ADAPTIVE INERTIAL AUTOCENTERING OF A RIGID ROTOR WITH UNKNOWN IMBALANCE SUPPORTED BY ACTIVE MAGNETIC BEARINGS

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ABSTRACT

An adaptive inertial autocentering method for rigid rotors with inertial mass imbalance supported by active magnetic bearings is presented. It results in rotation about the inertial center rather than the geometric center of the rotor, therefore drastically reducing the synchronous transmitted forces due to the imbalance. The adaptation algorithm can be interpreted as an observer. It provides an estimate of the inertial center position. Measurement of the rotor angle is not required. Experimental results illustrate the usefulness of such an approach. The method can be extended to operation at variable angular velocity. This is done in such a way that the stability can be discussed using the theory of linear time invariant systems. The convergence of the adaptation is made depend on the angular velocity, thus allowing fast convergence (with respect to time) at high angular velocities.

INTRODUCTION

Active magnetic bearings are of increasing importance for the support of rotors rotating at high speeds. Feedback of the rotor position and velocity, or equivalently proportional-derivative (PD) position feedback, permits its stable positioning. However, as rotating rigid bodies tend to rotate about their mass center and principle axis of inertia rather than about their geometric center, positioning the rotor geometric center leads to vibrations, namely periodic forces at constant rotor angular velocities. In case the application does not require the exact positioning of the geometric center, as is the case in vacuum pumps for instance, it may be useful to suppress these vibrations. This can be done either by feed-forward compensation of the resulting forces or by on-line identification — or adaptation — of the unknown imbalance parameters of the rotor. An interesting solution of this problem, called adaptive autocentering control, has been proposed in the paper [1].

In the present note, inspired by this latter paper, we propose a simplified adaptive autocentering method. A first step in the simplification is the use of complex notation. A full-order observer has been used in experiments in order to show that inertial autocentering can indeed be achieved by such an approach. Experimental results obtained with a spindle constructed at the German company AXOMAT are reported. An extension of the method is then proposed which has the following features:

- The order of the adaptation algorithm, which can be interpreted as a reduced-order observer, is reduced compared to the one in [1].
- The stability of the adaptation algorithm is proven for time-varying angular velocity.
- Measuring the rotor angle is not required; instead, it is sufficient to know the angular velocity and acceleration.
- The stabilization is based on the theory of linear time invariant systems — the stability proof is, thus, simplified.
- The convergence of the adaptation is made depend on the angular velocity. In particular this results in fast convergence (with respect to time) at high rotational speed.

The paper is organized as follows: We start with an explanation of the idea on the example of a rotating disc with imbalance. For notational convenience a complex variable notation is introduced for that purpose. Then we briefly state how the rigid body dynamics of a rotating shaft can be reduced to the equations of the disc and present experimental results obtained with the AXOMAT spindle and a full-order observer which can be shown to work for slowly varying angular velocity. Then the idea is extended to the case of varying rotational speed.

MATHEMATICAL MODEL OF A ROTATING DISC

Most often, for the support of a rotating shaft the magnetic bearings are arranged in two vertical planes — see Figure 2. Then, as in [1], in each of the bearing planes a rigid shaft can be represented as a rotating disc (or planar rotor); see Figure 1. For the dynamics of the mechanical system the way the forces are produced by the magnetic bearings is irrelevant: what is important, is the acceleration due to the bearing forces acting on the rotor in the plane considered. The notational effort is reduced by using complex notation. Then, with Newton's second law, the model of the rotor reads:

