IDENTIFICATION AND AUTOMATED CONTROLLER DESIGN FOR RIGID ROTOR AMB SYSTEMS*

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ABSTRACT
In this paper, the problem of automated controller design for AMB systems is addressed for the case of a rigid rotor with unknown properties that is to be supported by AMBs with known characteristics.
A two step procedure to solve this problem is presented. First, the unstable system is identified by a short sequence of step experiments. In a second step, a stabilizing controller is designed.

INTRODUCTION
Active magnetic bearings (AMBs) have many advantages over conventional bearings: contactless levitation, therefore absence of friction and wear, no need for lubrications, and very high rotational speeds.
Furthermore, they allow for adjustment of the damping, system monitoring and fault detection.
However, active magnetic bearings are not as widely spread in industry applications as these advantages might suggest. An important reason for this is the significant complexity of the complete plant in comparison with plants equipped with conventional bearings.
While for conventional designs the bearings can be chosen from a catalogue and standard techniques are available for connecting the bearings to the rotor, this procedure is not feasible for magnetic bearings. This is due to the fact that the controller required for operating the magnetic bearings critically depends on information about the rotor that is to be levitated, i.e. there is no generic controller one could sell with a pair of magnetic bearings that stabilizes all rotors the bearing could technically levitate.

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The state of the art approach to solving this problem consists in making an 'integral design' based on a man-made model of the rotor and the (known) model of the bearings. This procedure is time consuming and often involves more than one person.
To improve this situation and to make AMBs 'out of the box'-products like conventional bearings, an automatic startup algorithm capable of designing controllers for a given pair of magnetic bearings and arbitrary rotors (with reasonable mass) is required.
In this paper, an algorithm capable of performing this task for rigid rotors is presented. The algorithm consists of two parts. First, the parameters of the rotor model are identified by means of a sequence of step experiments. In the second step a controller to stabilize the system is designed.
In correspondence with the problem setting described above, it is assumed that all information on the bearings (stiffness coefficients k_s and k_i, air gap, clearance, maximum current) is known, and no information on the rotor (geometry, mass, moment of inertia) is available. The bearings are assumed to operate in differential driving mode and are considered linear.
The paper is organized as follows: First, the problem is formulated for a single, one-dimensional bearing, and the analytical solution is presented. Then, the model is extended to two dimensions. Finally, the procedure to lift a real rotor supported by two bearings is described.
At the end of the paper, experimental results obtained from a test rig are presented.

ONE-DIMENSIONAL CASE
Let us consider a one-dimensional bearing as schematically depicted in Figure 1, where x_0

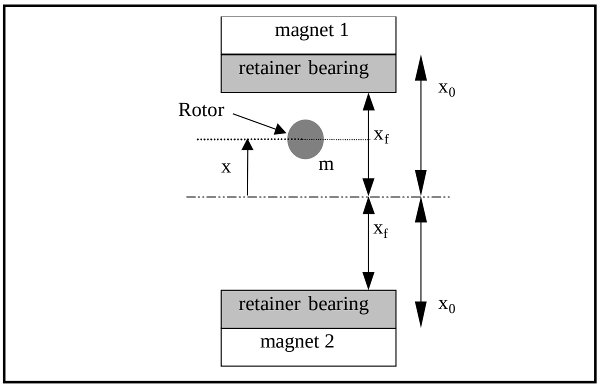

Figure 1: 1-DOF Magnetic Bearing

represents the distance from the bearing center to the magnets, x_f denotes the distance from the bearing center to the retainer bearings, and x denotes the position of the rotor with respect to the center position. Let further m denote the mass of the rotor, and k_s and k_i the force-displacement and force-current factors of the bearing for a given bias current $i_{0,ref}$, respectively.

Identification

With the above notation and i being the bearing current's deviation from the bias current, the equation describing the rotor's motion can be written as follows:

$$m \cdot \ddot{x} = k_s \cdot x + k_i \cdot i - m \cdot g \qquad (1)$$

In order to design a controller capable of stabilizing the system, it is necessary to identify the unknown mass m of the rotor. To this end, the following experiment is performed. First, the current is switched off such that the rotor rests at the position $x = -x_f$. Then, a current step of size I_s is applied to the upper magnet and the rotor is accelerated upwards until it hits the upper retainer bearing at $x = x_f$.

