A UNIFIED TRANSFER FUNCTION APPROACH TO CONTROL DESIGN FOR VIRTUALLY ZERO POWER MAGNETIC SUSPENSION

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ABSTRACT

This paper studied the virtually zero power control both theoretically and experimentally. A transfer function approach was used in the analysis and design of control system. The general structures of controllers achieving virtually zero power control were derived for both current- and voltage controlled magnetic suspension systems. A direct synthesis method of constructing virtually zero power controllers was developed based on the analysis. Several experiments were carried out with an apparatus with a single degree of freedom of motion. The experimental results show the effectiveness of the proposed synthesis method.

INTRODUCTION

The virtually zero power control has been used in such magnetic suspension systems as momentum wheels for spacecraft stabilization [1, 2] and carrier systems in clean room [3] because of its power-saving characteristics. It controls an electromagnet with a permanent magnet to maintain air gap length so that the attractive force by the permanent magnet balances the other static forces acting on the suspended object.
There are several methods of achieving virtually zero power control [1-5]. One of them introduces a current integrator into a minor feedback loop [2, 3]. Another uses an observer-based controller with a function of estimating the net static force acting on the suspended object [4]. However, the general properties of the virtually zero power controllers have not been clarified.
This paper adopts a transfer function approach to analyzing and designing virtually zero power controllers. The general structures of controllers achieving the zero power control are derived for both current- and voltage-controlled magnetic suspension systems. A direct synthesis method is developed based on the analyses. Several experiments are carried out with an experimental apparatus with a single degree of freedom of motion so as to demonstrate the effectiveness of the proposed synthesis method.

MODELS

Figure 1 shows a single-degree-of-freedom-of-motion model for analysis. A hybrid magnet consisting of a permanent magnet and a pair of electromagnets is used for generating suspension force. The suspended object with a mass of m is assumed to move only in the vertical direction translationally. The equation of motion is given by

$$m\ddot{z}(t) = k_s z(t) + k_i i_z(t) + w_z(t)\,, \qquad (1)$$

where z is the displacement of the suspended object, k_i and k_s are the gap- and current-force coefficients of the hybrid magnet, i_z is the control current, and w_z is disturbance force acting on the suspended object.
The coils of the electromagnets are assumed to be connected in series so that they can be treated as a single electromagnet. The electrical circuit equation associated with the electromagnet becomes

$$L\frac{di_z}{dt} + Ri_z + k_b\dot{z} = v_z(t)\,, \qquad (2)$$

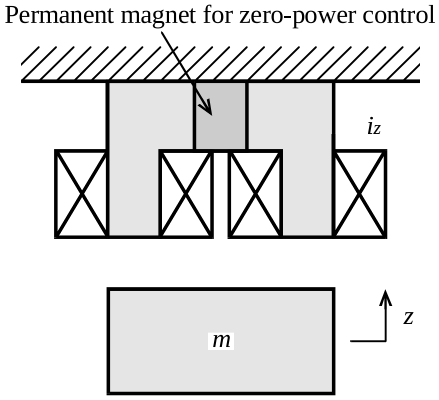


FIGURE 1: Basic model

where v_z is the voltage applied to the coil, L and R are the inductance and resistance of the coil, and k_b is the velocity-voltage coefficient.
The transfer function representation of the dynamics described by Eqs. (1) and (2) becomes

$$Z(s) = \frac{1}{t_o(s)}(a_{23}I_z(s) + d_0W_z(s)) \,, \tag{3}$$

$$(Ls + R)I_z(s) + k_b sX(s) = V_z(s) \,, \tag{4}$$

where each Laplace-transformed variable is denoted by its capital, and

$$t_o(s) = s^2 - a_{21} \,, \tag{5}$$

$$a_{21} = \frac{k_s}{m} \,, \qquad a_{23} = \frac{k_i}{m} \,, \qquad d_0 = \frac{1}{m} \,.$$

The magnetic suspension systems are classified into two types according to the power amplifier. One uses a power amplifier with current output so that the coil current can be treated as control input and the control system is designed based on Eq.(3). This type is called as current-controlled magnetic suspension system. The other type uses a power amplifier with voltage output so that the voltage applied to the coil is treated as a control input while the coil current is treated as a state variable; the control system is designed based on Eqs. (3) and (4). This type is called as voltage-controlled magnetic suspension system.

