INTRODUCTION TO SMART MAGNETIC BEARINGS DESIGN

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ABSTRACT
Magnetic bearing systems have been studied for a number of years. Only limited work, however, has been completed on the smart magnetic bearings technology. In this paper, we provide the overview of the concept and present the formulation for control and dynamics of rigid rotor supported on smart magnetic bearings.

INTRODUCTION
The overall goal of an AMB controller is to stabilize the plant and to reach optimal technical and economical performance. To achieve this, AMB system has to be optimized in an overall mechatronic design approach. The term "*mechatronic*" refers to [1] synergistic integration of mechanics, electronics, control, computer science, diagnostics, and intelligent systems in the design and manufacturing processes of "*mechatronic*" products or systems.

As a result of such approach, the resulting "*mechatronic*" products should be smart (or intelligent) ones. In the case of rotating machinery applications it means that smart AMB can provide on-line diagnostics and predictive maintenance information. It should also be able to carry out automatic reconfiguration of control laws in a case of machinery malfunction occurrence. Smart magnetic bearings should be able to adapt the control laws to rotor parameters and loads, to diagnose and predict the operational condition of rotating machinery, and to realize additional goals essential for specific rotating equipment. One of the examples is the dynamic compensation of sudden rotor unbalance due to the blade loss, or another one is the necessity in gyroscopes to calibrate the measurement path to reduce the angular velocity measurement error. In this paper we will describe an idea of smart radial magnetic bearings for rigid rotor.

A model for lateral motion of axially symmetric of rigid rotor supported by two radial magnetic bearings was reduced to modal model with complex variables. Next this model was divided into two subsystems connected by two lateral modes: translation and rotation. As an example, the voltage-control scheme was chosen. The control system (observer and controller gains) was designed independently for each mode. This approach allows one to express analytically the controller and observer gains as functions of values of the desired closed-loop poles and the plant model parameters. It appeared that gyroscopic effects strongly influence the controller and the observer parameters. It is desired to design the adaptive control (as a gain scheduling versus angular velocity) to adjust the controller. In this approach there is also a possibility to obtain information about modal external forces and moments which is quite useful in the navigation measurement instruments.

In the identification procedure the modal scheme is also used. Two lateral modes of rigid rotor are identified independently. In this method The identification method from [2] which is a modification of OKID method [3] is used in smart rigid rotor magnetic bearing system. In this method it is assumed that the sum of input and output numbers equals to the state vector dimension. The deadbeat observer is used to design the observer/controller model of the closed-loop system. In our case the Markov parameters are not calculated from the observer/controller system realization but the ARX model of observer/controller is identified. From this model we can directly calculate (a) the open-loop physical system realization, and (b) the observer gain physical realization. Such approach was used to obtain the physical state-space model of the open-loop system for voltage controlled magnetic bearings.

The connection of the control system design and the above identification procedure leads to the adaptive control system. This time, the off-line identification was replaced by the in-line identification procedure, assuming that the changes of the system parameters are slow. For the identified open-loop system parameters new control law parameters are calculated to obtain desired dynamics of the closed-loop system.

The in-line version of the above identification method is also useful for diagnostic purposes. In the physical model, the elements of matrices are usually a simple combination of system physical parameters (resistance, inductance, mass, moment of inertia, and so on) and therefore are useful for comprehensive diagnostics of the system. Trends in the physical parameter changes allows to forecast the future technical condition of the system.

STRUCTURE OF THE ROTOR-BEARING SYSTEM

There is some hardware and software selection flexibility in the design of magnetic bearing control system for rigid rotor [4]. We can select:

– the type and number of sensors,
– the type of controller (digital or analog),
– the type of control laws (voltage, current, magnetic flux, self-sensing),
– the structure of actuator (heteropolar, homopolar, number of poles).

There are some criteria in the choice of the rotor-bearing system structure related the application of rotating machinery. In the case of smart magnetic bearing system, we should choose such a structure, which will allows us reach the assumed goals: adaptive control, on-line diagnostics, and reconfiguration of the system. It is evident that we cannot use the current-control in such a system since the high current amplifier gains “hide” the coils parameters. It is also well known from redundant theory [5] that from the safety point of view, the bigger number of sensors, the better.