By applying the Laplace transform considering the initial condition $x = -x_f$, the rotor's response to the current step can be expressed in the frequency domain as:

$$m(s^2 X(s) + s x_f) = k_s X(s) + k_i I(s) - \frac{mg}{s} \qquad (2)$$

Rearranging terms yields

$$X(s) = \frac{-x_f \cdot s}{s^2 - \frac{k_s}{m}} + \frac{\frac{k_i}{m}}{s^2 - \frac{k_s}{m}} \cdot \frac{I_s}{s} - \frac{\frac{g}{s}}{s^2 - \frac{k_s}{m}} \qquad (3)$$

where the first factor of the second term describes the plant's transfer function from current to displacement and the first and third terms stem from the initial conditions and the influence of gravity, respectively. Investigation of the second term reveals that the system is of second order with poles at $p = \pm\sqrt{\frac{k_s}{m}}$ and a static gain of $k_{plant} = -\frac{k_i}{k_s}$, (4) and (5).

Transformation of (3) to the time domain and scaling by $1/x_f$ then yields for the step response of the rotor:

$$x(t) = \left(-1 + \frac{I_s x_0}{x_f i_0} - \frac{g m}{x_f k_s}\right) \cosh\left(\sqrt{\frac{k_s}{m}} t\right) - \left(\frac{I_s x_0}{x_f i_0} - \frac{g m}{x_f k_s}\right) \qquad (6)$$

As can be seen from equation (6), the function describing the rotor's path contains the unknown system pole p as an argument to the cosine hyperbolicus term. We will now describe a procedure to determine this parameter and the unknown mass m.

1. Perform a step experiment as described above with on-line measurement of the rotor's position.
2. From the time data obtained, extract the moments of take-off and contact with the upper retainer bearing.
3. Fit a function of type $x(t) = a \cdot \cosh(p_i \cdot t) - c$ to the position measurements made during the flight-phase of the rotor.

The parameter p_i resulting from this procedure then directly yields an estimate for the system's poles at $\pm p$. Based on p_i an estimate for the rotor mass m can be calculated using equation (4).

Current Step Size. It is important to understand that in the experiment described above the size of the current step implicitly defines the operating point of the (linear) bearing. A current step of $I_s A$ implies that a linear bearing with bias current $I_s/2A$ is subjected to a current step from $i{=}-I_s/2A$ to $i{=}I_s/2A$. The force-displacement factor k_s and the force-current factor k_i both depend on the bearing's bias current i_0, i.e.

$$k_s \sim i_0^2 \text{ and } k_i \sim i_0 . \qquad (7)$$

Therefore either the current step size I_s must be adjusted to equal twice the reference bias current $i_{0,ref}$ or the force-displacement and force-current factors used in the above calculations must be updated as follows:

$$k_{s,new} := \frac{k_{s,old}}{i_{0,ref}^2}\left(\frac{I_s}{2}\right)^2, \; k_{i,new} := \frac{k_{i,old}}{i_{0,ref}}\left(\frac{I_s}{2}\right). \qquad (8)$$

Controller Design
Once the rotor's mass has been determined, the system is fully identified. Its transfer function is

$$G(s) = \frac{\frac{k_i}{m}}{s^2 - \frac{k_s}{m}} \quad (9)$$

A controller for this system can be derived as follows. Since this system has a constant phase of –180 degrees, the controller must provide positive phase and sufficient gain in order to stabilize the unstable system pole. A straightforward solution of achieving stabilization for this type of plant is by means of a lead compensator. The structure of this element is as follows:

$$R(s) = k\frac{Ts+1}{\alpha Ts+1} \text{ with } T = \frac{1}{f\sqrt{\alpha p}},\ \alpha > 0\,.$$

The parameters k, f, and α must be chosen such that the system is robustly stabilized.
One can show that k corresponds directly to the gain margin of the closed loop system. To achieve stabilisation, the product of k and the static plant gain k_{plant} must be less than -1. A value of -1.5 for this product yields a gain margin of 3dB, which can be considered a minimum requirement.
An additional requirement for the controller is that its gain must be large enough to actually lift the rotor (sufficiently small sensitivity function at $\omega = 0$). This condition is not fulfilled by all stabilizing controllers.
From the equation

$$k_s \cdot x + k_i \cdot i = m \cdot g$$

which describes the balance of forces if the rotor is at rest at position x in the air gap and from the static force current relation defined by the controller $i = -kx$ (negative feedback), it follows that the gain required to hold the rotor statically at a position $x = -x_s$ in the air gap a controller gain k of

$$k = \frac{k_s}{k_i} + \frac{mg}{k_i x_s}$$

is required. A reasonable minimum choice for the static air gap is one quarter of the air gap from the bearing center, yielding

$$k = \frac{k_s}{k_i} + \frac{4mg}{k_i x_s}$$

The controller gain can then be chosen as the maximum of this gain and the minimum stabilizing gain described above.