$$\ddot{P} = a_p. \tag{1}$$

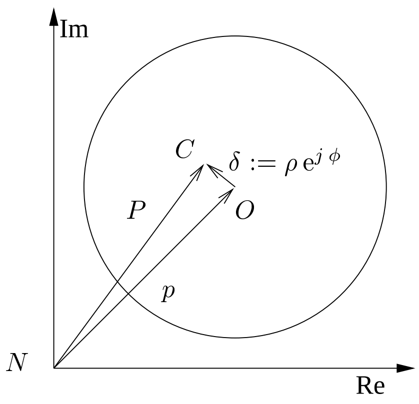

FIGURE 1: The disc with imbalance

Here $Y + j\,Z = P \in \mathbb{C}$ denotes the coordinates of the center of inertia C, $a_p \in \mathbb{C}$ is the acceleration due to the magnetic force and gravity. With $\rho \in \mathbb{R}$ the (unknown) distance between the geometric center O of the rotor, which is represented by $p \in \mathbb{C}$, and ϕ the angle between $\overline{OC}$ and the direction corresponding to the real axis, one has

$$P = p + \rho\, e^{j\phi}.$$

This can be further simplified as

$$P = p + \delta, \tag{2}$$

where $\delta = \rho\, e^{j\phi}$. The latter implies

$$\dot{\delta} = j\omega\delta \qquad \text{and} \qquad \ddot{\delta} = (-\omega^2 + j\dot{\omega})\delta, \tag{3}$$

where $\omega = \dot{\phi}$ is the angular velocity of the rotor.

MODEL OF THE SHAFT

Starting from the equations of motion of the rigid shaft we derive equations equivalent to (1). The setup is rather standard [3]. The model equations are:

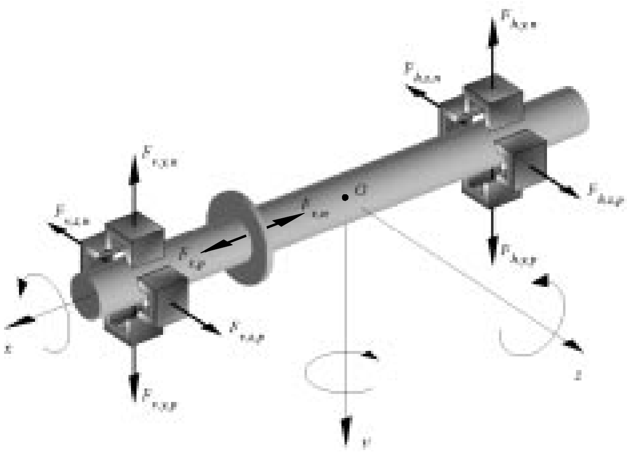

FIGURE 2: The shaft with its radial magnetic bearings.

$$\begin{aligned}
m\,\ddot{X} &= \underbrace{F_{x,p} - F_{x,n}}_{F_x} + m\,g_x \\
m\,\ddot{Y} &= \underbrace{F_{v,y,p} - F_{v,y,n}}_{F_{v,y}} + \underbrace{F_{h,y,p} - F_{h,y,n}}_{F_{h,y}} + m\,g_y \\
m\,\ddot{Z} &= \underbrace{F_{v,z,p} - F_{v,z,n}}_{F_{v,z}} + \underbrace{F_{h,z,p} - F_{h,z,n}}_{F_{h,z}} + m\,g_z \\
\Theta_2\,\ddot{\psi} &= -(l_{f,v} - X)F_{v,z} + (l_{f,h} + X)F_{h,z} - \Theta_1\,\dot{\phi}\,\dot{\theta} \\
\Theta_2\,\ddot{\theta} &= \quad (l_{f,v} - X)F_{v,y} - (l_{f,h} + X)F_{h,y} + \Theta_1\,\dot{\phi}\,\dot{\psi} \\
\Theta_1\,\ddot{\phi} &= D_\phi.
\end{aligned}$$

Here X, Y, and Z are the coordinates of the center of mass G of the shaft in a frame (with axes x, y, and z) fixed in space, at a point being considered as the "center" of the device. The angles ϕ, ψ, and θ describe the angular position of the axes of a body-fixed frame. The coil forces are denoted by $F_\bullet$, the motor torque as D_ϕ. (Here and in the sequel bullets ($\bullet$) are to be replaced by appropriate indices.) The shaft has mass m and moments of inertia Θ_1 and Θ_2; $l_{f,v}$ and $l_{f,h}$ are the distances between the symmetry planes of the bearings (where the forces $F_\bullet$ are produced) and the point G (see Figure 2).