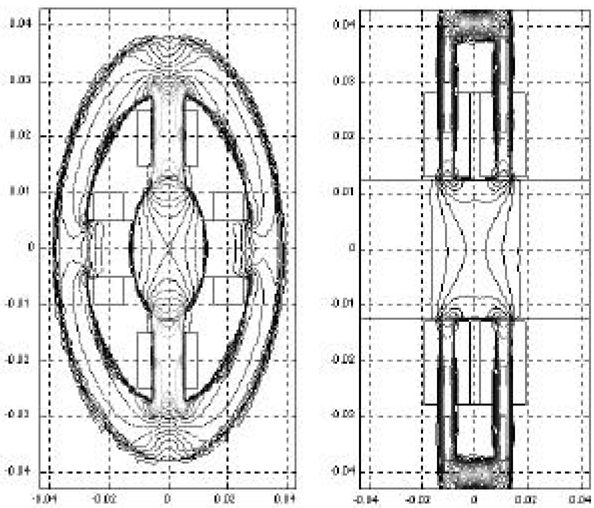

FIGURE 1. Magnetic flux density in the homopolar magnetic bearing

In any case we should carefully optimise the actuator structure to obtain the maximum load capacity. Computer-aided design methods (FEM) are very useful in this case. For example they allow us to shape the actuator structure by analysing the magnetic flux density as it is shown in Fig. 1 for the homopolar actuator [6]. As it is seen in Fig. 1. the permanent magnets and narrows edges strongly saturate magnetic paths. These places should be carefully shaped.

After some considerations we have chosen heteropolar voltage control system with measurement of rotor displacement and currents in the coils for further analysis.

CONTROL LAW FOR SINGLE AXIS

Consider the magnetic suspension with two opposite coils with voltages inputs: u_1, u_2 and measured outputs: currents i_1, i_2, and mass displacement x from the bearing center. After linearization of the system equations at the points: $x=0$, $i_1=i_o$, $i_2=i_o$ we obtain the state-space model of the open-loop system [2]:

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{A_c x} + \mathbf{B_c u} + \mathbf{B_F} F_z + \mathbf{F}_r\,,\\ \mathbf{y} &= \mathbf{C_c x}\,.\end{aligned} \qquad (1)$$

The above matrices are as follows:

$$\mathbf{x} = \begin{bmatrix} x \\ \dot{x} \\ i_1 \\ i_2 \end{bmatrix},\quad \mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix},\quad \mathbf{y} = \begin{bmatrix} x \\ i_1 \\ i_2 \end{bmatrix},\quad \mathbf{C_c} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{A_c} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ v_1 & 0 & v_2 & v_3 \\ 0 & v_4 & v_5 & 0 \\ 0 & v_6 & 0 & v_7 \end{bmatrix},\quad \mathbf{B_c} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ v_8 & 0 \\ 0 & v_9 \end{bmatrix},$$

$$\mathbf{B_F} = \begin{bmatrix} 0 & \frac{1}{m} & 0 & 0 \end{bmatrix}^T,\quad \mathbf{F}_r = \begin{bmatrix} 0 & \frac{F_{01} - F_{02}}{m} & 0 & 0 \end{bmatrix}^T,$$

$$v_1 = \frac{k_{s1} + k_{s2}}{m},\quad v_2 = \frac{k_{i1}}{m},\quad v_3 = -\frac{k_{i2}}{m},\quad v_4 = -\frac{k_{i1}}{L_{s1} + L_{o1}},$$

$$v_5 = -\frac{R_1}{L_{s1} + L_{o1}},\quad v_6 = \frac{k_{i2}}{L_{s2} + L_{o2}},\quad v_7 = -\frac{R_2}{L_{s2} + L_{o2}},$$

$$v_8 = \frac{k_{w1}}{L_{s1} + L_{o1}},\quad v_9 = \frac{k_{w2}}{L_{s2} + L_{o2}}.$$

where: F_z – external force, $\mathbf{F}_r$ - results from difference between the operation point forces, x_o – clearance, R_j - coil resistances, L_{sj} – leakage inductances, L_{oj} – air-gap inductances; k_{wj} - the amplifier gains, and $k_{sj} = (K_j i_o^2)/(2x_o^3)$, $k_{ij} = (K_j i_o)/(2x_o^2)$, $K_j = N_j^2 A m_o$, while index j indicates the coil and $j=1, 2$. In the last expression there is: N – active coil number in the electromagnet, A – electromagnet pole cross section, and m_o – magnetic permeability. Thus, the open loop system is a plant with two inputs and three outputs and with set values: $x=0$, $i_1=i_o$, $i_2=i_o$. It means that the control errors are: $x_b=-x$, $i_{1b} = i_o - i_1$, $i_{2b} = i_o - i_2$.

