IDENTIFICATION OF MASS UNBALANCE AND SENSOR RUNOUT ON A ROTOR EQUIPPED WITH MAGNETIC BEARINGS

Yoichi Kanemitsu
Kyushu University, Fukuoka, Japan, kanemitu@mech.kyushu-u.ac.jp
Sinya Kijimoto
Kyushu University, Fukuoka, Japan, kiji@mech.kyushu-u.ac.jp
Koichi Matsuda
Kyushu University, Fukuoka, Japan, matsuda@mech.kyushu-u.ac.jp
Park Tea Jin
Kyushu University, Fukuoka, Japan, tjpark@sky.mech.kyushu-u.ac.jp

ABSTRACT
In this paper, we propose a new identification method of the sensor runout and the unbalance on a rigid rotor supported by active magnetic bearings applying the incremental least square on-line method and perform some numerical simulations on the their identification applying to a rigid rotor model of a turbo-molecular pump. The paper also presents some results on the identification accuracy and effects of the rotor model error. From the numerical simulations we conclude that the proposed identification method is effective for the simultaneous identification of the unbalance of rotor and the sensor runout.

INTRODUCTION
An active magnetic bearing(AMB) is non-contact, frictionless and has the ability to actively control the bearing force and the journal eccentricity in the bearing. The application of AMB also has a possibility to compensate the mass unbalance on the rotor.

Two methods are generally adopted to reduce the unbalance vibration of the rotor levitated by AMB. One method is called the peak-gain method (PG method), in which the feedback gain of the AMB controller is extremely high only at the rotating speed frequency. Another method is called the feed foreword method (FF method)[1], in which the unbalance on the rotor is estimated from the measured vibration signals by the AMB proximity sensor and a compensating magnetic pull from the AMB is added to the rotor in opposite direction of the unbalance force so as to cancel out the mass unbalance force. The FF method is better than the PG method from a viewpoint of the system stability. But it is essential to estimate the unbalance correctly for the FF method. If there is any error in the unbalance estimation, unbalance vibration remains in proportion to the estimation error.

In a magnetically levitated rotor by AMB eddy current type proximity sensors are usually employed to measure the clearance between rotor and stator.

Since the eddy current type proximity sensor is sensitive to circumferential irregularity of conductivity and permeability of sensor target material on the rotor, the sensor output signal contains components proportional to the circumferential irregularity and the shaft displacement.

The sensor output caused by the circumferential irregularity is called electrical sensor runout[2]. The proximity sensor runout enters into the control circuit of the AMB and then the magnetic pull induced by the runout signal whirls the rotor which is supported by the AMB just as an unbalance on the rotor does. And moreover the undesirable sensor runout saturates the control current from the power source.

It is necessary for realization of minimal rotor whirl by the FF method to identify the sensor runout and the unbalance at the same time, compensate the unbalance force by the FF method and eliminate the sensor runout from the measured proximity signal of the AMB simultaneously. It is recommended for minimization of the control current to use the modified proximity signal without the runout for the AMB levitation control.

EQUATION OF MOTION OF RIGID ROTOR SUPPORTED BY ACTIVE MAGNETIC BEARINGS
A simple AMB model is shown in figure 1. AMB levitates a rotor by regulating the current in magnetic

coil and hold the bearing clearance (gap) constant. An attractive force from AMB is linearized with respect to the control current i and the gap change x as follows:

$$f = -Fi + Gx \tag{1}$$

where

$$F = \frac{\mu_0 S N^2}{4}\left\{\frac{i_{P0}}{x_{P0}^{\,2}} + \frac{i_{N0}}{x_{N0}^{\,2}}\right\},$$

$$G = \frac{\mu_0 S N^2}{4}\left\{\frac{i_{P0}^{\,2}}{x_{P0}^{\,3}} + \frac{i_{N0}^{\,2}}{x_{N0}^{\,3}}\right\}$$

F, G are called AMB control stiffness and negative position stiffness respectively.

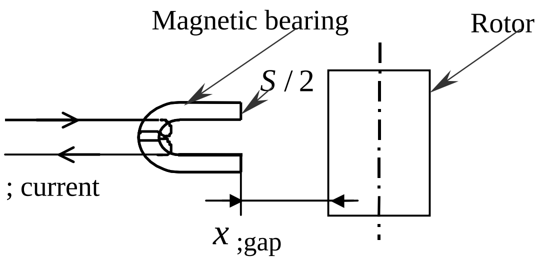


FIGURE 1: A Model of Active Magnetic Bearing

In this paper, we try to identify the unbalance and the sensor runout on a rotor of a small turbo-molecular pump levitated by AMB, of which photograph is shown in figure 2.