ANALYSIS

In discussing the virtually zero power control, the disturbance should be considered to be stepwise. It can be modeled as

$$W_z(s) = \frac{A_0}{s} \,. \tag{6}$$

In the following analysis, a transfer function approach will be applied, which has been effective in studying the fundamental properties of magnetic bearing controllers [6, 7].

Current-Controlled Suspension System

When linear control law is applied, the control input is generally represented as

$$I_z(s) = -\frac{h(s)}{g(s)} Z(s) \,, \tag{7}$$

where $g(s)$ and $h(s)$ are coprime polynomials in s.
The virtually zero power control operates to accomplish

$$\lim_{t \to \infty} i_z(t) = 0 \quad \textit{for stepwise disturbances.} \tag{8}$$

Substituting Eq.(7) into Eq.(3) gives

$$Z(s) = \frac{g(s)}{t(s)} d_0 W_z(s) \,, \tag{9}$$

where

$$t(s) = (s^2 - a_{21})g(s) + a_{23}h(s) \,. \tag{10}$$

Substituting Eq.(10) into Eq.(7) gives

$$I_z(s) = -\frac{h(s)}{t(s)} d_0 W_z(s) \,. \tag{11}$$

To achieve the control object (8), $h(s)$ must satisfy

$$h(s) = s\tilde{h}(s) \,, \tag{12}$$

where $\tilde{h}(s)$ is an appropriate polynomial. Figure 2a shows a general form of zero-power controller. It can be transformed as shown in Fig.2b because

$$I_z(s) = -\frac{s\tilde{h}(s)}{g(s)} Z(s)$$

$$= -\frac{1}{1+\frac{\tau}{s}} \cdot \frac{(s+\tau)\tilde{h}(s)}{g(s)} Z(s) \,, \tag{13}$$

where τ is a parameter introduced for the transformation.
Figure 2 indicates that there are two approaches for achieving virtually zero power control:

1. Feeding back the time derivative of displacement,
2. Introducing a minor feedback of the integral of current [2, 3].

The first approach is interpreted as constructing the feedback signal from the velocity of the suspended object [1] because velocity is the time derivative of displacement.
In the following we refer to controllers shown by Fig.2a as DD (Derivative feedback of Displacement) type and those shown by Fig.2b as IC (Integral feedback of Current) type.

Voltage-Controlled Suspension System

In most voltage-controlled magnetic suspension systems,

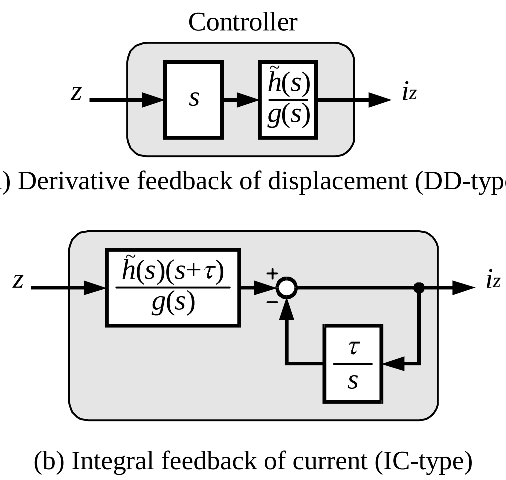


FIGURE 2: General forms of controllers with the performance of virtually zero power control

the displacement of the suspended object and the current flowing through the coil are detected. Therefore, when linear control law is applied, the control input is generally represented as

$$V_z(s) = -(M(s)Z(s) + P(s)I_z(s))$$
$$= -(\frac{n(s)}{m(s)}Z(s) + \frac{q(s)}{p(s)}I_z(s)), \qquad (14)$$

where $M(s)$ and $P(s)$ are transfer functions of the compensator, and

$m(s)$, $n(s)$: coprime polynomials,
$p(s)$, $q(s)$: coprime polynomials.

Substituting Eq.(14) into Eq.(4) and rearranging it gives

$$I_z(s) = -\frac{M(s) + K_b s}{P(s) + Ls + R} Z(s) . \qquad (15)$$

Representing the transfer function in Eq.(15) as

$$\frac{M(s) + K_b s}{P(s) + Ls + R} = \frac{h(s)}{g(s)}, \qquad (16)$$

we obtain the same equation as Eq.(7) so that we can apply the same approaches.