In many applications for control purposes, there are used averaged values of current, i.e. control current $i = (i_1 - i_2)/2$, and operation point current $i_o = (i_1 + i_2)/2$, where i_1, i_2 are currents in the opposite coils.
Directly from Eqs.(1) we have:

$$\dot{\mathbf{x}}_p = \mathbf{A}_p\mathbf{x}_p + \mathbf{B}_p\mathbf{u}_p + \mathbf{B_F F}_z + \mathbf{F}_r, \quad \mathbf{y}_p = \mathbf{C}_p\mathbf{x}_p. \tag{2}$$

The above matrices are as follows:

$$\mathbf{x}_p = \begin{bmatrix} x \\ \dot{x} \\ i \\ i_o \end{bmatrix}, \ \mathbf{u}_p = \begin{bmatrix} u \\ u_o \end{bmatrix}, \ \mathbf{y}_p = \begin{bmatrix} x \\ i_1 \\ i_2 \end{bmatrix}, \ \mathbf{C}_p = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix},$$

$$\mathbf{A}_p = \begin{bmatrix} 0 & 1 & 0 & 0 \\ v_{p1} & 0 & v_{p2} & v_{p3} \\ 0 & v_{p4} & v_{p5} & 0 \\ 0 & v_{p6} & 0 & v_{p7} \end{bmatrix}, \ \mathbf{B}_p = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ v_{p8} & v_{p9} \\ v_{p9} & v_{p8} \end{bmatrix},$$

$v_{p1} = v_1$, $v_{p2} = v_2 - v_3$, $v_{p3} = v_2 + v_3$ $v_{p4} = (v_4 - v_6)/2$, $v_{p5} = (v_5 + v_7)/2$, $v_{p6} = (v_4 + v_6)/2$, $v_{p7} = (v_5 + v_7)/2$, $v_{p8} = (v_8 + v_9)/2$, $v_{p9} = (v_8 - v_9)/2$.

It is evident that the average currents and voltages in the model (2) are not useful to split the system into two SISO subsystems which can elevate the control law design.
In the classical approach, the average currents $i_1 = i_o + i$, $i_2 = i_o - i$ are introduced to the nonlinear model which is linearized at the points x=0, i=0. This leads to the open-loop model:

$$\dot{\mathbf{x}}_r = \mathbf{A}_r\mathbf{x}_r + \mathbf{B}_r\mathbf{u}_r + \mathbf{B_F F}_z + \mathbf{F}_r, \quad \mathbf{y}_r = \mathbf{C}_r\mathbf{x}_r. \tag{3}$$

There is only one difference between the models (2) and (3), an element of matrix **A** is: $v_{r3} = 0$. Since $v_{r3} = 0$, we can decompose the system (3) into two SISO subsystems. In the subsystem II we assumed that:

$$v_{r6} = -\frac{1}{2}\left(\frac{k_{i1}}{L_{s1} + L_{o1}} - \frac{k_{i2}}{L_{s2} + L_{o2}}\right) \cong 0,$$

and in both subsystems that:

$$v_{r9} = \frac{1}{2}\left(\frac{k_{w1}}{L_{s1} + L_{o1}} - \frac{k_{w2}}{L_{s2} + L_{o2}}\right) \cong 0.$$

Let us compare models (3), (2), and (1). We can notice that the model (1) is useful in the diagnostics procedure since elements of matrices are in the simple form, while in the models (2) and (3) they are in the averaged form. The model (2) and (3) cover each other when both coils have got the same parameters. The model (3) is useful for control law design since it can be split into SISO subsystems:

$$\text{I).} \quad \ddot{x} = v_{r1}x + v_{r2}i, \quad \dot{i} = v_{r4}\dot{x} + v_{r5}i + v_{r8}u, \tag{4}$$

$$\text{II).} \quad \dot{i}_o = v_{r7}i_o + v_{r8}u_o, \tag{5}$$

where: $v_{r1} = \frac{k_{s1} + k_{s2}}{m}$, $v_{r2} = \frac{k_{i1} + k_{i2}}{m}$,

