.

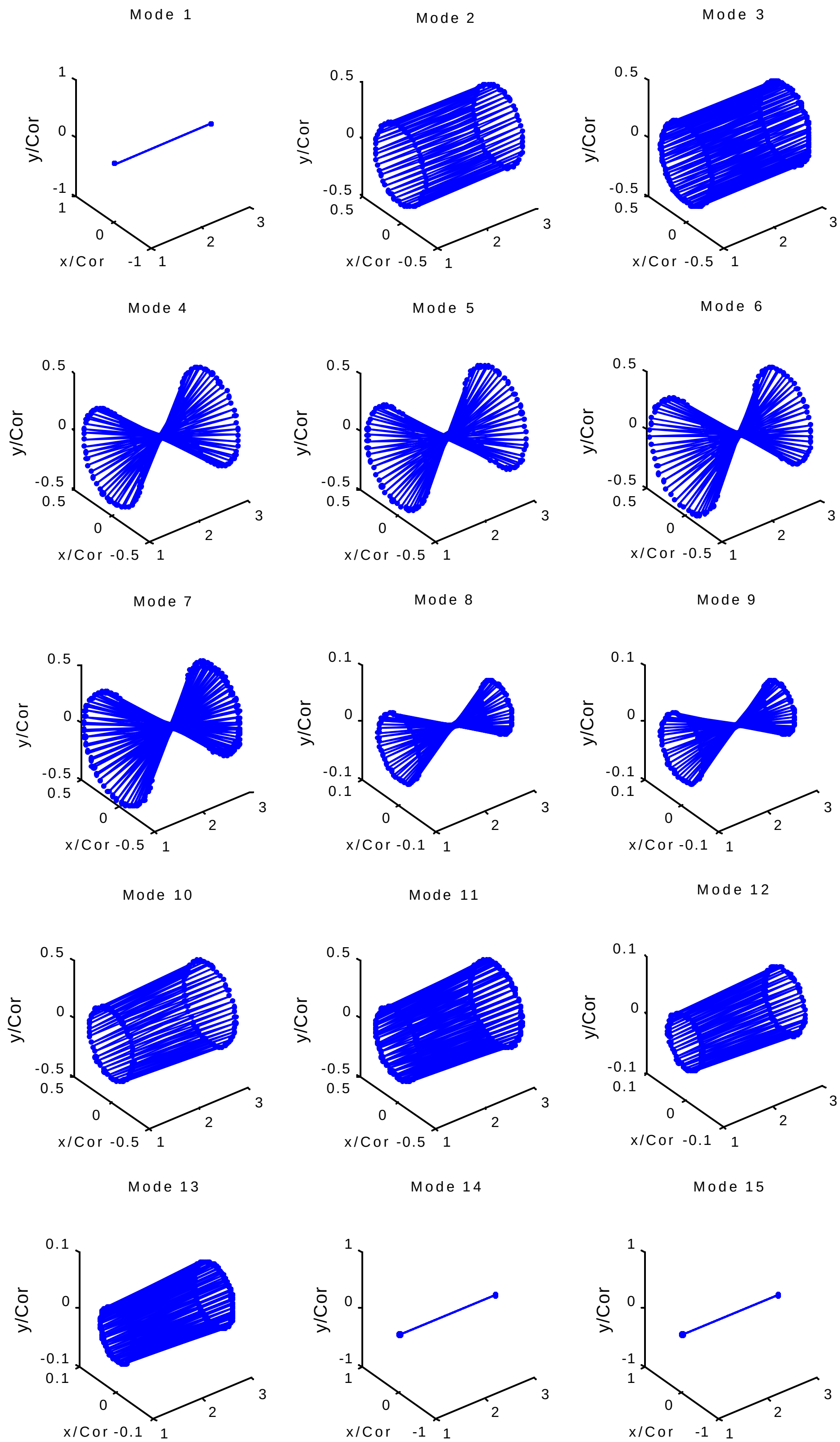

FIGURE 4: Dimensionless Eigenmodes of the Rotor-Active Magnetic Bearing System
(*x*-displacements in the *x* direction, *y*-displacements in the *y* direction, C_{0r} -radial clearance of journal bearings)

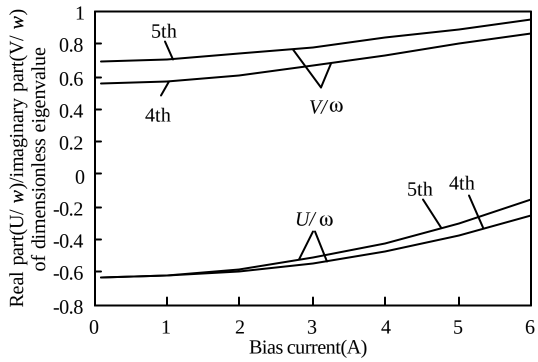

FIGURE 5: The Fourth- and Fifth- Order Eigenvalues versus Bias Current of Thrust Bearing
(*U*-real part of eigenvalue, *V*-whirling frequency)

CONCLUSIONS

(1)The system equations of motion for a rigid rotor supported by journal and thrust bearings are formulated. The effect of noncollocation of sensors and actuators, the effect of thrust bearing on the lateral vibration, the electromechanical coupling and the coupling between the two journal bearings can be taken into account.

(2)The negative moment stiffness of thrust bearing can increase the frequencies of some of the conical lateral whirl, and reduce the corresponding damping, and as a result may deteriorate system stability. Therefore sufficient attentions should be paid to this effect in the analysis and design of rigid rotor systems equipped with a magnetic thrust bearing in order to ensure system reliability.

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