According to the first approach, the feedback control signal should be constructed from the time derivative of displacement. It indicates that the transfer function $M(s)$ is selected to satisfy

$$M(s) = \frac{s\tilde{n}(s)}{m(s)}, \qquad (17)$$

where $\tilde{n}(s)$ is a polynomial. Substituting Eq.(17) into Eq.(15) gives

$$I_z(s) = -\frac{(\tilde{n}(s) + K_b m(s))p(s)}{(q(s) + Ls + R)m(s)} \cdot sZ(s) . \qquad (18)$$

If $q(s)$ is selected so that $q(s) + Ls + R$ has no zero at the origin, the control object (8) is achieved.

Figure 3a shows a general form of virtually zero power controllers. It can also be transformed as shown by Fig.3b where τ is a parameter introduced for the transformation. This figure shows that zero power control can be achieved by introducing a local integral feedback. It means to feed back the integral of the control input itself [5] or equivalently the control *voltage* applied to the coil when the control input is constructed only from the displacement, *that is*

$$P(s) = 0 . \qquad (19)$$

According to the second approach, the integral of the coil current is fed back. It suggests that the transfer function $P(s)$ is selected as

$$P(s) = \frac{q(s)}{s\tilde{p}(s)}, \qquad (20)$$

where $\tilde{p}(s)$ is a polynomial. Substituting Eq.(20) into Eq.(15) gives

$$I_z(s) = -\frac{(n(s) + K_b sm(s))\tilde{p}(s)}{\{q(s) + (Ls + R)s\tilde{p}(s)\}m(s)} \cdot sZ(s) . \qquad (21)$$

When $m(s)$ is selected to have no zero at the origin, the control object (8) is also achieved.

Another interesting and simple approach is to feed back only the coil current, *that is*

$$M(s) = 0 . \qquad (22)$$

Substituting Eq.(21) into Eq.(15) gives

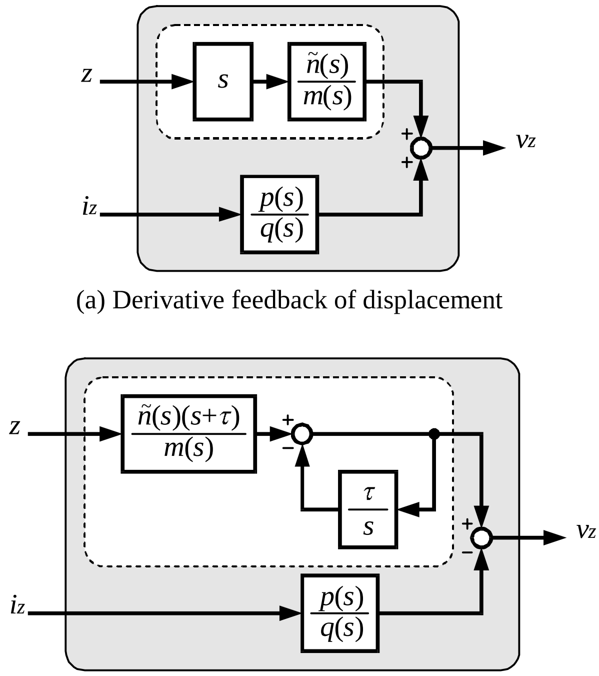
$$I_z(s) = -\frac{K_b s}{P(s) + Ls + R} Z(s) . \qquad (23)$$

If $P(s)$ is selected so that $P(s) + Ls + R$ has no zero at the origin, the control object (8) is achieved. This means that the self-sensing suspension has the performance of virtually zero power control automatically [8].

CONTROL SYSTEM DESIGN

This section will present a direct synthesis method of constructing controllers with the performance of virtually zero power control. The current-controlled suspension system is treated here.

Assuming that compensators are restricted to proper rational functions, a second- or higher-order compensator is necessary for assigning the closed-loop



(a) Derivative feedback of displacement

(b) Local integral feedback

FIGURE 3: General forms of controllers with the performance of virtually zero power control for the voltage-controlled magnetic suspension system

poles arbitrarily. From Eqs.(7) and (8), a second-order compensator achieving virtually zero power control can be represented as

$$I_z(s) = -\frac{s(\tilde{h}_2 s + \tilde{h}_1)}{s^2 + g_1 s + g_0 s} Z(s) . \tag{24}$$