$$v_{r4} = -\frac{1}{2}\left(\frac{k_{i1}}{L_{s1} + L_{o1}} + \frac{k_{i2}}{L_{s2} + L_{o2}}\right),$$

$$v_{r5} = v_{r7} = -\frac{1}{2}\left(\frac{R_1}{L_{s1} + L_{o1}} + \frac{R_2}{L_{s2} + L_{o2}}\right),$$

$$v_{r8} = \frac{1}{2}\left(\frac{k_{w1}}{L_{s1} + L_{o1}} + \frac{k_{w2}}{L_{s2} + L_{o2}}\right).$$

Let us assume that the state feedback control law for I subsystem is in the form:

$$u = -[k_1 \ k_2 \ k_3][x \ \ \dot{x} \ \ i]^T \tag{6}$$

For desired poles p_1, p_2, p_3 of the I closed-loop subsystem we have [7]:

$$k_1 = -\frac{v_{r1}(p_1 + p_2 + p_3) + p_1 p_2 p_3}{v_{r2}v_{r8}},$$
$$k_2 = \frac{v_{r1} + v_{r2}v_{r4} + p_1p_2 + p_1p_3 + p_2p_3}{v_{r2}v_{r8}}, \tag{7}$$
$$k_3 = (v_{r5} - p_1 - p_2 - p_3)/v_{r8}.$$

The control law for subsystem II is in the form:

$$u_o = -k_o i_o, \tag{8}$$

where the control gain for the assumed closed-loop pole p_o is simply:

$$k_o = p_o + v_{r7}. \tag{9}$$

Combining the control laws (6) and (8) by introducing $u_1 = u_o + u$, $u_2 = u_o - u$, we obtain the control law for the model (1):

$$\mathbf{u} = -\mathbf{Kx}, \tag{10}$$

where:

$$\mathbf{K} = \begin{bmatrix} k_1 & k_2 & (k_o + k_3)/2 & (k_o - k_3)/2 \\ -k_1 & -k_2 & (k_o - k_3)/2 & (k_o + k_3)/2 \end{bmatrix}. \tag{11}$$

The full-order observer for subsystem I can be designed in a “classical” way but in our case we should design the reduced order observer to detect only mass velocity since other elements of the state vector are measured directly. It is realized by digital differentiation of the mass displacement.

CONTROL LAW FOR RIGID ROTOR

Consider symmetrical rigid rotor laterally supported by two magnetic bearings as it is shown in Fig. 2.

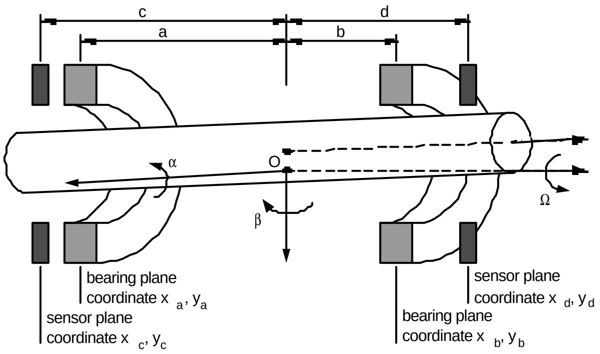

FIGURE 2. Rotor-sensors-magnetic bearings system configuration

The rotor motion can be described by modal coordinates [7] mass center translational coordinates x, y, and rotattional coordinates α, β, which can be arranged into modal coordinate vector: $\mathbf{q} = [x \quad \alpha \quad y \quad \beta]^T$. The rotor motion can also be expressed in terms of the coordinates of the rotor center at the bearing planes: $\mathbf{q}_b = [x_a \quad x_b \quad y_a \quad y_b]^T$ [8] or by coordinates of the rotor center at measurement planes $\mathbf{q}_m = [x_c \quad x_d \quad y_c \quad y_d]^T$. These coordinates can be transformed one to another according to the following formulae:

$$\mathbf{q}_b = \mathbf{T}_1 \mathbf{q}, \quad \mathbf{q}_m = \mathbf{T}_2 \mathbf{q}, \tag{12}$$

where:

$$\mathbf{T}_1 = \begin{bmatrix} \mathbf{T}_b & \mathbf{0} \\ \mathbf{0} & \mathbf{T}_b \end{bmatrix}, \quad \mathbf{T}_2 = \begin{bmatrix} \mathbf{T}_m & \mathbf{0} \\ \mathbf{0} & \mathbf{T}_m \end{bmatrix}, \quad \mathbf{T}_b = \begin{bmatrix} 1 & a \\ 1 & b \end{bmatrix}, \quad \mathbf{T}_m = \begin{bmatrix} 1 & c \\ 1 & d \end{bmatrix}. \tag{13}$$

The modal coordinates -approach leads to the analytical formulae for control laws and will be used in our considerations. To simplify formulae we assume that the bearing and measurement planes coincide.