In order to make a numerical model of AMB, the transfer function of the controller has been measured and approximated with PID type controller which is represented as Equation (2) and the measured and the estimated transfer functions are shown in figure 3. The solid line and the dotted line represent the measured transfer function and the approximated one. The estimated PID gain is also tabulated in table 1

$$i = k_P \hat{x} + k_I \int \hat{x}\, dt + k_D \frac{d\hat{x}}{dt} \tag{2}$$

The rotor of the turbo-molecular pump is levitated by 2 AMBs and has a massive cylinder with rotor blades on the top end. Now the cylinder is replaced by a disk for the convenience of the verification experiment of the proposing identification method. The tested rotor is drawn in figure 4.

Equation of motion of the rigid rotor shown in figure 4 and levitated by 2 AMBs is given as following equation.

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \omega \boldsymbol{M}_1 \dot{\boldsymbol{x}} + \boldsymbol{F}\boldsymbol{i} + \boldsymbol{G}\boldsymbol{x}_s = \boldsymbol{U}_i \tag{3}$$

where $\boldsymbol{x}, \boldsymbol{x}_s, \boldsymbol{x}_r, \boldsymbol{i}$ are the actual rotor displacement vector, the measured displacement vector at AMB sensor, the sensor runout vector and the AMB control current vector.

TABLE 1: Estimated control gain of AMB from measured transfer function

	Bearing 1	Bearing 2
k_p (A/m)	7810	6950
k_I (A/ms)	98300	49300
k_D (As/m)	9.83	4.4

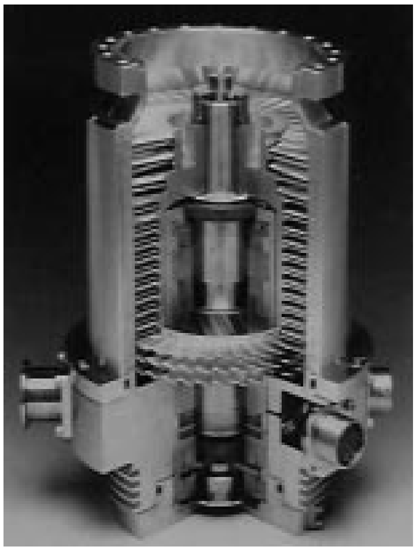


FIGURE 2: Photograph of the turbo molecular pump rotor

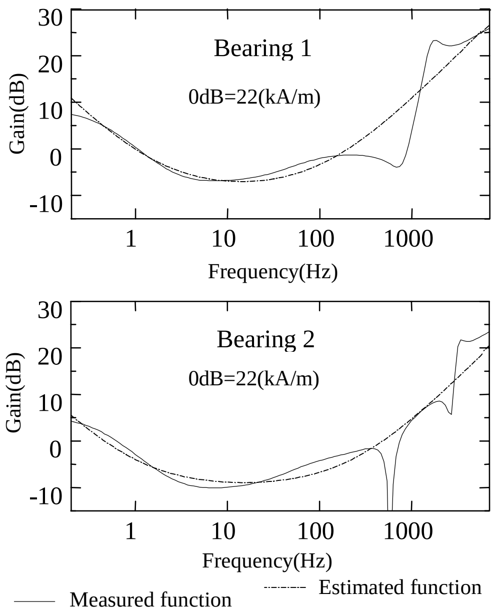


FIGURE 3: Measured and estimated transfer function

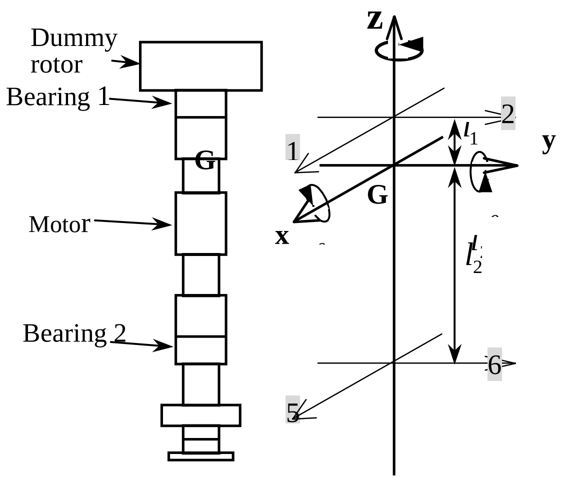


FIGURE 4: Rotor Model of Turbo-molecular Pump (1,2,5,6 indicate the upper bearing in x direction and y-direction and the the lower bearing x-direction and y-direction respectively. G is the center of gravity of the rotor)