From Eq.(10), the characteristic polynomial of the closed-loop system is obtained as

$$t(s) = s^4 + g_1 s^3 + (-a_0 + g_0 + b_0 \tilde{h}_2) s^2 + (-a_0 g_1 + b_0 \tilde{h}_1) s - a_0 g_0 . \tag{25}$$

To obtain a system with a characteristic equation of the form

$$t_d(s) = (s^2 + 2\varsigma_1 \omega_1 s + \omega_1^2)(s^2 + 2\varsigma_2 \omega_2 s + \omega_2^2) = s^4 + d_3 s^3 + d_2 s^2 + d_1 s + d_0 , \tag{26}$$

we can match coefficients to obtain

$$g_1 = d_3 , \tag{27}$$

$$g_0 = -\frac{d_0}{a_0} , \tag{28}$$

$$\tilde{h}_2 = \frac{1}{b_0}(d_2 + \frac{d_0}{a_0} + a_0) , \tag{29}$$

$$\tilde{h}_1 = \frac{1}{b_0}(d_1 + a_0 d_3) . \tag{30}$$

When compensators are restricted to strictly proper rational functions, a third- or higher-order compensator is necessary for arbitrary pole assignment. A third-order compensator achieving zero power control can be represented as

$$I_z(s) = -\frac{s(\tilde{h}_2 s + \tilde{h}_1)}{s^3 + g_2 s^2 + g_1 s + g_0} Z(s) . \tag{31}$$

To obtain a system with a characteristic equation of the form

$$t_d(s) = s^5 + d_4 s^4 + d_3 s^3 + d_2 s^2 + d_1 s + d_0 , \tag{32}$$

we can match coefficients to obtain

$$g_2 = d_4 , \tag{33}$$

$$g_1 = a_0 + d_3 , \tag{34}$$

$$g_0 = -\frac{d_0}{a_0} , \tag{35}$$

$$\tilde{h}_2 = \frac{1}{b_0}(d_2 + a_0 g_2 - g_0) , \tag{36}$$

$$\tilde{h}_1 = \frac{1}{b_0}(d_1 + a_0 g_1) . \tag{37}$$

EXPERIMENTS

Figure 4 shows an experimental apparatus with a single-degree-of-freedom motion. It has an arm as a suspended object, and a hybrid magnet composed of a permanent magnet and electromagnets. The permanent magnets is made of NdFeB materials.

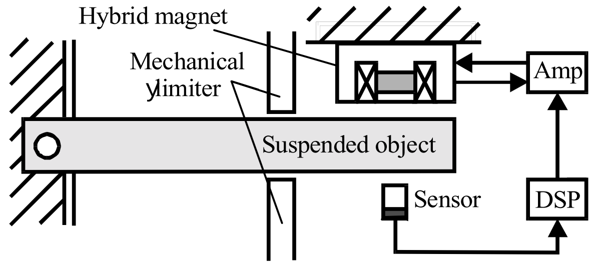


FIGURE 4: Experimental apparatus

It is to be noted that permanent magnets providing bias flux can be built in the target iron of the suspended object instead of the stator magnet. Such configuration has an effect of widening the operation range because repulsive force can be generated between the suspended object and the electromagnet [9].

The motion of the arm is restricted by a mechanical device (limiter) to avoid collision with the magnet and the sensor. Designed control laws are implemented with a DSP-based digital controller.

In the following experiments, the arm was on the lower-side limiter when control started or the power switch was turned on. It has been confirmed that both DD-type and IC-type controllers succeeded in pulling the suspended object up to an equilibrium position.

The response of a designed system to a sudden change of static force acting on the arm is shown in Fig.5. A DD-type second-order compensator given by Eq.(24) is implemented. The closed-loop poles are selected as