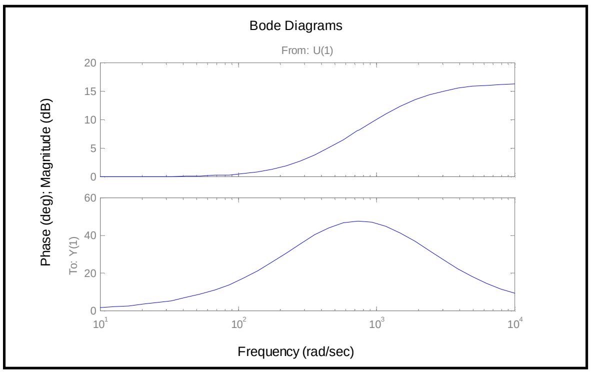

Figure 2: Bode Plot of Controller

The parameter α can be used to define the maximum phase lift of the compensator. This is a tradeoff between good damping and reasonable high frequency gain. The smaller the parameter α is selected, the higher the maximum phase lift. However, with smaller parameters alpha the high frequency gain of the controller also increases. A good compromise is $\alpha = 0.15$. This yields a phase lift of nearly 50 degrees at frequency $f \cdot p$ and an increase in high frequency gain of 15dB.
Finally, the coefficient f can be used to shift the controller in the frequency domain such that the phase margin is as large as possible. It is worth noting that the optimal f only depends on k and not on α. Figure 2 shows a bode plot of the controller for $\alpha = 0.15$, $k=1$, $p=500$, and $f=1.5$.

EXTENSION TO TWO-DIMENSIONAL CASE
We will now extend the procedure described above to two dimensions, i.e. we will consider a single magnetic bearing with two degrees of freedom and the x- and y axes rotated by 45 degrees with respect to the vertical axis, see Figure 3. The rotor can be imagined as disc-like. This investigation is of theoretical nature and lays the basis for analysis of the full rotor case.

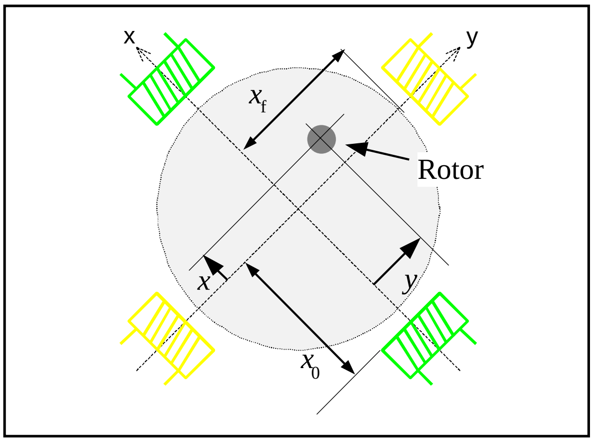

Figure 3: Two-Dimensional Bearing

Identification
Compared to the one-dimensional case described above, the main difference in the system model is the existence of two independent axes. In order to achieve the desired upward acceleration of the rotor, a current step must be applied to both bearings at the same time. The movement along each of two axes can then be analyzed separately. The model description is analogous to that of the one-dimensional case, with the difference that both the air gap x_f and the gravitational constant are reduced by a factor of $\sqrt{2}$. As a result, the equation of motion becomes

$$m(s^2X(s) + s\frac{x_f}{\sqrt{2}}) = k_s X(s) + k_i I(s) - \frac{mg}{s\sqrt{2}}$$

and the transfer function from current to displacement for each of the system's axes again is that given in (9). As before, the pole can be directly estimated by fitting a function of type $x(t) = a \cdot \cosh(p_i \cdot t) - c$ to the position measurement, and the mass can be extracted from the estimated pole.

Controller Design
The two dimensional system can be controlled by designing an individual controller for each of the two axes. Since the transfer function from current to displacement along each of the axes is identical to the one dimensional case and the factor $\sqrt{2}$ cancels out in the calculation of the minimum gain required for lifting the rotor to a certain position, the design procedure remains identical.