For the treatment of the imbalance the position in x-direction is not important. Therefore, we restrict ourselves to the systems described by the complex variables

$$\chi := \psi + j\theta \qquad \text{and} \qquad P := Y + jZ.$$

The introduction of these complex variables allows us to write the system equations as well as the equations of the controller and the observers in a compact form. The acceleration of the center of mass (due to bearing forces and gravity) in radial directions then reads

$$a_p := a_y + j\, a_z$$

with

$$\begin{aligned} a_y &= \frac{1}{m}\left(F_{v,y} + F_{h,y}\right) + g_y \\ a_z &= \frac{1}{m}\left(F_{v,z} + F_{h,z}\right) + g_z . \end{aligned} \tag{4}$$

The angular acceleration due to bearing forces can be written as

$$a_\chi := a_\psi + j\, a_\theta$$

with

$$\begin{aligned} a_\psi &= \frac{1}{\Theta_2}\left(-(l_{f,v} - X)F_{v,z} + (l_{f,h} + X)F_{h,z}\right) \\ a_\theta &= \frac{1}{\Theta_2}\left((l_{f,v} - X)F_{v,y} - (l_{f,h} + X)F_{h,y}\right) . \end{aligned}$$

With a_p, a_χ, P, and χ the system equations read

$$\ddot{P} = a_p \tag{5}$$

$$\ddot{\chi} = j\omega(t)\frac{\Theta_1}{\Theta_2}\dot{\chi} + a_\chi . \tag{6}$$

Equation (6) is a linear differential equation with a time-varying coefficient, while (5) is time invariant.

STABILIZING ADAPTIVE FEEDBACK

Feedback is required for the stabilization of the rotor position. However, if the objective is the control of the position P of the inertial center C a difficulty arises, because the rotor imbalance, and thus P (or equivalently δ), is unknown.

If P was known one could use the feedback

$$a_p = -k_1\dot{P} - k_0 P \tag{7}$$

with $0 < k_0, k_1 \in \mathbb{R}$ (Different parameters, depending on ω will be used below.) in order to get the stable system:

$$\ddot{P} + k_1\dot{P} + k_0 P = 0.$$

Assume now the position p of O were measured (which will most often be the case) and that in addition $\dot{p}$ were available by measurement, numerical differentiation, or an observer. Moreover, let $\hat{\delta}$ be the estimate of the imbalance δ which will be provided by the adaptation system to be derived in the sequel. Then, instead of (7), the feedback

$$a_p = -k_1\dot{p} - k_0 p - (k_1 j\omega + k_0)\,\hat{\delta} \tag{8}$$

can be used. This feedback results in

$$\begin{aligned} \ddot{p} + k_1\dot{p} + k_0 p &= k_1\dot{\delta} + k_0\delta - (k_1 j\omega + k_0)\,\hat{\delta} \\ &= (k_1 j\omega + k_0)\,\tilde{\delta}, \end{aligned} \tag{9}$$

with the estimation error $\tilde{\delta} = \delta - \hat{\delta}$. Thus, if the estimation error exponentially converges to zero, the position P of C and

$$a_p = -k_1\dot{P} - k_0 P + (k_1 j\omega + k_0)\,\tilde{\delta}$$

will converge to zero, too.

In the sequel we present two possibilities to obtain estimates of δ (and its time derivative $\dot{\delta}$). The first one is a simple observer estimating position and velocity of the center of mass as well as its deviation δ from the geometric center. Stability of this observer is guaranteed only for constant or slowly varying angular velocity ω. The second possibility consists of an adaptation scheme which can be used with non-constant ω.

Full-order Observer

In this section we assume the angular velocity ω to be constant. Then both (5) and (6) are linear and time invariant.

This section is dedicated to the description of what has been implemented to obtain the experimental results of the next section. To avoid steady state position errors (caused e. g., by modelling errors) the observer is extended such that constant disturbance accelerations can be estimated. The estimates can be used in the controller to compensate the disturbances.

