CHARACTERISTICS OF THRUST MAGNETIC BEARING AND ITS EFFECT ON RADIAL ONES IN THE SYSTEM

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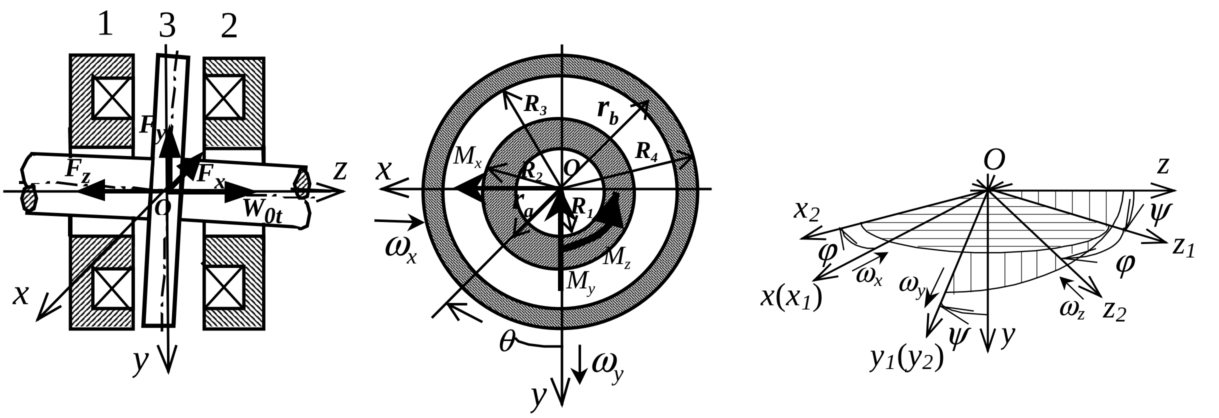
ABSTRACT
Static and dynamic mechanical characteristics of a thrust magnetic bearing are studied due to the runner disk inclination. Application refers to a thrust magnetic bearing for a turbo-expander/compressor. The static tilt of the runner disk has remarkable influence on the mechanical characteristics of thrust magnetic bearing, changes the static load distribution between two radial magnetic bearings and exerts violent coupling effect among a thrust magnetic bearing and two radial magnetic bearings. Such a finding can be used for the coupled electromechanical dynamics analysis of rotor system equipped with magnetic bearings.

INTRODUCTION
Since 1990, an important discovery in rotor-hydrodynamic bearing system dynamics is the coupled action of thrust bearing upon the lateral vibration in a rotor system. In the pioneering research of Mittwollen *et al*. (1990)[1], a series of rotor-dynamic coefficients of thrust magnetic bearing were defined and used for the investigation of the action of thrust bearing upon the lateral vibration in which the static tilt of the runner was neglected. Afterwards, in the research of Yu Lie *et al.* (1995)[2], a general analysis method was developed to investigate the coupled dynamics of a rotor equipped with journal and thrust hydrodynamic bearings simultaneously. Considerations included the effects of static tilt parameters of the rotor on rotor-dynamic coefficients of thrust hydrodynamic bearing and the action of thrust hydrodynamic bearing upon the system dynamics. However, up to now, the effects of static tilt parameters of the rotor on rotor-dynamic coefficients of thrust magnetic bearing and the coupled action of thrust magnetic bearing upon the lateral vibration in a rotor system have not been studied thoroughly in a public literature. Few literatures dealt with the mechanical characteristic of a thrust magnetic bearing in the past. The influence of the time constants in the control loop on the maximal achievable dynamic stiffness of thrust magnetic bearing were investigated in P.K.Budig *et al.* (1994)[3]. For convenience, the axial vibration is regarded as an independent degree of freedom in great majority of literatures [4~7]. And it is supposed that the runner disk parallels with the magnets. Only two stiffness coefficients of force-displacement and force-current are concerned in the rotor-dynamic coefficients of thrust magnetic bearing. The dynamic characteristic of the thrust magnetic bearing is qualitatively similar to that of thrust hydrodynamic bearing in many aspects except for the force coming from magnetic field. The effect of thrust magnetic bearing upon the lateral vibration in a rotor system is not negligible. When the effect is studied, the stiffness coefficients of thrust magnetic bearing are more than two. Therefore it is very necessary to research the mechanical characteristics of a thrust magnetic bearing comprehensively.