EXTENSION TO THE FULL ROTOR CASE
We will now consider a full rotor supported by two magnetic bearing, see Figure 4. The rotor's motion in each of the two planes is described by the equations

$$\begin{pmatrix} m & 0 \\ 0 & I_p \end{pmatrix}\begin{pmatrix} \ddot{x} \\ \ddot{\alpha} \end{pmatrix} = T_1\begin{pmatrix} k_{sa} & 0 \\ 0 & k_{sb} \end{pmatrix}\begin{pmatrix} x_a \\ x_b \end{pmatrix} + T_1\begin{pmatrix} k_{ia} & 0 \\ 0 & k_{ib} \end{pmatrix}\begin{pmatrix} i_a \\ i_b \end{pmatrix} - \begin{pmatrix} mg \\ 0 \end{pmatrix} \qquad (10)$$

where m is the rotor's total mass, I_p is its polar moment of inertia, k_{sa} and k_{sb} are the force-displacement factors of bearing A and bearing B, and k_{ia} and k_{ib} represent the corresponding force-current factors. The matrix T_1 describes the transformation of the forces generated by the bearings to center of gravity. This system of equations can be transformed to bearing coordinates by means of a similarity transformation using the matrix T_2. The matrix T_1 and the transformation are defined by:

$$T_1 = \begin{pmatrix} 1 & 1 \\ -a & b \end{pmatrix} \text{ and } \begin{pmatrix} x \\ \alpha \end{pmatrix} = T_2\begin{pmatrix} x_a \\ x_b \end{pmatrix} \text{ w. } T_2 = \frac{1}{a+b}\begin{pmatrix} b & a \\ -1 & 1 \end{pmatrix}.$$

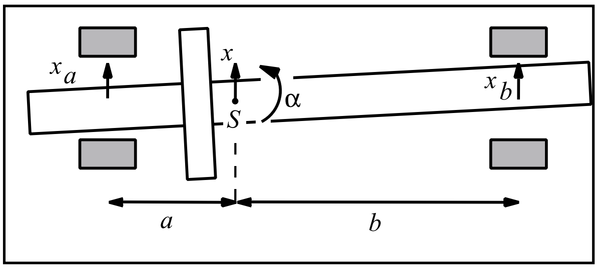

Figure 4: Full rotor (single plane)

The above system then becomes

$$\begin{pmatrix} m_1 & m_3 \\ m_3 & m_2 \end{pmatrix}\begin{pmatrix} \ddot{x}_a \\ \ddot{x}_b \end{pmatrix} = \begin{pmatrix} k_{sa} & 0 \\ 0 & k_{sb} \end{pmatrix}\begin{pmatrix} x_a \\ x_b \end{pmatrix} + \begin{pmatrix} k_{ia} & 0 \\ 0 & k_{ib} \end{pmatrix}\begin{pmatrix} i_a \\ i_b \end{pmatrix} - T_2'\begin{pmatrix} mg \\ 0 \end{pmatrix} \qquad (11)$$

This representation exists for any rigid rotor.

Identification
To obtain a suitable model for controller design, the three unknown masses m_1, m_2 and m_3 must be determined. This can be done as follows:

Determination of m_1 and m_2. The diagonal entries of the mass matrix can be determined as follows: While one bearing remains switched off, the step experiment described in the previous section is applied to the other bearing. As a consequence, the rotor remains at rest on one side, while at the other side it is moved upwards. In terms of equation (11), this means for the case that the step experiment is performed on bearing A,

$$m_1\ddot{x}_a = k_{sa}x_a + k_{ia}i_a - \frac{b}{a+b}mg \qquad (12)$$

Up to the gravity term, this equation is identical to the one derived in the section describing the two dimensional case. The difference in the last term has no influence on the transfer function to be identified. It merely is a difference in the disturbing gravity force. Therefore, the identification approach developed there can be directly applied, yielding the mass coefficient m_1. The coefficient m_2 can be identified from application of the same procedure to bearing B.
Based on this information and the controller design method described in the last section, controllers capable of lifting each side of the rotor individually (with the other bearing switched off) can be designed. It must be understood, however, that these controllers can not be guaranteed to stabilize the rotor when both bearings are active. While the two individual systems have essentially one pole, the completely levitated rotor has two unstable poles for each plane (tilt- and translational mode), which can not be derived from the poles observed with one of the bearings switched off.