We only treat the design of an observer for the coordinate P of the center of mass. The coordinates of the geometric center are assumed to be measured. The observer design for the angular coordinates χ is only slightly more involved.

The differential equation for the center of mass P is

$$\ddot{P} = a_p + d_p, \tag{10}$$

where $a_p = a_y + j\, a_z$ is the acceleration of the center of mass due to bearing forces $F_\bullet$ and gravity (see eq. (4)) and d_p is a constant disturbance acceleration, i. e.,

$$\dot{d}_p = 0,$$

which is estimated to compensate for modelling errors (analogous to what is usually done by an integral part of the controller).

The relation between the geometric center p and the center of mass is

$$P = p + \rho\, \mathrm{e}^{j\omega t + \phi_0} = p + \delta_p,$$

where ρ is the distance between the geometric center and the center of mass. Both ρ and ϕ_0 are unknown. We only know that the complex variable differential equation

$$\dot{\delta}_p = j\omega\delta_p$$

holds for δ_p.

The observer for the position of the center of mass can be written as a simulator of the system extended by a stabilizing injection of the observation error $\tilde p = p - (\hat P - \hat\delta_p)$:

$$\begin{aligned} \dot{\hat P} &= \hat P_v & &+ l_1 \tilde p \\ \dot{\hat P}_v &= a_p + \hat d_p & &+ l_2 \tilde p \\ \dot{\hat d}_p &= & &l_3 \tilde p \\ \dot{\hat \delta}_p &= j\,\omega\,\hat\delta_p & &+ l_4 \tilde p. \end{aligned} \tag{11}$$

With the third equation of (11) we estimate the constant disturbance ($\dot{d}_p = 0$) and with the fourth one the deviation δ_p of the center of mass P from the geometric center p.

The complex observer gains $l_1, \ldots, l_4$ can be chosen such that the differential equation for the observer error $\tilde p$ is stable with eigenvalues independent of the angular velocity ω.

Using the estimated position $\hat P$, the corresponding velocity $\hat P_v$, and the constant disturbance estimate $\hat d_p$ in the feedback

$$a_p = -\hat d_p - k_1 \hat P_v - k_0 \hat P \tag{12}$$

with controller gains $k_0, k_1 > 0$ the center of mass is stabilized at the origin of a frame fixed in space.