$\boldsymbol{M}, \boldsymbol{M}_1, \boldsymbol{U}_i, \boldsymbol{F}, \boldsymbol{G}$ are the mass matrix, the gyro-moment matrix, the rotor unbalance matrix, the control stiffness matrix and the negative position stiffness matrix .

$$\boldsymbol{x} = [x \quad y \quad \theta_x \quad \theta_y]^T, \boldsymbol{x}_s = [x_1 \quad x_2 \quad x_5 \quad x_6]^T$$

$$\boldsymbol{x}_r = [x_{1r} \quad x_{2r} \quad x_{5r} \quad x_{6r}]^T, \hat{\boldsymbol{x}} = [\hat{x}_1 \quad \hat{x}_2 \quad \hat{x}_5 \quad \hat{x}_6]^T$$

$$\boldsymbol{i} = [i_1 \quad i_2 \quad i_5 \quad i_6]^T \ \boldsymbol{M} = \mathrm{diag}(m, m, I_d, I_d)$$

$$\boldsymbol{M}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_p \\ 0 & 0 & -I_p & 0 \end{bmatrix}, \boldsymbol{F} = \begin{bmatrix} F_1 & 0 & F_5 & 0 \\ 0 & F_2 & 0 & F_6 \\ 0 & -F_2 l_1 & 0 & F_6 l_2 \\ F_1 l_1 & 0 & -F_5 l_2 & 0 \end{bmatrix}$$

$$\boldsymbol{G} = \begin{bmatrix} -G_1 & 0 & -G_5 & 0 \\ 0 & -G_2 & 0 & -G_6 \\ 0 & G_2 l_1 & 0 & -G_6 l_2 \\ -G_1 l_1 & 0 & G_5 l_2 & 0 \end{bmatrix}$$

$$\boldsymbol{U}_i = \begin{bmatrix} m\varepsilon\omega^2 \cos(\omega t + \phi) \\ m\varepsilon\omega^2 \sin(\omega t + \phi) \\ -(I_d - I_p)\tau\omega^2 \sin(\omega t + \psi) \\ (I_d - I_p)\tau\omega^2 \cos(\omega t + \psi) \end{bmatrix}$$

$$\hat{\boldsymbol{x}} = \boldsymbol{x}_s + \boldsymbol{x}_r \tag{4}$$

The shaft displacement at the center of gravity and the sensor position of the AMBs is given from the geometrical relation of the shaft as follows:

$$\boldsymbol{x} = \boldsymbol{L}\boldsymbol{x}_s = \boldsymbol{L}(\hat{\boldsymbol{x}} - \boldsymbol{x}_r) \tag{5}$$

$$\boldsymbol{L} = \frac{1}{l_1 + l_2} \begin{bmatrix} l_2 & 0 & l_1 & 0 \\ 0 & l_2 & 0 & l_1 \\ 0 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 \end{bmatrix}$$

As we have assumed the controller of the AMB to be the PID controller, the control current of the AMB are described as follows:

$$\boldsymbol{i} = \boldsymbol{K}_D \dot{\hat{\boldsymbol{x}}} + \boldsymbol{K}_P \hat{\boldsymbol{x}} + \boldsymbol{K}_I \int \hat{\boldsymbol{x}} dt \tag{6}$$

where

$\boldsymbol{K}_D = \mathrm{diag}(K_{D1}, K_{D2}, K_{D5}, K_{d6}), \boldsymbol{K}_P = \mathrm{diag}(K_{P1}, K_{P2}, K_{P5}, K_{P6})$

$\boldsymbol{K}_I = \mathrm{diag}(K_{I1}, K_{I2}, K_{I5}, K_{I6})$

Now we adopt the estimated values in table 1 as the controller gain k_{Di}, K_{Pi}, K_{Ii} $(i = 1,2,5,6)$.

Substituting equations (4)(5)(6) for $\boldsymbol{x}_s, \boldsymbol{x}, \boldsymbol{i}$ of equation (3), we obtain a following equation of motion:

$$\boldsymbol{ML}\ddot{\hat{\boldsymbol{x}}} + (\omega\boldsymbol{M}_1\boldsymbol{L} + \boldsymbol{FK}_D)\dot{\hat{\boldsymbol{x}}} + (\boldsymbol{FK}_P + \boldsymbol{G})\hat{\boldsymbol{x}} + \boldsymbol{FK}_I \int \hat{\boldsymbol{x}} dt$$
$$= \boldsymbol{U}_i + \boldsymbol{ML}\ddot{\boldsymbol{x}}_r + \omega\boldsymbol{M}_1\boldsymbol{L}\dot{\boldsymbol{x}}_r + \boldsymbol{G}\boldsymbol{x}_r \tag{7}$$

The unbalance matrix $\boldsymbol{U}_i$ is represented as follows:

$$\boldsymbol{U}_i = \omega^2 \boldsymbol{M}_2 \begin{bmatrix} \varepsilon \cos(\omega t + \phi) \\ \varepsilon \sin(\omega t + \phi) \\ -\tau \sin(\omega t + \psi) \\ \tau \cos(\omega t + \psi) \end{bmatrix} \tag{8}$$

where $\boldsymbol{M}_2 = \mathrm{diag}(m, m, I_d - I_{P,} I_d - I_P)$

ε, ϕ are mass eccentricity and its phase, τ, ψ are inclination of the central principal axis of inertia and its phase and ω is rotating speed.