Let us introduce the complex coordinates:

$$\mathbf{p} = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} x + jy \\ \alpha + j\beta \end{bmatrix}, \quad \mathbf{p}_b = \begin{bmatrix} p_{b1} \\ p_{2} \end{bmatrix} = \begin{bmatrix} x_a + jy_a \\ x_b + jy_b \end{bmatrix}, \tag{14}$$

and coordinates $\bar{\mathbf{p}}, \bar{\mathbf{p}}_b$ are complex conjugate of coordinates $\mathbf{p}$, $\mathbf{p}_b$, respectively. Following the introduced notation first of Eqs. (12) reduces to the form:

$$\mathbf{p}_b = \mathbf{T}_b \mathbf{p}, \tag{15}$$

and two equations for axisymmetric rotor motion are obtained:

$$m\ddot{p}_1 = F_e + F_s, \quad I_y \ddot{p}_2 + jI_z \Omega \dot{p}_2 = M_e + M_s, \tag{16}$$

where: m- rotor mass, $I_x = I_y$, I_z- inertia moments against the axes x,y,z, respectively, Ω - rotor angular velocity, $F_e = F_{ex} + jF_{ey}$, $M_e = M_{ex} + jM_{ey}$ - electromagnetic forces and their moments reduced to the rotor mass center, $F_s = F_{sx} + jF_{sy}$, $M_s = M_{sx} + jM_{sy}$ - external forces and its moments reduced to the rotor mass center, respectively. Magnetic forces and external forces applied in planes can also be presented in complex notation as:

$$\mathbf{F}_b = \begin{bmatrix} F_{b1} \\ F_{b2} \end{bmatrix} = \begin{bmatrix} F_{xa} + jF_{ya} \\ F_{xb} + jF_{yb} \end{bmatrix}, \quad \mathbf{F}_{sb} = \begin{bmatrix} F_{sb1} \\ F_{sb2} \end{bmatrix} = \begin{bmatrix} F_{sxa} + jF_{sya} \\ F_{sxb} + jF_{syb} \end{bmatrix}. \tag{17}$$

Modal generalized forces from Eqs. (16) can be calculated from the above forces using the following formulae:

$$[M_e \quad F_e]^T = \mathbf{T}_b^{-1} \mathbf{F}_b, \quad [M_s \quad F_s]^T = \mathbf{T}_b^{-1} \mathbf{F}_{sb}, \tag{18}$$

It is important to notice that Eqs. (16) are completely decoupled since the input generalized forces are also decoupled. Therefore the magnetic forces should be expressed in terms of modal displacements and modal currents:

$$\mathbf{M}_c \ddot{\mathbf{p}} + \mathbf{C}_c \dot{\mathbf{p}} + \mathbf{K}_c \mathbf{p} + \mathbf{K}_\Delta \bar{\mathbf{p}} = \mathbf{K}_i \mathbf{i} + \mathbf{K}_{i\Delta} \bar{\mathbf{i}} + \mathbf{F}_z \tag{19}$$

where:

$$\mathbf{i} = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} i_{x1} + ji_{y1} \\ i_{x2} + ji_{y2} \end{bmatrix}, \quad \mathbf{M}_c = \begin{bmatrix} m & 0 \\ 0 & I_y \end{bmatrix}, \quad \mathbf{C}_c = jI_z \Omega \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\mathbf{K}_c = \begin{bmatrix} -K_{10} & 0 \\ 0 & -K_{20} \end{bmatrix}, \quad \mathbf{K}_\Delta = \begin{bmatrix} -K_{1\Delta} & 0 \\ 0 & -K_{2\Delta} \end{bmatrix},$$