Determination of m_3. In order to determine the remaining parameter m_3, it is important to notice that

$$\frac{b}{a+b} m = m_1 + m_3 \tag{13}$$

This can be seen from the following thought experiment: let the rotor fall freely, with both bearings switched off. Then k_{sa} and k_{ia} are 0, and the first line of equation (11) becomes

$$m_1 g + m_3 g = \frac{b}{a+b} m\, g ,$$

from which the above statement follows immediately. Analogously, the second line of equation (11) it follows that

$$\frac{a}{a+b} m = m_2 + m_3 .$$

After having determined stabilizing controllers for each of the individual bearings with the other bearing switched off, we are able to bring the rotor to the bearing center by means of a (very slow) integrator. Then the gravity term in equation (12) can be determined from measurements of the control currents. Based on this measurement and the known mass m_1, m_3 can be determined from equation (13) and the system is then fully identified.

Controller Design

After the mass matrix has been identified, the unstable system poles can be directly calculated. To this end, the homogeneous system (without external forces) must be considered:

$$\begin{pmatrix} m_1 & m_3 \\ m_3 & m_2 \end{pmatrix} \begin{pmatrix} \ddot{x}_a \\ \ddot{x}_b \end{pmatrix} - \begin{pmatrix} k_{s\,a} & 0 \\ 0 & k_{s\,b} \end{pmatrix} \begin{pmatrix} x_a \\ x_b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

The system poles can then be calculated from the eigenvalues λ_1 and λ_2 of the matrix

$$-\begin{pmatrix} m_1 & m_3 \\ m_3 & m_2 \end{pmatrix}^{-1} \begin{pmatrix} k_{s\,a} & 0 \\ 0 & k_{s\,b} \end{pmatrix} \tag{14}$$

The system has four poles that are located on the real axis at

$$p_{1,2} = \pm\sqrt{\lambda_1}\, i \text{ and } p_{3,4} = \pm\sqrt{\lambda_2}\, i$$

A controller for the above system can be designed as follows. By means of a similarity transformation based on the matrix of eigenvectors of matrix (14), T_3, system (11) can be diagonalized, i.e. without the gravity force

$$\begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix} \begin{pmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{pmatrix} = \begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} + T_3' \begin{pmatrix} k_{i\,a} & 0 \\ 0 & k_{i\,b} \end{pmatrix} \begin{pmatrix} i_a \\ i_b \end{pmatrix}.$$

If now the input of the system is multiplied by the inverse of the last matrix product in the above equation, the resulting new system is completely decoupled, i.e. it consists of two independent SISO systems just like the one described in the one dimensional case. To each of the channels the control procedure described above can be applied. The final controller consists then of the product of the diagonal augmentation of these two controllers and the inverse matrix mentioned before.

Adaptation of operating point. In some cases, it may be interesting to design very stiff controllers or controllers with particularly low bandwidth. The latter case is of particular interest when the algorithm is applied to flexible rotors where the controller should just barely levitate the rotor without destabilizing higher frequency eigenmodes. (The flexible rotor could then be identified by identification techniques described in [2] and a more sophisticated controller could be designed based on this model.)

A key parameter to achieving these goals is the bias current i_0 the bearing is operated with. Although the step current required to perform the experiment described above is rather large, it is nevertheless possible to design controllers for very low bias currents. For the one-bearing experiments, it follows from (4) and (7) that if the bias current is scaled by a factor of k, the new system pole is located at $k \cdot p$, i.e. the system pole linearly depends on the bias current. This allows for direct transformation of poles obtained from experiments with a (usually large) bias current to arbitrary other operating points. This scalability makes the method applicable to all operating points, even those with very low bias current where no step experiment can be performed. In particular, this means that no additional experiments are required if it is decided to readjust the bias current after the step experiment has been carried out.

For the full rotor case, things are similar. If both bias currents are scaled by the same factor, the two system poles will each also be scaled by that factor. For the general case where the two bias currents are scaled by different factors, the poles can be extracted from the analytical solution of the eigenvalue problem described above. In addition to adjusting the absolute position of the poles, their relative position can be adjusted as well within certain limits.