The sensor runout vector $\boldsymbol{x}_r$ is represented as follows:

$$\boldsymbol{x}_r = \begin{bmatrix} \lambda_1 \cos(\omega t + \phi_1) \\ \lambda_1 \sin(\omega t + \phi_1) \\ \lambda_2 \cos(\omega t + \phi_2) \\ \lambda_2 \sin(\omega t + \phi_2) \end{bmatrix} \tag{9}$$

where λ, ϕ are the synchoronous components of the sensor runout and their phases and subscripts 1,2 identify 2 radial bearings.

IDENTIFICATION METHOD

The identification method is as follows:

(1)Laplace tramsforms of the Equation of motion

Taking Laplace transforms of equation (7), we obtain the left-hand term of the equation as

$$\left[\boldsymbol{ML}s^2 + (\omega\boldsymbol{M}_1\boldsymbol{L} + \boldsymbol{FK}_D)s + (\boldsymbol{FK}_P + \boldsymbol{G}) + \boldsymbol{FK}\frac{1}{s}\right]\hat{\boldsymbol{X}}(s)$$

(10a)

where $L[\bullet]$ is the Laplace transforms and $L(\hat{\boldsymbol{x}}) = \hat{\boldsymbol{X}}(s)$.

The right-hand term of equation (7) by the Laplace transforms is given as follows:

First term of the right-hand term

$$L[\boldsymbol{U}_i]=\frac{\omega^2}{s^2+\omega^2}\boldsymbol{M}_2\begin{bmatrix}\varepsilon(s\cos\phi-\omega\sin\phi)\\ \varepsilon(\omega\cos\phi+s\sin\phi)\\ -\tau(\omega\cos\psi+s\sin\psi)\\ \tau(s\cos\psi-\omega\sin\psi)\end{bmatrix}$$

$$=\boldsymbol{M}_2\left[s\omega\boldsymbol{H}_{01}+\omega^2\boldsymbol{H}_{02}\right]U(s)\boldsymbol{E} \qquad (10b)$$

where

$$U(s)=L[\sin(\omega t)]=\frac{\omega}{s^2+\omega^2}$$

$$\boldsymbol{E}=\begin{bmatrix}\boldsymbol{E}_0\\ \boldsymbol{E}_1\end{bmatrix},\ \boldsymbol{E}_0=\begin{bmatrix}\varepsilon\cos\phi\\ \varepsilon\sin\phi\\ \tau\cos\psi\\ \tau\sin\psi\end{bmatrix},\ \boldsymbol{E}_1=\begin{bmatrix}\lambda_1\cos\phi_1\\ \lambda_1\sin\phi_1\\ \lambda_2\cos\phi_2\\ \lambda_2\sin\phi_2\end{bmatrix}$$

$$\boldsymbol{H}_{01}=\begin{bmatrix}\begin{matrix}1&0&0&0\\0&1&0&0\\0&0&0&-1\\0&0&1&0\end{matrix}&\boldsymbol{0}\end{bmatrix},\ \boldsymbol{H}_{02}=\begin{bmatrix}\begin{matrix}0&-1&0&0\\1&0&0&0\\0&0&-1&0\\0&0&0&-1\end{matrix}&\boldsymbol{0}\end{bmatrix}$$

$\boldsymbol{0}$ is 4×4 zero matrix.