CHARACTERISTIC COEFFICIENTS
Fig.1 is the schematics of a thrust magnetic bearing in Cartesian coordinates. The movement of the runner disk in space is defined by the speed v_x、v_y and v_z of its center of mass and three rotational velocities ω_x、ω_y and ω_z. The F_x、F_y and F_z are the components of resultant force of magnets 1 and 2 attracting the runner disk 3. The M_x、M_y and M_z are the components of couple moment. When the runner disk tilts, the definition of φ and ψ in *O-xyz* inertial coordinates is shown in Fig.1(*c*)



(a) Thrust magnetic bearing (b) Structural parameters (c) Tilting parameters of runner disk

FIGURE 1: Schematic of a Thrust Magnetic Bearing

Static Characteristics

Suppose W_{0t} is the axial external load supported by the runner disk in static balance point (z_0, φ_0, ψ_0). The i_{z0} is correspondent control current. And the static point of operation is defined as $s_0(z_0, \varphi_0, \psi_0, i_{z0})$. Then the load capacity of the thrust magnetic bearing is the total attraction on the runner disk:

$$\boldsymbol{F_0} = -F_{x0}\boldsymbol{i} - F_{y0}\boldsymbol{j} - F_{z0}\boldsymbol{k} \tag{1}$$

with

$$\begin{pmatrix} F_{x0} \\ F_{y0} \\ F_{z0} \end{pmatrix} = F_0 \begin{Bmatrix} \sin\varphi_0 \\ \cos\varphi_0 \sin\psi_0 \\ \cos\varphi_0 \cos\psi_0 \end{Bmatrix} \approx F_0 \begin{Bmatrix} \varphi_0 \\ \psi_0 \\ 1.0 \end{Bmatrix} \tag{2}$$

$$F_0 = \iint_\Omega p_1\big|_{s_0} r dr d\theta - \iint_\Omega p_2\big|_{s_0} r dr d\theta \tag{3}$$

where p_1 and p_2 are the magnetic stresses produced by magnets 1 and 2 respectively. The integration region Ω includes the internal and external annular domains in Fig.1(b). The couple moment from the magnetic stress is

$$\boldsymbol{M_0} = -M_{x0}\boldsymbol{i} - M_{y0}\boldsymbol{j} - M_{z0}\boldsymbol{k} \tag{4}$$

where

$$\begin{Bmatrix} M_{x0} \\ M_{y0} \\ M_{z0} \end{Bmatrix} = \begin{Bmatrix} -\iint_\Omega p_1\big|_{s_0} r^2 \cos\theta dr d\theta + \iint_\Omega p_2\big|_{s_0} r^2 \cos\theta dr d\theta \\ -\iint_\Omega p_1\big|_{s_0} r^2 \sin\theta dr d\theta + \iint_\Omega p_2\big|_{s_0} r^2 \sin\theta dr d\theta \\ M_{y0}\psi_0 - M_{x0}\varphi_0 \end{Bmatrix} \tag{5}$$

Rotor-dynamic Coefficients

The increment of $\boldsymbol{F}$ in the case of small perturbation can be expressed by linear force stiffness coefficients:

with

$$\begin{Bmatrix} \Delta F_x \\ \Delta F_y \\ \Delta F_z \end{Bmatrix} = \begin{bmatrix} k_{xz} & k_{x\varphi} & k_{x\psi} & k_{xiz} \\ k_{yz} & k_{y\varphi} & k_{y\psi} & k_{yiz} \\ k_{zz} & k_{z\varphi} & k_{z\psi} & k_{ziz} \end{bmatrix} \begin{Bmatrix} \Delta z \\ \Delta\varphi \\ \Delta\psi \\ \Delta i_z \end{Bmatrix} \tag{7}$$