$$\mathbf{K}_i = \begin{bmatrix} K_{i1} & 0 \\ 0 & K_{i2} \end{bmatrix}, \quad \mathbf{K}_{i\Delta} = \begin{bmatrix} K_{i\Delta} & 0 \\ 0 & K_{i\Delta} \end{bmatrix},$$

and $K_{10} = (K_{x1} + K_{y1})/2$, $K_{20} = (K_{x2} + K_{y2})/2$,

$K_{1\Delta} = (K_{x1} - K_{y1})/2$, $K_{2\Delta} = (K_{x2} - K_{y1})/2$,

$K_{i1} = (K_{ix1} + K_{iy1})/2$, $K_{i2} = (K_{ix2} + K_{iy2})/2$,

$K_{i\Delta} = (K_{ix1} - K_{iy1})/2$, $K_{i\Delta} = (K_{ix2} - K_{iy1})/2$,

In the above formulae $\mathbf{i}$ denotes vector of complex modal currents, $\bar{\mathbf{i}}$ denotes complex modal currents conjugate with $\mathbf{i}$, K_{x1}, K_{y1}, K_{x2}, K_{y2}, are modal negative stiffnesses of the uncontrolled magnetic bearings, and K_{ix1}, K_{iy1}, K_{ix2}, K_{iy2}, are modal current stiffnesses. These coefficients can be calculated by transformation (12) or (15) from the stiffnesses for particular control axes which is presented by Eqs. (4) and (5). The modal currents are driven by modal voltages according to the following formula:

$$\mathbf{u}+\bar{\mathbf{u}}=\mathbf{K}_i\dot{\mathbf{p}}+\mathbf{K}_{i\Delta}\dot{\bar{\mathbf{p}}}+\mathbf{L}\dot{\mathbf{i}}+\mathbf{L}_\Delta\dot{\bar{\mathbf{i}}}+\mathbf{R}\mathbf{i}+\mathbf{R}_\Delta\bar{\mathbf{i}} \qquad (20)$$

$$\mathbf{u}_o+\bar{\mathbf{u}}_o=\mathbf{L}\dot{\mathbf{i}}_o+\mathbf{L}_\Delta\dot{\bar{\mathbf{i}}}_o+\mathbf{R}\mathbf{i}_o+\mathbf{R}_\Delta\bar{\mathbf{i}}_o \qquad (21)$$

where:

$$\mathbf{u}=\begin{bmatrix}u_1\\u_2\end{bmatrix}=\begin{bmatrix}u_{x1}+ju_{y1}\\u_{x2}+ju_{y2}\end{bmatrix},\ \mathbf{u}_o=\begin{bmatrix}u_{o1}\\uo_2\end{bmatrix}=\begin{bmatrix}u_{ox1}+ju_{oy1}\\u_{ox2}+ju_{oy2}\end{bmatrix},$$

$$\mathbf{L}=\begin{bmatrix}L_1&0\\0&L_2\end{bmatrix},\ \mathbf{L}_\Delta=\begin{bmatrix}L_{1\Delta}&0\\0&L_{2\Delta}\end{bmatrix},$$

$$\mathbf{R}=\begin{bmatrix}R_1&0\\0&R_2\end{bmatrix},\ \mathbf{R}_\Delta=\begin{bmatrix}R_{1\Delta}&0\\0&R_{2\Delta}\end{bmatrix},$$

and
$$L_1=(L_{ox1}+L_{sx1}+L_{oy1}+L_{sy1})/2,$$
$$L_\Delta=(L_{ox1}+L_{sx1}-L_{oy1}-L_{sy1})/2,$$
$$L_2=(L_{ox2}+L_{sx2}+L_{oy2}+L_{sy2})/2,$$
$$L_\Delta=(L_{ox2}+L_{sx2}-L_{oy2}-L_{sy2})/2,$$
$$R_1=(R_{x1}+R_{y1})/2,\ R_2=(R_{x2}+R_{y2})/2,$$
$$R_\Delta=(R_{x1}-R_{y1})/2,\ R_\Delta=(R_{x2}-R_{y1})/2.$$