EXPERIMENTAL RESULTS

To prove the effectiveness of the method, tests have been carried out on a test rig. The total rotor mass was 3.154kg and its moment of inertia was 0.0215 kgm^2 The two bearings were identical with nominal force-current factors k_i of *39.1 N/A* and force-displacement factors k_s of *293 N/mm* (based on a bias current of

$i_{0,ref}$ = 3*A*). The rotor's center of gravity was at 157 *mm* from the left end of the rotor, yielding values of 10.97 *mm* for *a* and 11.73 *mm* for *b*.
The rotor was lifted in each of the bearings by means of a step experiment as described above. Figure 5 shows the path of the rotor in the air gap of bearing A. The positions and currents measured during the experiment are depicted in Figure 6. From the position data, the moments of take-off and contact with the with the upper part of the retainer bearing have been identified. For the system under discussion, the flight phase lasted only 4.7 *ms*. This time sequence has then been extracted from the data and fit to the function predicted by the analytical analysis. Figure 7 shows the result of the curve fitting. From the curve fit, a pole at 535.1 *rad/s* was identified. Based on the known value for k_s, the corresponding mass m_1 has been calculated to be 1.023 *kg*.
Based on the identified pole, a stabilizing controller for one bearing was designed according to the guidelines given above. Then, the controller was switched on and augmented with a slow integrator. With the rotor in the bearing center, the current was measured to be 2.051 *A*, yielding a value of 1.405 *kg* for $m_1 + m_3$. Then, the procedure was repeated for bearing B. Table 1 shows a summary of the results.

	PoleA	PoleB	m1	m2	m3
Model	504.2	482.5	1.152	1.258	0.369
Identified	535.1	467.3	1.023	1.342	0.382

Based on the identified parameters, the poles of the complete rotor system have been calculated to be at 97.8 *Hz* and 67.8 *Hz* for the translational and tilt mode, respectively. The theoretical values are at 93.95 *Hz* and 68.2 *Hz*. A controller designed based on this data for i_0 = 0.75 *A* successfully stabilized the rotor.

CONCLUSIONS

In this paper, a new method for automated AMB controller design for rigid rotors has been presented. A procedure to identify the parameters of the rotor has been described. A method to design stabilizing controllers based on the identified poles has been outlined. Experimental results have been presented to prove the effectiveness of the method.

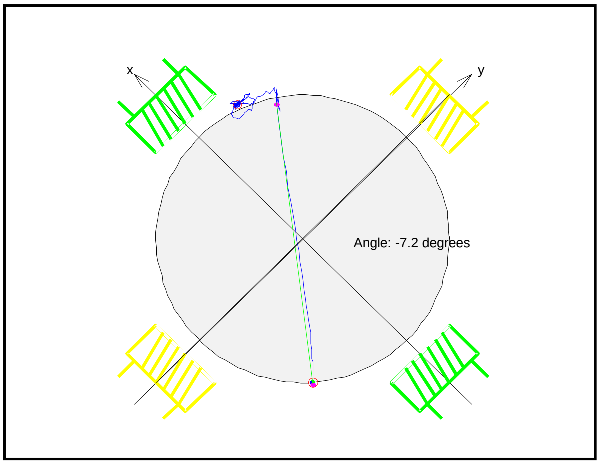

Figure 5: Path of rotor in bearing A

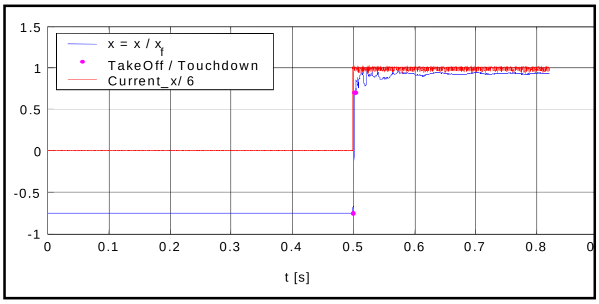

Figure 6: Measured current and displacement

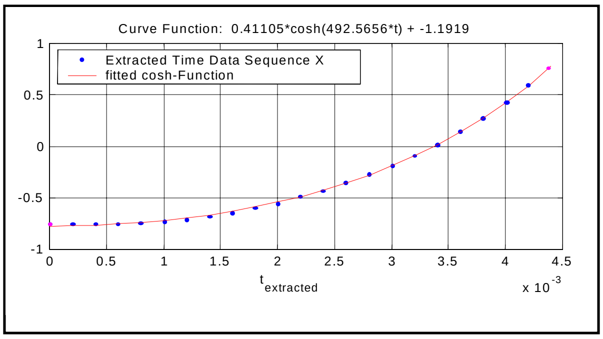

Figure 7: Measured position and curve fit

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