In Eq.(7), the k_{ls} ($l=x, y, z$; $s=z, \varphi, \psi, i_z$) is defined as the force stiffness coefficients in l direction, corresponding to the displacement or angular displacement or control current

$$k_{ls} = \frac{\partial F_l}{\partial s} \tag{8}$$

and it is called as stiffness coefficient of force-displacement or force-angular displacement or force-current with

$$k_{ls} = \frac{\partial F}{\partial s}\Big|_{s_0} = \iint_\Omega \frac{\partial p_1}{\partial s}\Big|_{s_0} r dr d\theta - \iint_\Omega \frac{\partial p_2}{\partial s}\Big|_{s_0} r dr d\theta$$

$$(s=z, \varphi, \psi, i_z) \tag{9}$$

Similarly, introducing moment stiffness coefficients the increment of moment can be expressed as:

$$\Delta\boldsymbol{M_0} = -\Delta M_{x0}\boldsymbol{i} - \Delta M_{y0}\boldsymbol{j} - \Delta M_{z0}\boldsymbol{k} \tag{10}$$

with

$$\begin{Bmatrix} \Delta M_x \\ \Delta M_y \\ \Delta M_z \end{Bmatrix} = \begin{bmatrix} k_{mxz} & k_{mx\varphi} & k_{mx\psi} & k_{mxiz} \\ k_{myz} & k_{my\varphi} & k_{my\psi} & k_{myiz} \\ k_{mzz} & k_{mz\varphi} & k_{mz\psi} & k_{mziz} \end{bmatrix} \begin{Bmatrix} \Delta z \\ \Delta\varphi \\ \Delta\psi \\ \Delta i_z \end{Bmatrix} \tag{11}$$

Where

$$k_{mls} = \frac{\partial M_l}{\partial s} \quad (l=x, y, z; \quad s=z, \varphi, \psi, i_z) \tag{12}$$

is stiffness coefficient of moment-displacement or moment-angular displacement or moment-current with

$$k_{mxs}=\frac{\partial M_x}{\partial s}=-\iint_\Omega \frac{\partial p_1}{\partial s}\Big|_{s_0} r^2\cos\theta dr d\theta+\iint_\Omega \frac{\partial p_2}{\partial s}\Big|_{s_0} r^2\cos\theta dr d\theta$$

$$(s=z,\ \varphi,\ \psi,\ i_z) \qquad (13)$$

$$k_{mys}=\frac{\partial M_y}{\partial s}=-\iint_\Omega \frac{\partial p_1}{\partial s}\Big|_{s_0} r^2\sin\theta dr d\theta+\iint_\Omega \frac{\partial p_2}{\partial s}\Big|_{s_0} r^2\sin\theta dr d\theta$$

$$(s=z,\ \varphi,\ \psi,\ i_z) \qquad (14)$$

CALCULATION AND DISCUSSION

Fig.2 is the schematic of a rotor-magnetic bearing system in a turboexpander/compressor. A set of parameters from the thrust magnetic bearing in the calculation are R_1=33mm, R_2=47mm, R_3=67.2mm, R_4=75mm, W_{0t}=1500N, the thrust bearing clearance c_{0t}=0.5mm, the number of coil N=143, the bias current I_{0t}=4.0A. The distance between two radial magnetic bearings is l=0.275m and the weight of rotor is 123N。

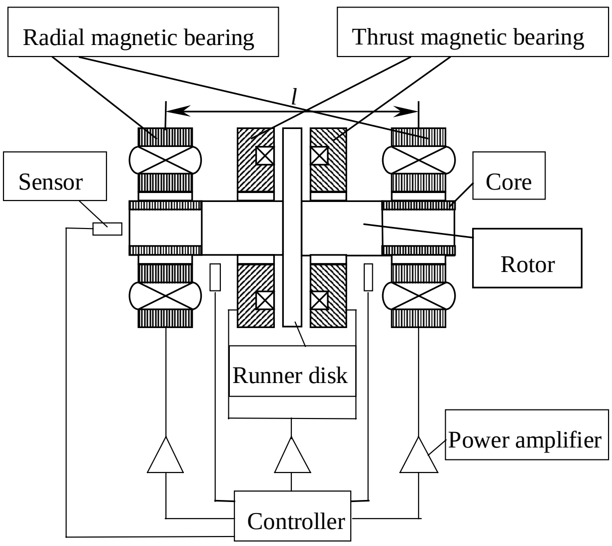


FIGURE 2: 5-axis controlled rotor-magnetic bearing system in a turbo-expander/compressor

Effects of Static Deflection of Runner Disk on Thrust and Radial Magnetic Bearings

Fig.3 shows the variation of the static characteristics of the thrust magnetic bearing versus static tilt parameters $\bar{\varphi}_0$ and $\bar{\psi}_0$ ($\bar{z}_0$=0， $\bar{i}_{z0}$=0.25，"–" indicates non-dimension).