Complex matrices $\mathbf{M_c}$, $\mathbf{C_c}$, $\mathbf{K_c}$, etc., represent the isotropic properties of rotor bearing system whereas the complex matrices $\mathbf{K_D}$, $\mathbf{K_{iD}}$, etc., represent anisotropic properties of the bearings. The control effort should be twofold [9]. The first part of control action $\bar{\mathbf{u}}$ should be solely devoted to make the system isotropic and then the second part of control action $\mathbf{u}$ should be applied to isotropic system to finally control the system. The current in Eq. (19) can be expressed as:

$$\bar{\mathbf{i}}=\mathbf{K}_{i\Delta}^{-1}\mathbf{K}_\Delta\bar{\mathbf{p}}, \qquad (22)$$

to make the rotor bearing system isotropic. Introducing this expression to the Eq. (20), we obtain the steady-state control voltage $\bar{\mathbf{u}}$ in the following form:

$$\bar{\mathbf{u}}=\mathbf{R}_\Delta\mathbf{K}_{i\Delta}^{-1}\mathbf{K}_\Delta\bar{\mathbf{p}}. \qquad (23)$$

In similar way we reduce the operation point complex modal model (21) to the isotropic one by steady-state control voltage:

$$\bar{\mathbf{u}}_o=\mathbf{R}_\Delta\bar{\mathbf{i}}_o. \qquad (24)$$

When subsystem given by Eq. (19) and Eq. (20) is isotropic then it can be split into two modal subsystems.

Translational motion state-space model. The structure of this model is similar to the single axis models (4) and (5). Therefore the control laws are calculated according to (6) and (8).

Rotational motion state-space model. This model has got the similar structure to the translation motion model except state matrix which has got additional nonzero element generated by gyroscopic effect. Analytical formulae for calculation of the control law gain for this model are given in [7].

Complete modal control system of the rigid rotor motion is shown in Fig. 3.

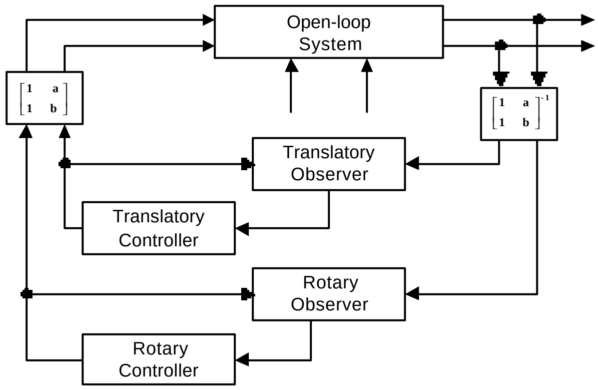

FIGURE 3. Modal control system

ON-LINE IDENTIFICATION OF CONTROLLER/OBSERVER ARX MODEL

An identification procedure of magnetic bearing physical parameters was shown in [2]. On-line version of this procedure will be useful in diagnostics and adaptive control of rigid rotor motion. According to the method described in [2] we design an ARX model of the observer/controller system:

$$\mathbf{y_u}(k)=\sum_{i=1}^{p}\mathbf{a}_i\mathbf{y_u}(k-i)+\sum_{i=1}^{p}\mathbf{b}_i\mathbf{v}(k-i), \qquad (25)$$

where: $\mathbf{a_i}$, $\mathbf{b_i}$ are ARX model parameters. Stacking up Eq. (25) for different k one can form a matrix equation:

$$\mathbf{y_v}(k)=\mathbf{P}\mathbf{V_v}(k-1), \qquad (26)$$

where:

$$\mathbf{y_v}(k)=[\mathbf{y_u}(1)\quad \mathbf{y_u}(2)\quad \ldots\mathbf{y_u}(p)\quad \cdots\quad \mathbf{y_u}(k)],$$
$$\mathbf{P}=[\mathbf{a}_1\quad \mathbf{b}_1\quad \mathbf{a}_2\quad \mathbf{b}_2\quad \ldots\quad \mathbf{a}_p\quad \mathbf{b}_p], \qquad (27)$$

$$\mathbf{V_v}(k-1)=\begin{bmatrix}\mathbf{y_u}(0)&\cdots&\mathbf{y_u}(p-1)&\cdots&\mathbf{y_u}(k-1)\\ \mathbf{v}(0)&\cdots&\mathbf{v}(p-1)&\cdots&\mathbf{v}(k-1)\\ 0&\cdots&\mathbf{y_u}(p-2)&\cdots&\mathbf{y_u}(k-2)\\ 0&\cdots&\mathbf{v}(p-2)&\cdots&\mathbf{v}(k-2)\\ \vdots&\ddots&\vdots&\ddots&\vdots\\ 0&\cdots&\mathbf{y_u}(0)&\cdots&\mathbf{y_u}(k-p)\\ 0&\cdots&\mathbf{v}(0)&\cdots&\mathbf{v}(k-p)\end{bmatrix}.$$

