POWER OPTIMAL CURRENT CONTROL SCHEME FOR BEARINGLESS PM MOTORS

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ABSTRACT

Bearingless motors offer new possibilities in drive technology and open up new design approaches to applications where conventional bearings can hardly be employed. However, especially for low power applications a very cost-effective and highly efficient motor design is desirable.

In this paper a current control scheme is presented that ensures operation of the bearingless motor with minimized resistive power losses. Moreover, for motor designs where all the winging systems are used both for radial force and torque generation this control scheme may also ensure production of arbitrary radial forces and torque even if the sum of the phase currents is equal to zero. As a result, star connection of the motor phases is possible and hence the number of required power switches for driving the motor can be reduced.

INTRODUCTION

In the last few years bearingless motors have been applied to many different industrial applications [2]-[4], [6], [7]-[10]. These applications cover a wide output power range from a few watts up to hundreds of kilo watts. Usually, the design considerations for a drive depend upon the type of the bearingless motor but are also strongly influenced by the power range. For applications in the upper power range, normally, separate three-phase windings for torque and force generation are used [7]. In this case conventional industrial three-phase servo amplifiers are applicable. For low power applications, however, a very inexpensive mechanical motor design can be accomplished when the windings are used for force and torque generation at the same time and concentrated windings are employed rather than distributed ones [1], [11]. To supply these motors, usually, one full bridge converter per phase is required. However, a substantial reduction in the complexity of the power converter can be obtained if the sum of the phase currents can be kept zero under any load condition. This makes star connection of the motor phases possible and thus the number of power switches can be halved. Moreover, for small motors the efficiency of a drive plays an increasingly important role in the motor design. For this reason permanent magnet type bearingless motors are often applied to low power applications. Further improvement of the efficiency can be achieved when the resistive power losses are minimized by means of an appropriate control scheme.

In the following sections an new control scheme for bearingless PM motors with concentrated windings is presented. The derivation of this control scheme is based on an optimization problem which makes it possible to vary the characteristics of the motor by modifying the constraints.

FORCE AND TORQUE MODEL

Before the optimization problem can be formulated it is necessary to derive a mathematical model which describes generation of the radial levitation forces and torque. The exact force and torque model for bearingless motors with PM excitation has the quadratic form [11]

$$\mathbf{F}_{r} = \begin{bmatrix} \mathbf{i}_{1}^{T} & \mathbf{0} \\ \mathbf{0} & \mathbf{i}_{1}^{T} \end{bmatrix} \begin{bmatrix} \mathbf{M}_{Qx}(\mathbf{x}_{r}, \boldsymbol{\varphi}) \\ \mathbf{M}_{Qy}(\mathbf{x}_{r}, \boldsymbol{\varphi}) \end{bmatrix} \mathbf{i}_{1} + \mathbf{M}_{L}(\mathbf{x}_{r}, \boldsymbol{\varphi}) \mathbf{i}_{1} + \mathbf{M}_{C}(\mathbf{x}_{r}, \boldsymbol{\varphi})$$

$$T_{r} = \mathbf{i}_{1}^{T} \mathbf{N}_{Q}(\mathbf{x}_{r}, \boldsymbol{\varphi}) \mathbf{i}_{1} + \mathbf{N}_{L}(\mathbf{x}_{r}, \boldsymbol{\varphi}) \mathbf{i}_{1}, \qquad (2)$$

where \mathbf{F}_r and T_r denote the vector of the levitation forces in the stator coordinate system and the electromagnetic torque respectively. The stator currents of the *m* phase stator system are represented by

$$\mathbf{i}_1 = \begin{bmatrix} i_1 & i_2 & \cdots & i_m \end{bmatrix}^T.$$

The rotor position in radial direction is written as

$$\mathbf{x}_r = \begin{bmatrix} x_r \\ y_r \end{bmatrix}$$

and the rotor angle is denoted φ . For a bearingless PM motor with a large air gap and when rare earth permanent magnets are used, the elements of the quadratic terms \mathbf{M}_{Qx} , \mathbf{M}_{Qy} and \mathbf{N}_Q are small in comparison with the elements of the other matrices. Therefore these terms can be neglected and the levitation force and torque model can be found as

$$\mathbf{F}_r = \mathbf{M}_L(\mathbf{x}_r, \mathbf{\phi}) \,\mathbf{i}_1 + \mathbf{M}_C(\mathbf{x}_r, \mathbf{\phi}) \tag{3}$$

$$T_r = \mathbf{N}_L(\mathbf{x}_r, \boldsymbol{\varphi}) \mathbf{i}_1.$$
(4)

In equation (3) the matrix \mathbf{M}_{C} is only a function of the rotor position. Normally, this matrix becomes zero when the rotor is in the centered position. In this case equations (3) and (4) can be combined into one expression

$$\mathbf{Q} = \mathbf{T}_m \, \mathbf{i}_1, \tag{5}$$

with

$$\mathbf{Q} = \begin{bmatrix} F_{rx} \\ F_{ry} \\ T_r \end{bmatrix}$$

and

$$\mathbf{T}_m = \begin{bmatrix} \mathbf{M}_L(\mathbf{x}_r, \boldsymbol{\varphi}) \\ \mathbf{N}_L(\mathbf{x}_r, \boldsymbol{\varphi}) \end{bmatrix}.$$