The $\bar{\varphi}_0$ and $\bar{\psi}_0$ have some influence on the static load capacity. Comparing with the case of $\bar{\varphi}_0$= $\bar{\psi}_0$=0, $\bar{F}_{z0}$=0.4236 (that is in the case of neglecting the influence of static tilt parameters)，when $\bar{\varphi}_0$= $\bar{\psi}_0$=±0.4,

factor of 9.2%.

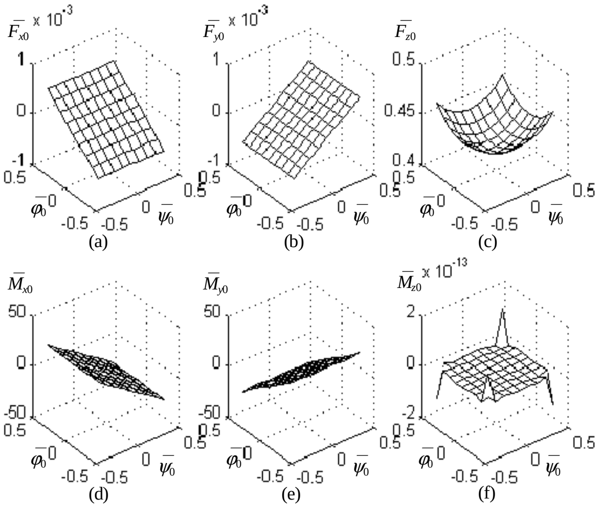


FIGURE 3: Variations of the static characteristics of the thrust magnetic bearing versus static tilt parameters $\bar{\varphi}_0$ and $\bar{\psi}_0$ ($\bar{z}_0$=0, $\bar{i}_{z0}$=0.25)

The couple moment caused by the static tilt of runner disk changes the static load distribution of the radial magnetic bearings and exerts violent coupling effect between the thrust and radial magnetic bearings. The components of moment $\bar{M}_{x0}$ and $\bar{M}_{y0}$ increase with $\bar{\varphi}_0$ and $\bar{\psi}_0$. When the $\bar{\varphi}_0$= $\bar{\psi}_0$=±0.4, the absolute value of $\bar{M}_{x0}$ and $\bar{M}_{y0}$ is 23.428. The corresponding dimensional quantities are 31.880Nm. The resultant of couple moment M is 45.085Nm. This additional couple moment must be supported by two radial magnetic bearings at the distance of l (Fig.2). Each must support additionally the load 163.947N. The additional force load is 2.67 times as much as the gravity load (61.5N) of the rotor on each radial magnetic bearing. Therefore, the couple moment of the thrust magnetic bearing changes the static load distribution, static current and rotor-dynamic coefficients of two radial magnetic bearings and then causes remarkable influence on the dynamic performance of the whole rotor-magnetic bearing system [8].

The coupling effect of the load capacity components $\bar{F}_{x0}$ and $\bar{F}_{y0}$ on the radial magnetic bearings is very small. For example, when the $\bar{\varphi}_0$= $\bar{\psi}_0$=0.4， $\bar{F}_{y0}$=5.382×10^{-4}， the corresponding dimensional quantity is F_{y0}=1.83N. It is only about 1.5% of the gravity load of the rotor. Their coupling effects on the radial magnetic bearing and on lateral vibration are negligible. Because $\bar{M}_{z0}$ is almost equal to 0, the effect of F_x, F_y and M_z is negligible in the analysis of rotor dynamics and the effects of F_z, M_x and M_y are only considered. They can be expressed linearly as

$$\begin{Bmatrix} \Delta F_z \\ \Delta M_x \\ \Delta M_y \end{Bmatrix} = \begin{bmatrix} k_{zz} & k_{z\varphi} & k_{z\psi} & k_{ziz} \\ k_{mxz} & k_{mx\varphi} & k_{mx\psi} & k_{mxiz} \\ k_{myz} & k_{my\varphi} & k_{my\psi} & k_{myiz} \end{bmatrix} \begin{Bmatrix} \Delta z \\ \Delta\varphi \\ \Delta\psi \\ \Delta i_z \end{Bmatrix} \quad (15)$$

Effects of Static Deflection of Runner Disk on the Rotor-dynamic Coefficients

Fig.4 shows variations of rotordynamic coefficients of the thrust magnetic bearing versus the static tilt parameters $\bar{\varphi}_0$ and $\bar{\psi}_0$ ($\bar{z}_0$=0， $\bar{i}_{z0}$=0.25).