Second term of the right-hand term

$$\boldsymbol{ML}L[\ddot{x}_r]=\frac{-\omega^2}{s^2+\omega^2}\boldsymbol{ML}\begin{bmatrix}\lambda_1(s\cos\phi_1-\omega\sin\phi_1)\\ \lambda_1(\omega\cos\phi_1+s\sin\phi_1)\\ \lambda_2(s\cos\phi_2-\omega\sin\phi_2)\\ \lambda_2(\omega\cos\phi_2+s\sin\phi_2)\end{bmatrix}$$

$$=\boldsymbol{ML}\left[s\omega\boldsymbol{H}_{11}+\omega^2\boldsymbol{H}_{12}\right]U(s)\boldsymbol{E} \qquad (10c)$$

where $\boldsymbol{I}_n=\mathrm{diag}(1,1,1,1)$

$$\boldsymbol{H}_{11}=[\boldsymbol{0}\quad -\boldsymbol{I}_n],\ \boldsymbol{H}_{12}=\begin{bmatrix}\boldsymbol{0}&\begin{matrix}0&1&0&0\\-1&0&0&0\\0&0&0&1\\0&0&-1&0\end{matrix}\end{bmatrix}$$

Third term of the right-hand term

$$\omega\boldsymbol{M}_1\boldsymbol{L}L[\dot{x}_r]=\frac{\omega^2}{s^2+\omega^2}\boldsymbol{M}_1\boldsymbol{L}\begin{bmatrix}-\lambda_1(\omega\cos\phi_1+s\sin\phi_1)\\ \lambda_1(s\cos\phi_1-\omega\sin\phi_1)\\ -\lambda_2(\omega\cos\phi_2+s\sin\phi_2)\\ \lambda_2(s\cos\phi_2-\omega\sin\phi_2)\end{bmatrix}$$

$$=\omega\boldsymbol{M}_1\boldsymbol{L}\left[s\boldsymbol{H}_{21}+\omega\boldsymbol{H}_{22}\right]U(s)\boldsymbol{E} \qquad (10d)$$

where $\boldsymbol{H}_{21}=-\boldsymbol{H}_{12}$, $\boldsymbol{H}_{22}=\boldsymbol{H}_{11}$

Fourth term of the right-hand term

$$\boldsymbol{G}L[\boldsymbol{x}_r]=\frac{1}{s^2+\omega^2}\boldsymbol{G}\begin{bmatrix}\lambda_1(s\cos\phi_1-\omega\sin\phi_1)\\ \lambda_1(\omega\cos\phi_1+s\sin\phi_1)\\ \lambda_2(s\cos\phi_2-\omega\sin\phi_2)\\ \lambda_2(\omega\cos\phi_2+s\sin\phi_2)\end{bmatrix}$$

$$=\boldsymbol{G}\left[s\frac{1}{\omega}\boldsymbol{H}_{31}+\boldsymbol{H}_{32}\right]U(s)\boldsymbol{E} \qquad (10e)$$

where $\boldsymbol{H}_{31}=[\boldsymbol{0}\quad \boldsymbol{I}_n]$, $\boldsymbol{H}_{32}=\boldsymbol{H}_{21}$

In the end, we obtain the Laplace transforms of equation (7) as follows:

$$\left[\boldsymbol{MLs}^2+(\omega\boldsymbol{M}_1\boldsymbol{L}+\boldsymbol{FK}_D)s+(\boldsymbol{FK}_P+\boldsymbol{G})+\boldsymbol{FK}_I\frac{1}{s}\right]\hat{\boldsymbol{X}}(s)$$

$$=[\boldsymbol{H}_1(\omega)s+\boldsymbol{H}_2(\omega)]U(s)\boldsymbol{E}$$

(11)

where

$$\boldsymbol{H}_1(\omega)=\omega\boldsymbol{M}_2\boldsymbol{H}_{01}+\omega\boldsymbol{MLH}_{11}+\omega\boldsymbol{M}_1\boldsymbol{LH}_{21}+\frac{1}{\omega}\boldsymbol{GH}_{31}$$

$$\boldsymbol{H}_2(\omega)=\omega^2\boldsymbol{M}_2\boldsymbol{H}_{02}+\omega^2\boldsymbol{MLH}_{12}+\omega^2\boldsymbol{M}_1\boldsymbol{LH}_{22}+\boldsymbol{GH}_{32}$$

(2) Discrete-time system from continuous time system using the Bilinear z transformation

Substituting equation (12) to equation (11) to take the bilinear z transform of equation.(11) , we obtain the equation in the z plane as equation (13):

$$s=2\left(1-z^{-1}\right)/T\left(1+z^{-1}\right) \qquad (12)$$

$$\left[\boldsymbol{W}_0+\boldsymbol{W}_1z^{-1}+\boldsymbol{W}_2z^{-2}+\boldsymbol{W}_3z^{-3}\right]\hat{\boldsymbol{X}}(z)$$
$$=\left[\boldsymbol{U}_0+\boldsymbol{U}_1z^{-1}+\boldsymbol{U}_2z^{-2}+\boldsymbol{U}_3z^{-3}\right]U(z)\boldsymbol{E} \qquad (13)$$