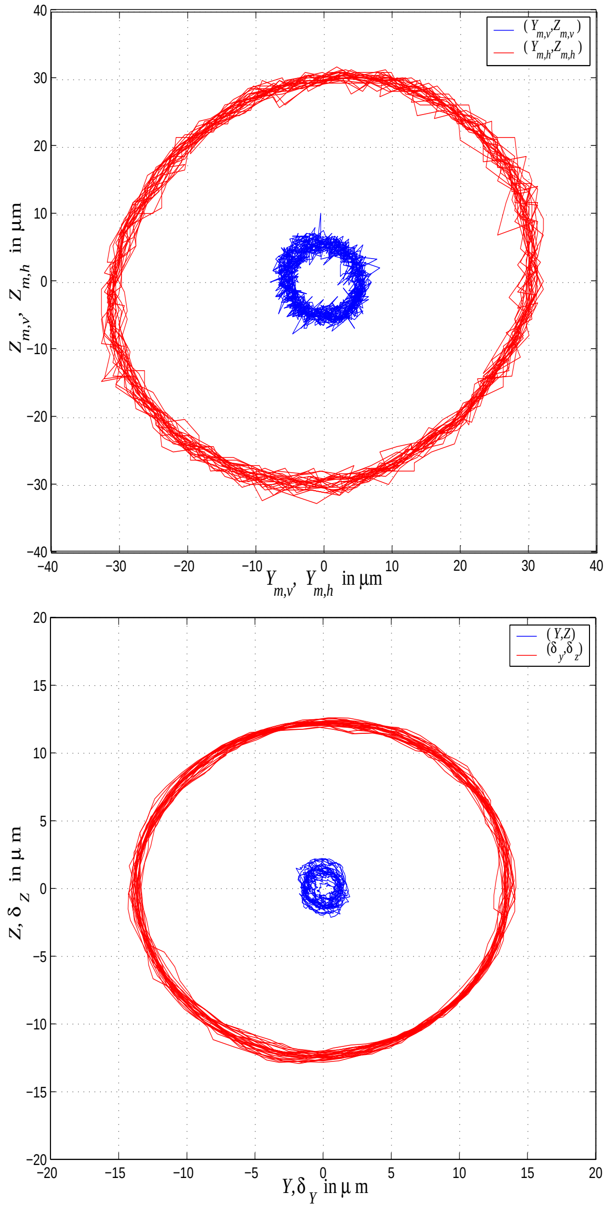

FIGURE 3: Shaft with additional imbalance rotating at 15000rpm. Top: measured path in the measurement planes. Bottom: estimated path of the geometric center with respect to center of mass

Experimental Results

The observer (11) together with the feedback (12) has been implemented on a dSPACE DS1103 controller board. On the test bed of AXOMAT verified the usefulness of the proposed observer.

In this experiment the imbalance of the shaft was considerably increased by adding an extra mass. In Figure 3 the position in the v-measurement plane is shown while the shaft rotates at 15000rpm. As expected, the geometric center position in this plane moves along a circular path while the coil currents show only slight variation. This means, the periodic bearing forces are small (see Figure 4). Note that in Figure 4 the current $i_{v,y,p}$ is zero as a consequence of the flatness-based control method used — see [4, 2]. The top plot in Figure 4 shows the time dependence of the position in the v-measurement plane.

EXTENSION TO VARIABLE SPEED

For the design of the observer (11) we have assumed $\omega = \text{const.}$. Now we want to propose an adaptation scheme which allows to treat the case where ω is a (sufficiently smooth) function of time. We will present the idea using the equations of the disc from the beginning of the paper.

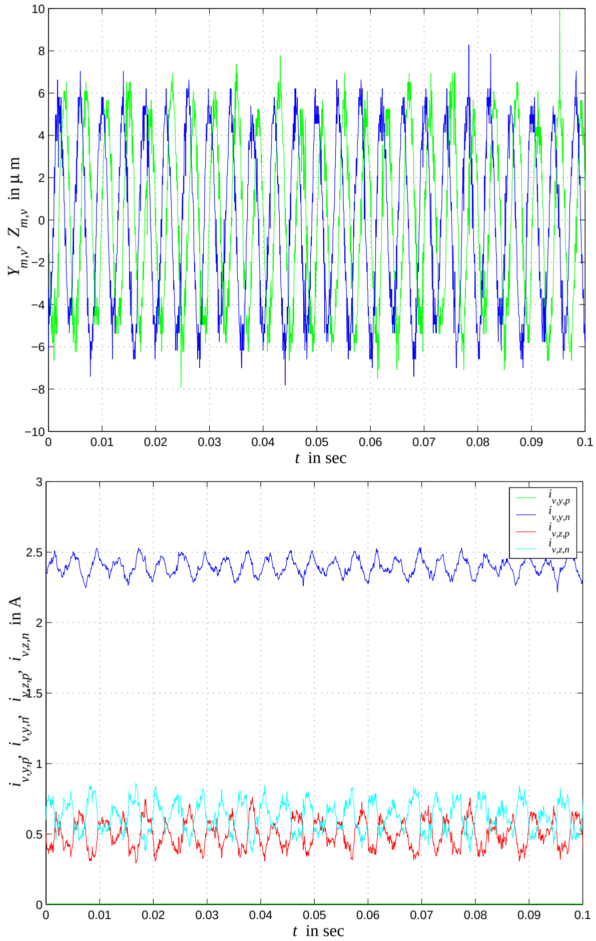

FIGURE 4: Shaft with additional imbalance rotating at 15000rpm. Top: measured position (as function of time) in the v-measurement plane. Bottom: currents in the coils of the v-bearing