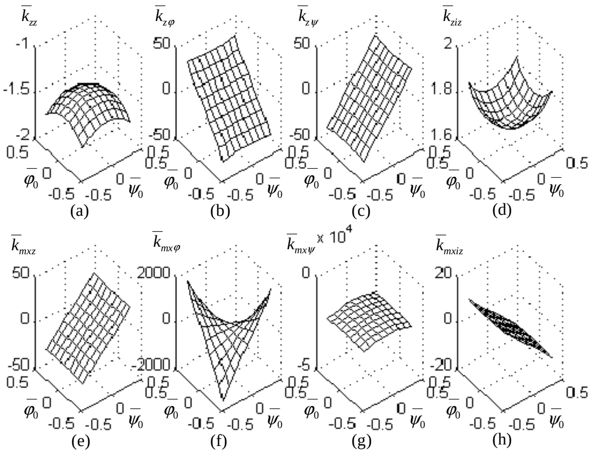


FIGURE 4: Variations of rotordynamic coefficients of thrust magnetic bearing versus the static tilt parameters $\bar{\varphi}_0$ and $\bar{\psi}_0$ ($\bar{z}_0$=0， $\bar{i}_{z0}$=0.25)

The Static tilt parameters $\bar{\varphi}_0$ and $\bar{\psi}_0$ have strong influence on the rotor-dynamic coefficients. Comparing with $\bar{\varphi}_0$= $\bar{\psi}_0$=0， $\bar{k}_{zz}$=−1.4401， $\bar{k}_{ziz}$= 1.6943， $\bar{k}_{mx\psi}$=−1.8393×10^4， $\bar{\varphi}_0$= $\bar{\psi}_0$= ±0.4 causes $\bar{k}_{zz}$=−1.7153, $\bar{k}_{ziz}$=1.8502, $\bar{k}_{mx\psi}$=−2.2009×10^4 , increasing respectively by factors of 19.1%, 9.2% and 19.7%. And the $\bar{k}_{z\varphi}$, $\bar{k}_{z\psi}$, $\bar{k}_{mxz}$, $\bar{k}_{mx\varphi}$ and $\bar{k}_{mxiz}$ change respectively from 0 to ±36.054, ±36.054, ±28.103, ±1867.5 and ±11.0247.

The thrust magnetic bearing decreases the stability of the lateral vibration in the rotor system. It is because the $\bar{k}_{mx\psi}$ (or $\bar{k}_{my\varphi}$) is always negative in any case [8].

The stiffness coefficients of thrust magnetic bearing are more than two. Two stiffness coefficients k_{zz} and k_{ziz} were concerned only and the static tilt parameters $\bar{\varphi}_0$ and $\bar{\psi}_0$ were not considered in the past great majority of literatures [4~7]. However，even if in the ideal case of $\bar{\varphi}_0$= $\bar{\psi}_0$=0, at least $\bar{k}_{mx\psi}$ and $\bar{k}_{my\varphi}$ are not equal to 0 in addition to $\bar{k}_{zz}$ and $\bar{k}_{ziz}$. When the runner disk tilts，all 12 stiffness coefficients in Eq.(15) are not equal to 0. Therefore, at least the four stiffness coefficients $\bar{k}$, $\bar{k}$, $\bar{k}$ and $\bar{k}$ must be considered in ideal case of the runner disk. At least twelve stiffness coefficients in Eq.(15) must be considered in general case of the $\bar{\varphi}_0$= $\bar{\psi}_0$≠0[8].

CONCLUSIONS

The static tilt of the runner disk has remarkable influence on the static and dynamic mechanical characteristics of the thrust magnetic bearing. The emerging coupling moment changes the static load capacity distribution of the radial magnetic bearings and exerts violent coupling effect between thrust and radial magnetic bearings. Therefore the coupling effect of thrust magnetic bearing must be considered in the study of dynamics of a rotor-magnetic bearings system.

The thrust magnetic bearing decreases the stability of the lateral vibration in the rotor system.

When the dynamics of a practical rotor-magnetic bearing system is studied, at least four stiffness coefficients of the thrust magnetic bearing must be considered in ideal case of $\bar{\varphi}_0$= $\bar{\psi}_0$=0. And at least twelve stiffness coefficients must be considered in general case of tilting runner disk.

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