Exciting the closed-loop system by known (measured) signal $\mathbf{r}(k)$ and measuring the output $\mathbf{y}(k)$ and control $\mathbf{u_f}(k)$ signals, make possible for one to calculate the

ARX model parameters through the batch least-squares method:

$$\mathbf{P}(k) = \mathbf{y}_v(k)\mathbf{V}_v^T(k-1)[\mathbf{V}_v(k-1)\mathbf{V}_v^T(k-1)]^{-1}. \quad (28)$$

When new data $\mathbf{v}(k+1)$ and $\mathbf{y}_u(k+1)$ are taken, the parameter matrix $\mathbf{P}(k)$ described in Eq. (28) must be updated to satisfy the following equation:

$$\mathbf{P}(k+1) = \mathbf{y}_v(k+1)\mathbf{V}_v^T(k)[\mathbf{V}_v(k)\mathbf{V}_v^T(k)]^{-1}, \quad (29)$$

where:

$$\mathbf{y}_v(k+1) = \begin{bmatrix} \mathbf{y}_v(k) & \mathbf{y}_u(k+1) \end{bmatrix}, \quad (30)$$

and

$$\mathbf{V}_v(k) = [\mathbf{V}_v(k-1) \quad \mathbf{v}_v(k)], \quad (31)$$

where:

$$\mathbf{v}_v(k) = \begin{bmatrix} \mathbf{y}_u(k) \\ \mathbf{v}(k) \\ \mathbf{y}_u(k-1) \\ \mathbf{v}(k-1) \\ \dots \\ \mathbf{y}_u(k+1-p) \\ \mathbf{v}(k+1-p) \end{bmatrix}. \quad (32)$$

Such formulation of the updated matrices leads to the Recursive Least-Squares formula [6] for identification of an ARX model parameters. Let us first define:

$$\mathbf{S}_v(k) = [\mathbf{V}_v(k)\mathbf{V}_v^T(k)]^{-1}, \quad (33)$$

now, define the following quantities:

$$\mathbf{G}_v(k) = \frac{\mathbf{v}_v^T(k)\mathbf{S}_v(k-1)}{1+\mathbf{v}_v^T(k)\mathbf{S}_v(k-1)\mathbf{v}_v(k)}, \quad (34)$$

$$\hat{\mathbf{y}}_u(k+1) = \mathbf{P}(k)\mathbf{v}_v(k). \quad (35)$$

Taking into account Eqs. (30) and (31), Eqs. (33) and (29) can be rewritten as:

$$\mathbf{S}_v(k) = \mathbf{S}_v(k-1)[\mathbf{I} - \mathbf{v}_v(k)\mathbf{G}_v(k)], \quad (36)$$

$$\mathbf{P}(k+1) = \mathbf{P}(k)[\mathbf{y}_u(k+1) - \hat{\mathbf{y}}_u(k+1)]\mathbf{G}_v(k). \quad (37)$$

To start the recursion calculations, the matrices $\mathbf{S}(0)$ and $\mathbf{P}(1)$ can be assigned as $[d \times \mathbf{I}_s]$ and $[\mathbf{0}_{p \times s}]$, respectively, where d is a large positive number.

ADAPTIVE CONTROL

Combining the on-line identification procedure of open loop system physical parameters with control laws given above in analytical way we obtain the adaptive control system. It should be notice that modal decoupling can be applied for the reduction of the identification model since parameters of each mode model can be identified independently. It reduces the computation effort as well for adaptive control system as well for diagnostics system.

DIAGNOSTICS SYSTEM

When we able to identify on-line the physical parameters of the open-loop system, we can introduce the diagnostics system in which the crossing of the limit values of the chosen parameters is signaled.

CONCLUSIONS

The analytical method for calculation of control law for rigid rotor magnetic bearing system is shown in the paper. The elements of gain matrix are a function of plant physical parameters and of assumed closed-loop system poles. The identification method of physical parameters is shown in paper [2]. This methods leads to adaptive control system and to diagnostics system. Magnetic bearings with in-built such systems can be called a smart magnetic bearings.

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