Adaptation Algorithm for Variable Rotational Speed

The estimate $\hat{\delta}$ of δ is provided by the reduced-order observer (or adaptation algorithm)

$$\begin{aligned} \dot{\zeta} &= \gamma\hat{\delta} - \lambda a - \dot{\lambda}\dot{p} \qquad (13)\\ &= \gamma(\zeta + \lambda\dot{p}) \\ &\quad + \lambda\left(k_1\dot{p} + k_0 p + (k_1 j\omega + k_0)(\zeta + \lambda\dot{p})\right) - \dot{\lambda}\dot{p} \\ \hat{\delta} &= \zeta + \lambda\dot{p}. \end{aligned}$$

The possibly non-constant parameters γ and λ will be determined on the basis of the error dynamics. The derivative of $\hat{\delta}$ is

$$\begin{aligned} \dot{\hat{\delta}} &= \dot{\zeta} + \dot{\lambda}\dot{p} + \lambda\ddot{p} \\ &= \gamma\hat{\delta} - \lambda a - \dot{\lambda}\dot{p} + \dot{\lambda}\dot{p} + \lambda(\ddot{P} - \ddot{\delta}) \\ &= \gamma\hat{\delta} + \lambda(\omega^2 - j\dot{\omega})\delta, \end{aligned}$$

the second equation following from the right hand side of (13) and (2), the third one from (1), (3), and (8). The estimation error $\tilde{\delta} = \delta - \hat{\delta}$ satisfies

$$\dot{\tilde{\delta}} = \dot{\delta} - \dot{\hat{\delta}} = j\omega\delta - \gamma\hat{\delta} - \lambda(\omega^2 - j\dot{\omega})\delta\,.$$

Defining the parameter λ by

$$(\omega^2 - j\dot{\omega})\lambda = j\omega - \gamma$$

one obtains $\dot{\tilde{\delta}} = \gamma\tilde{\delta}$.

Then

$$\tilde{\delta}(t) = \exp\left(\int_{s=0}^{s=t} \gamma\, ds\right)\tilde{\delta}(0),$$

and clearly, $\tilde{\delta}$ can be made converge to zero exponentially by choosing the real part $\Re(\gamma)$ negative:

$$|\tilde{\delta}(t)| \leq \exp\left(\int_{s=0}^{s=t} \Re\left(\gamma(s)\right)\, ds\right) |\tilde{\delta}(0)|.$$

Using

$$\gamma = j\omega + \gamma_0(\omega^2 - j\dot{\omega})$$

yields

$$\lambda = -\gamma_0.$$

The requirement $\Re(\gamma) < 0$ then means

$$\omega^2\Re(\gamma_0) + \dot{\omega}\Im(\gamma_0) = \Re(\gamma) < 0.$$

Using the Rotor Angle instead of Time

It is worth reconsidering the closed-loop equation (9). For this, consider ϕ as a "new time":

$$\frac{d\phi}{dt} = \omega \qquad \text{or, equivalently} \qquad \phi = \int_{t_0}^{t} \omega(s)\, ds.$$

Observe that the new time is reversed if $\omega < 0$. Therefore, the angular velocity ω must not change sign during operation for this to make sense. Denoting the derivatives with respect to ϕ by primes

$$\dot{P} = \frac{dP}{dt} = \frac{dP}{d\phi}\frac{d\phi}{dt} = P'\omega.$$

Analogously,

$$\ddot{P} = \frac{d\dot{P}}{d\phi}\omega = (P''\omega + P'\omega')\omega.$$

The closed-loop equation (9) can thus be rewritten as

$$\omega^2 P'' + \omega'\omega P' + k_1\omega P' + k_0 P = (k_1 j\omega + k_0)\,\tilde{\delta}.$$

With

$$k_0 = \omega^2\kappa_0 \qquad \text{and} \qquad k_1 = \omega\kappa_1 - \omega',$$

after dividing by ω^2 it results

$$P'' + \kappa_1 P' + \kappa_0 P = \left(j\kappa_1 - j\frac{\omega'}{\omega} + \kappa_0 \right) \tilde{\delta}. \qquad (14)$$