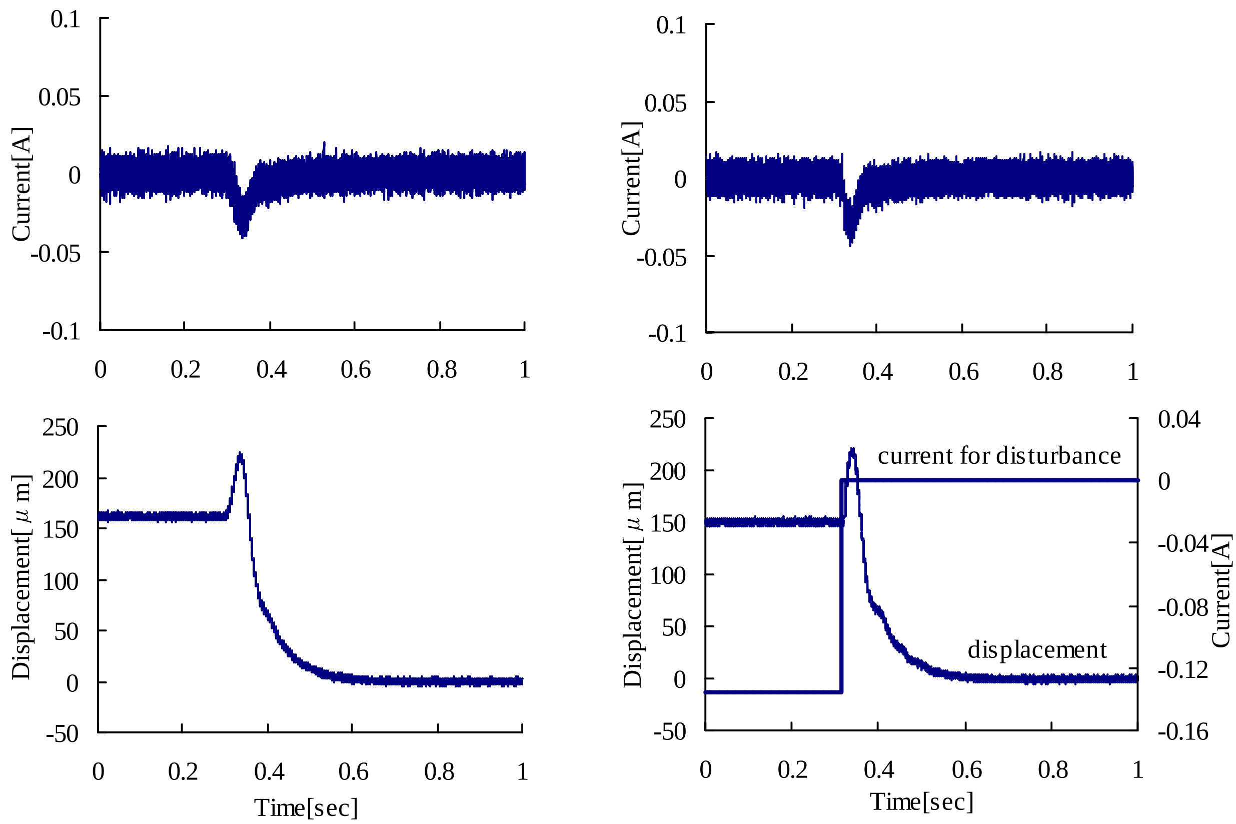
$$\omega_1 = 2\pi \times 15 \text{ (1/s)}, \quad \zeta_1 = 0.4 ,$$

$$\omega_2 = 2\pi \times 28 \text{ (1/s)}, \quad \zeta_2 = 4.5 .$$

In Fig.5a an external force of about 20N was made to act on the arm in the negative direction by using a weight initially. The weight was removed to measure step responses. It is observed that the coil current has no DC component independently of static load force. It demonstrates that the virtually zero power control is realized.

Figure 5b shows responses when a stepwise signal is added to control signal; only the component for control is shown as *current* in the upper graph. Comparing Fig.5a and Fig.5b, we find that the responses are similar. Therefore, it is reasonable to estimate dynamic characteristics by superimposing test signals on the control signal.

Figure 6 compares the frequency response of the DD-type system with that of the corresponding IC-type; the input and output signals are the superimposed signal and the control component of the coil current. The low-frequency characteristics indicate that the DC component is removed from the control component. The



(a) Response to a stepwise disturbance produced by a weight

(b) Response to a stepwise disturbance produced by a superimposed signal

FIGURE 5: Step responses to a stepwise disturbance force acting on the susepended object

equivalence between the two controllers is demonstrated well because there is no significant difference between Fig.6a and Fig.6b.

CONCLUSION

The virtually zero power control was studied both theoretically and experimentally. A transfer function approach was used in the analysis and design of control system.

The analytical study on the current-controlled magnetic suspension system showed that there are two basic approaches for achieving virtually zero power control; one constructs the feedback signal from the time derivative of the displacement of the suspended object, and the other introduces a minor feedback of the integral of the coil current.