where

$$\begin{cases}\boldsymbol{W}_0=8\boldsymbol{ML}+4T(\omega\boldsymbol{M}_1\boldsymbol{L}+\boldsymbol{FK}_D)+2T^2(\boldsymbol{FK}_P+\boldsymbol{G})+T^3\boldsymbol{FK}\\ \boldsymbol{W}_1=-24\boldsymbol{ML}-4T(\omega\boldsymbol{M}_1\boldsymbol{L}+\boldsymbol{FK}_D)+2T^2(\boldsymbol{FK}_P+\boldsymbol{G})+3T^3\boldsymbol{FK}\\ \boldsymbol{W}_2=24\boldsymbol{ML}-4T(\omega\boldsymbol{M}_1\boldsymbol{L}+\boldsymbol{FK}_D)-2T^2(\boldsymbol{FK}_P+\boldsymbol{G})+3T^3\boldsymbol{FK}\\ \boldsymbol{W}_3=-8\boldsymbol{ML}+4T(\omega\boldsymbol{M}_1\boldsymbol{L}+\boldsymbol{FK}_D)-2T^2(\boldsymbol{FK}_P+\boldsymbol{G})+T^3\boldsymbol{FK}\end{cases}$$

$$\begin{cases}\boldsymbol{U}_0=4T\boldsymbol{H}_1(\omega)+2T^2\boldsymbol{H}_2(\omega),\boldsymbol{U}_1=-4T\boldsymbol{H}_1(\omega)+2T^2\boldsymbol{H}_2(\omega)\\ \boldsymbol{U}_2=-4T\boldsymbol{H}_1(\omega)-2T^2\boldsymbol{H}_2(\omega),\boldsymbol{U}_3=4T\boldsymbol{H}_1(\omega)-2T^2\boldsymbol{H}_2(\omega)\end{cases}$$

T is the sampling period

As z is unit delay, following relations are given:

$$z^{-i}\hat{\boldsymbol{X}}_k(z)=\boldsymbol{X}_{k-i}(z)=\hat{\boldsymbol{x}}(k-i)$$

$$z^{-i}U_k(z)=U_{k-i}(z)=u(k-i)$$

Substituting the upper equations to equation (13), we obtain the equation in the discrete-time system

$$\boldsymbol{W}_0\hat{\boldsymbol{x}}(k)+\boldsymbol{W}_1\hat{\boldsymbol{x}}(k-1)+\boldsymbol{W}_2\hat{\boldsymbol{x}}(k-2)+\boldsymbol{W}_3\hat{\boldsymbol{x}}(k-3)$$
$$=[\boldsymbol{U}_0u(k)+\boldsymbol{U}_1u(k-1)+\boldsymbol{U}_2u(k-2)+\boldsymbol{U}_3u(k-3)]\boldsymbol{E} \quad (14)$$

Replacing the left-hand term and the parenthesized term in the right-hand term of equation (14) by $\boldsymbol{W}(k),\boldsymbol{U}(k)$ and substituting $\boldsymbol{W}(k),\boldsymbol{U}(k)$ to equation (14), we obtain equation (15).

$$\boldsymbol{W}(k)=\boldsymbol{W}_0\boldsymbol{x}(k)+\boldsymbol{W}_1\boldsymbol{x}(k-1)+\boldsymbol{W}_2\boldsymbol{x}(k-2)+\boldsymbol{W}_3\boldsymbol{x}(k-3)$$
$$\boldsymbol{U}(k)=\boldsymbol{U}_0u(k)+\boldsymbol{U}_1u(k-1)+\boldsymbol{U}_2u(k-2)+\boldsymbol{U}_3u(k-3)$$
$$\boldsymbol{W}(k)=\boldsymbol{U}(k)\boldsymbol{E}\quad(k=1\cdots n) \qquad (15)$$

In equation (15), the left-hand term $\mathbf{W}(k)$ is determined from the dynamic parameters of the rotor and AMB, the sampling period and the measured shaft displacement including the sensor runout and also the $\boldsymbol{U}(k)$ in the right-hand term is determined from the dynamic parameters of the rotor and AMB, the sampling period and the rotating speed. $\boldsymbol{E}$ is a unknown vector, of which element are unknown unbalance and sensor runout and be identified by the incremental least-square method.