This equation describes the closed-loop dynamics with respect to ϕ. It is asymptotically stable at $\omega > 0$ if κ_1 and κ_0 are chosen as

$$\kappa_1 = -(\lambda_1 + \lambda_2) \qquad \text{and} \qquad \kappa_0 = \lambda_1 \lambda_2,$$

with λ_1, λ_2 complex numbers with negative real parts (which need not be complex conjugate, P being complex). By the time reversion, positive real parts are required at $\omega < 0$. (One may use $\kappa_1 = -\text{sign}(\omega)(\lambda_1 + \lambda_2)$.)

If the angular velocity ω is constant, the coefficients on the right hand side of (14) are independent of ω — this is different in (9). As a consequence, with this choice of the feedback gains k_0 and k_1 at constant ω the closed-loop behavior with respect to ϕ is independent of ω. While the estimation error $\tilde{\delta}$ is multiplied by ω on the right hand side of (9), which leads to a stronger excitation of the dynamics of P at larger constant ω, this effect is eliminated in (14). The convergence is now exponential in the rotor angle ϕ. If $\tilde{\delta} = 0$ an error on P is, thus, reduced by the same amount during one rotor turn for any ω.

Let us now consider the behavior of the adaptation scheme with respect to the new time ϕ. From $\dot{\tilde{\delta}} = \gamma\tilde{\delta}$ it follows

$$\tilde{\delta}' = \frac{\gamma}{\omega}\tilde{\delta} = (j + \gamma_0(\omega - j\omega'))\,\tilde{\delta},$$

hence:

$$\tilde{\delta}(\phi) = e^{j\phi} \exp\left(\int_{s=0}^{s=\phi} \gamma_0\left(\omega(s) - j\omega'(s)\right)\, ds \right) \tilde{\delta}(0).$$

The convergence of the error w.r.t. ϕ depends on the complex parameter $j + \gamma_0(\omega - j\omega')$, the real part of which is $\omega\Re(\gamma_0) + \omega'\Im(\gamma_0)$. It follows that with $\omega > 0$ the error dynamics is exponentially stable w.r.t. ϕ if $\omega^2\Re(\gamma_0) + \dot{\omega}\Im(\gamma_0) = \Re(\gamma) < 0$ — which confirms the result obtained in time t before.

A useful choice is

$$\gamma_0 = \frac{\alpha}{\omega}, \qquad \text{with} \qquad 0 > \alpha \in \mathbb{R}.$$

With this choice

$$\begin{aligned}
\tilde{\delta}(\phi) &= e^{j\phi} \exp\left(\int_{s=0}^{s=\phi} \alpha\left(1 - j(\ln\omega)'(s)\right) ds \right) \tilde{\delta}(0) \\
&= e^{j\phi} \exp\left(\alpha\phi - j\alpha \left(\ln\left(\frac{\omega}{\omega(0)} \right) \right) \right) \tilde{\delta}(0) \\
&= e^{\alpha\phi} e^{j\phi} \left(\frac{\omega}{\omega(0)} \right)^{-j\alpha} \tilde{\delta}(0).
\end{aligned}$$

The convergence is thus determined by α:

$$|\tilde{\delta}(\phi)| = e^{\alpha\phi} |\tilde{\delta}(0)|.$$

CONCLUSIONS AND FURTHER WORK

Observer-based adaptive inertial autocentering has been shown to be useful. The experimental results were, however, obtained with an observer assuming constant rotation speed. An extension has been proposed which should offer a way to overcome this "constant-speed-drawback". For the implementation the proposed scheme will have to be discretized. In this respect it is important that the problem has been reduced to one which can be treated using the theory of linear time-invariant systems. By this linearity it is also possible to draw conclusions on the separability of the adaptation and the feedback dynamics. Investigating the practical usefulness of the approach is part of current research.

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