It was also shown that both the approaches can be applied to the voltage-controlled magnetic suspension system. Moreover, there are two special cases. In one of them the feedback signal is constructed only from the displacement. In this case, zero power control can be achieved by introducing a minor feedback of the integral of voltage applied to the coil. In the other case, the feedback signal is constructed only from the coil current. Such position-sensorless operation achieves zero power control automatically.

A synthesis method of constructing controllers with the performance of virtually zero power control was developed based on the analysis. Several experiments on the current-controlled magnetic suspension system were carried out. The experimental results showed the equivalence of both types of virtually zero power controllers and the effectiveness of the proposed synthesis method.

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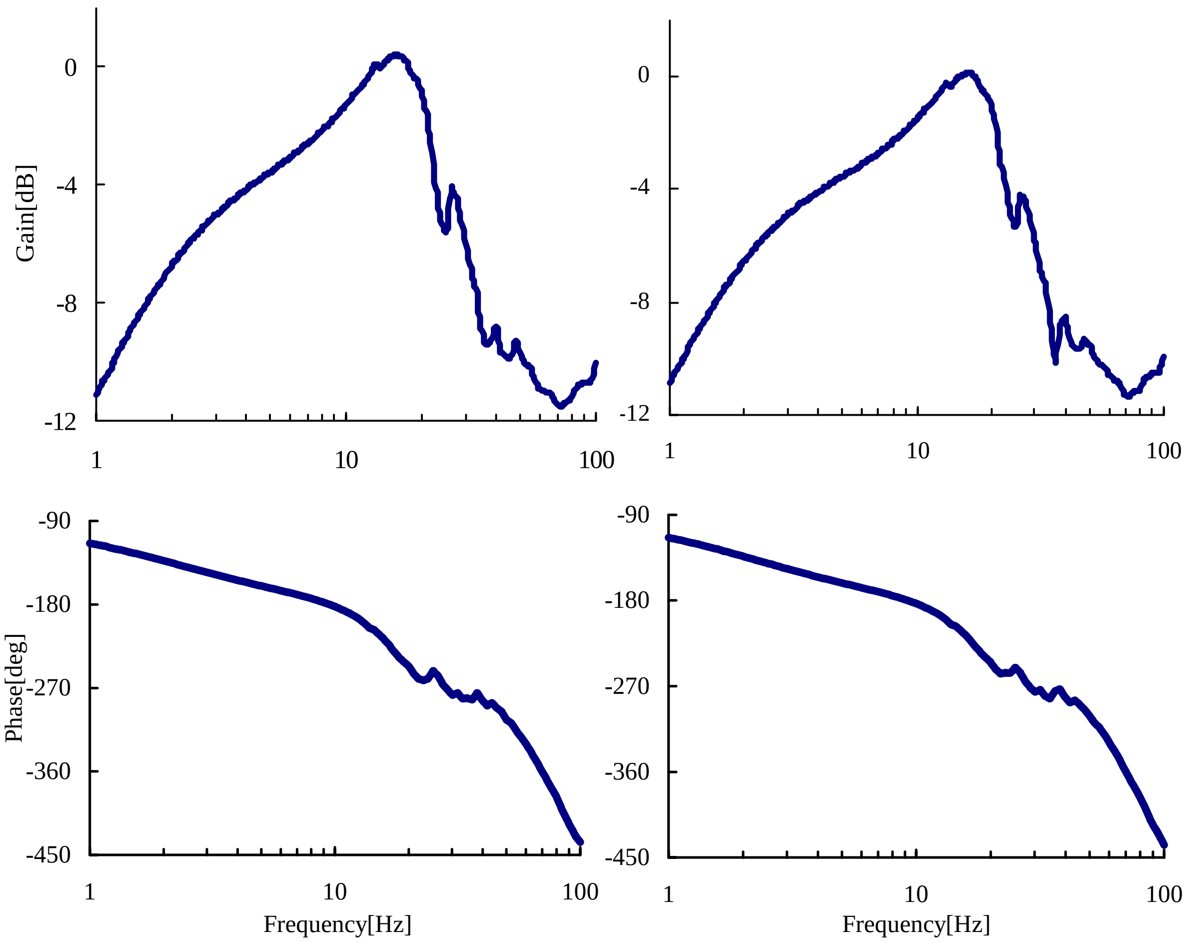

(a) Derivative feedback of displacement (b) Integral feedback of current

FIGURE 6: Frequncy responses of the virtually zero power control systems

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