(3) the incremental least-square method

As we may use n sets of $\boldsymbol{W}(k), \boldsymbol{U}(k)$ $(k = 1 \cdots n)$ and get 4n equations in equation (16) to identify the unknown vector $\boldsymbol{E}$ so that we employ the incremental least-square method to estimate the unbalance and the sensor runout.

$$\boldsymbol{W} = \boldsymbol{U}\boldsymbol{E} \tag{16}$$

where

$$\boldsymbol{W} = [\boldsymbol{W}(1) \quad \boldsymbol{W}(2) \quad \cdots \quad \boldsymbol{W}(n)]^T$$

$$\boldsymbol{U} = [\boldsymbol{U}(1) \quad \boldsymbol{U}(2) \quad \cdots \quad \boldsymbol{U}(n)]^T$$

In order to minimize the estimation error, a following quadratic cost function J is introduced and $\boldsymbol{E}$ is determined by minimizing the cost function J.

$$J = \boldsymbol{R}^T \boldsymbol{R}$$

where $\boldsymbol{R} = \boldsymbol{W} - \boldsymbol{U}\boldsymbol{E}$ is the estimation error.

Its minimum satisfies

$$\frac{\partial J}{\partial \boldsymbol{E}} = -2\boldsymbol{U}^T\boldsymbol{W} + 2\boldsymbol{U}^T\boldsymbol{U}\boldsymbol{E} = \boldsymbol{0}$$

, which gives

$$\boldsymbol{E} = \left(\boldsymbol{U}^T\boldsymbol{U}\right)^{-1}\boldsymbol{U}^T\boldsymbol{W}$$

provided of course the inverse $\left(\boldsymbol{U}^T\boldsymbol{U}\right)^{-1}$ exists.

The incremental least square on-line method is applied to identify. The incremental least square on-line algorithm [3][4] is given as follows:

$$\hat{\boldsymbol{E}}(k) = \hat{\boldsymbol{E}}(k-1) + \boldsymbol{P}(k)\boldsymbol{U}^T(k)\boldsymbol{q}(k) \tag{17}$$

where

$$\boldsymbol{q}(k) = \boldsymbol{W}(k) - \boldsymbol{U}(k)\hat{\boldsymbol{E}}(k-1)$$

$$\boldsymbol{Q}(k) = \left[\boldsymbol{I} + \boldsymbol{U}(k)\boldsymbol{P}(k-1)\boldsymbol{U}^T(k)\right]^{-1}$$

$$\boldsymbol{P}(k) = \boldsymbol{P}(k-1) - \boldsymbol{P}(k-1)\boldsymbol{U}^T(k)\boldsymbol{Q}(k)\boldsymbol{U}(k)\boldsymbol{P}(k-1)$$

IDENTIFICATION TEST RESULTS USING SIMULATION DATA

Numerical simulation of the rotor vibration excited by the unbalance including the measurement error caused by the AMB sensor runout is carried out to evaluate the proposed method for estimation of the unbalance and the sensor runout on the rotor. The Runge-Kutta method is employed and the condition of the numerical simulation is tabulated in table 2. 1000 data on the rotor displacement are calculated at each bearing at 500rpm and 1500rpm. Sampling period is set to be 1.7msec. The calculated vibrations at both bearings are shown in figure 5. The abscissa shows sampled data number

TABLE 2: Condition of numerical simulation

	Rotating speed(rpm)	Sampling period(ms)	Number of data
First data	500	1.7	500
Second data	1500	1.7	500

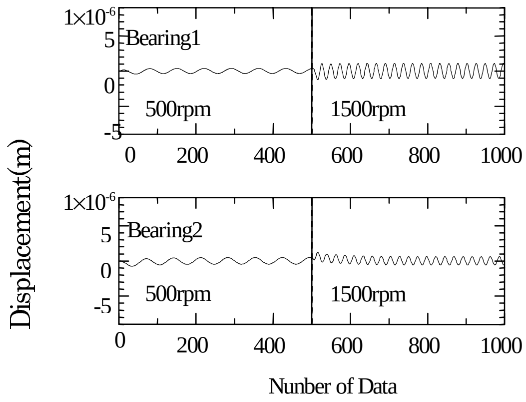
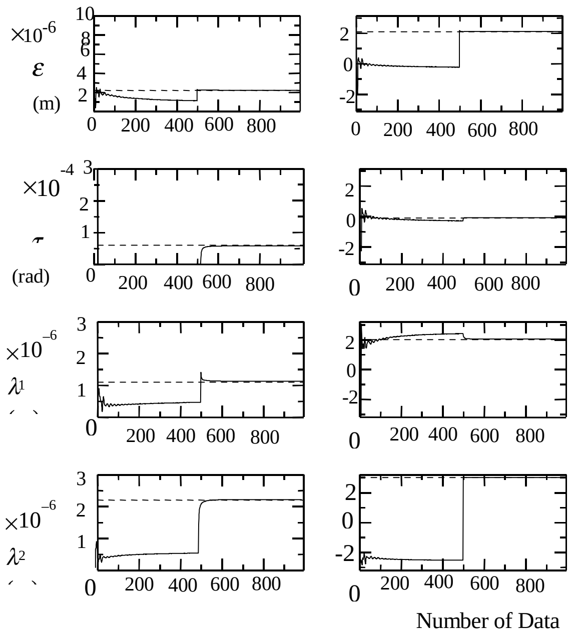


FIGURE 5: Time series data of calculated rotor displacement

The unbalance and the sensor runout are identified by the algorithm expressed in equation (17), using the simulated vibration data. The convergent process of the identification of the unbalance and the sensor runout is shown in figure 6. In this figure, the left side figures and the right side figures show the amplitude and the phase angles of the unbalance and the sensor runout respectively. Solid lines indicate the identified values of the identifying parameters and dotted straight lines are given values, namely static unbalance



-------Given value ——Identified value
(a) Gain (b) Phase(rad)

FIGURE 6: Identification process using the numerical simulation results

$\varepsilon = 2.0\mu\text{m}$, coupled unbalance $\tau = 0.6\times10^{-4}\,\text{rad}$, sensor runout of bearing 1 $\lambda_1 = 1.1\mu m$ and sensor runout of bearing 2 $\lambda_2 = 2.2\mu m$.

Just after a change of the rotating speed from 500rpm to 1500rpm, all gains and phase angles of the identified parameters converge to the given values rapidly. From this numerical result, we confirm that the unbalance and the sensor runout on the AMB rotor must be identified by the proposed rotor model and the incremental least-square method.

EFFECTS OF MODEL ERRORS ON IDENTIFICATION ACCURACY

Following the numerical identification test of the unknown parameters by using the known dynamic model of the rotor-AMB system and the simulated vibration data, we investigate effects of model errors of the rotor-AMB system on the identification accuracy.

The errors of control gains of the AMB are taken into consideration as the model errors and the numerical identification tests have been carried out.

In the identification test, static and dynamic unbalances are set at $37.6\mu\text{m}$ and $9.18\times10^{-6}\,\text{rad}$, and the control gains in the numerical identification are changed in the range of ±30% of them in the numerical simulation of the rotor unbalance vibration.

Figure 6 & 7 show the numerical identification results of the static unbalance and the dynamic unbalance.

Figure 6 shows that the estimation error of the static unbalance is below 2% even if the gain of the AMB regulator is estimated with 30% error and figure 7 also shows that the estimation error of the dynamic unbalance is below 0.2% even if the gain of the AMB regulator is estimated with 30% error

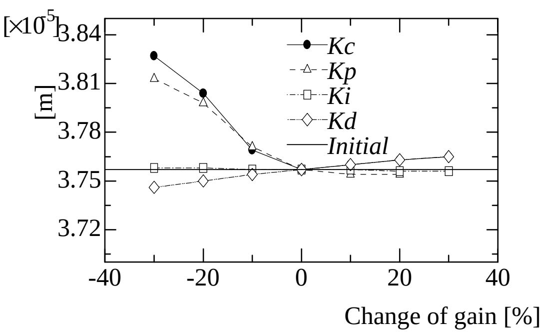


FIGURE 6: Variation of Estimated Static Unbalance by Model Errors

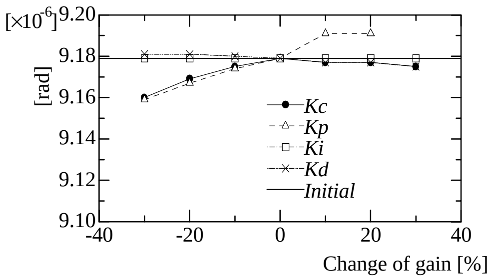


FIGURE **7**: Variation of Estimated Dynamic Unbalance by Model Errors

CONCLUSIONS

We have developed the equation of motion of the rigid rotor supported by active magnetic bearings in consideration of the static and coupled unbalances and the sensor runout and proposed the a identification method of the unbalance and the sensor runout simultaneously.

The numerical simulations carried out here on unbalance response of the rotor have shown that the proposed method is effective for identifying the unbalance and the sensor runout on the AMB rotor.

REFERENCES

(1) Y.Kanemitsu, Non-contact Sensor of Shaft Vibration and its Problem, Journal of JSME, 80-706, (1977)

(2) Y.Kanemitsu, et.al., Real Time Balancing of a Flexible Rotor supported by magnetic bearing, proc. 2nd int. symp.on magnetic bearings, (1990)

(3) S.Sagara, et al., System Identification, SICE (1981)

(4) T.Nakamizo, Signal Analysis and System Identification, korona-sya (1988)

(5) S.Beale, et al., Adaptive Forced Balancing for Magnetic Bearing Systems, Proc. 3rd Int. Symp.on Magnetic Bearings (1992)