OPTIMIZATION PROBLEM

For the operation of a bearingless motor it is necessary to supply the motor phases with such currents that desired radial forces and torque are generated. In a mathematical sense this requires that equation (5) has to be solved for the phase currents \mathbf{i}_1 . However, a unique solution of the form

$$\mathbf{i}_1 = \mathbf{T}_m^{-1} \mathbf{Q}$$

can only be found if the bearingless motor has three phases. A typical bearingless motor has more than three phases. This means that any desired radial force and torque might then be realized by many different sets of phase currents. Since many solutions might be possible it is the task to find a set of currents that generates the required radial forces and torque in the best way. One feasible solution to find the best set of currents is to minimize the resistive power losses. This leads to the following optimization problem:

$$\min_{\mathbf{i}_1} \mathbf{i}_1^T \mathbf{R}_1 \mathbf{i}_1 \tag{6}$$

subject to

$$\mathbf{T}_m \, \mathbf{i}_1 - \mathbf{Q} = \mathbf{0},\tag{7}$$

where \mathbf{R}_1 is the matrix of the phase resistances. A possible way of solving this optimization problem is to combine the desired force and torque constraints (7) into the cost function via scaling by Lagrange multipliers, denoted by λ

$$L = \mathbf{i}_1^T \mathbf{R}_1 \mathbf{i}_1 + \boldsymbol{\lambda}^T (\mathbf{T}_m \mathbf{i}_1 - \mathbf{Q}).$$

For an optimum, the partial derivatives of *L* with respect to both \mathbf{i}_1 and λ must be equal to zero:

$$\frac{\partial L}{\partial \mathbf{i}_1} = 2 \mathbf{i}_1^T \mathbf{R}_1 + \boldsymbol{\lambda}^T \mathbf{T}_m = \mathbf{0}$$

$$\frac{\partial L}{\partial \boldsymbol{\lambda}} = (\mathbf{T}_m \mathbf{i}_1 - \mathbf{Q})^T = \mathbf{0}$$
(8)

The phase currents \mathbf{i}_1 that minimizes the resistive power losses can be found by solving the linear system (8) as

$$\mathbf{i}_1 = \mathbf{K}_m(\mathbf{x}_r, \mathbf{\phi}) \mathbf{Q},$$

with the decoupling matrix



FIGURE 1: Bearingless single-phase motor

$$\mathbf{K}_m(\mathbf{x}_r, \boldsymbol{\varphi}) = \mathbf{R}_1^{-1} \mathbf{T}_m^T \left(\mathbf{T}_m \mathbf{R}_1^{-1} \mathbf{T}_m^T \right)^{-1}.$$

EXTENDED OPTIMIZATION PROBLEM FOR THE BEARINGLESS SINGLE-PHASE MOTOR

A very simple mechanical design of a bearingless motor can be obtained through an arrangement consisting of only four concentrated coils as pictured in figure 1. Since there is no interconnection between the coils the system has four phases and thus (5) has fewer equations than unknown currents. Therefore the proposed optimization problem can be applied to this motor design.

In addition to the constraint (7) it is possible to impose an extra constraint of the form

$$\begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix} \mathbf{i}_1 = \mathbf{1}^T \mathbf{i}_1 = 0 \tag{9}$$

to force the sum of the phase currents to zero. Owing to the symmetric motor design, all four coils have the same resistances, thus the resistance matrix can be simplified to

$$\mathbf{R}_1 = R_1 \mathbf{I},$$

where R_1 denotes the coil resistance and **I** is the unit matrix. With this simplification and by means of the constraints (7) and (9) a solution to the optimization problem can be found as

$$\mathbf{i}_1 = \mathbf{K}_m(\mathbf{x}_r, \mathbf{\phi})\mathbf{Q},$$

with



FIGURE 2: Decoupling matrix \mathbf{K}_m for the bearingless single-phase motor when the rotor is in the centered position

$$\mathbf{K}_{m}(\mathbf{x}_{r}, \boldsymbol{\varphi}) = \left(\mathbf{T}_{m}^{T} - \frac{1}{m}\mathbf{1}\mathbf{1}^{T}\mathbf{T}_{m}^{T}\right).$$

$$\cdot \left(\mathbf{T}_{m}\left(\mathbf{T}_{m}^{T} - \frac{1}{m}\mathbf{1}\mathbf{1}^{T}\mathbf{T}_{m}^{T}\right)\right)^{-1},$$
(10)

where *m* denotes the number of motor phases. The elements of the decoupling matrix \mathbf{K}_m for the bearingless single-phase motor calculated with equation (10) are shown as a function of the rotor angle φ in figure 2. From this figure it can be seen that the sum of each column is equal to zero for any rotor angle. This proves that the additional constraint (7) is satisfied.

From the third column of this matrix it can further be seen that torque generation is not possible for the four rotor angles

$$\varphi = \left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}.$$

Thus the motor behaves like a single-phase motor. For this reason the motor shown in figure 1 is called bearingless single-phase motor.

CURRENT CONTROL SCHEME

Full stabilization of a rigid rotor requires six degrees of freedom to be stabilized. However, for certain applications with no demand for stiff stabilization in axial and tilting angle direction it is possible to stabilize



FIGURE 3: Current control scheme for the bearingless single-phase motor

three degrees of freedom passively by means of the reluctance forces of the permanent magnets. For these applications only three degrees of freedom must be stabilized actively.

To achieve the highest possible efficiency, bearingless motors are normally supplied with voltage source converters. However, the current control scheme requires a current source instead of a voltage source. A feasible way of overcoming this problem is to use a nested current control loop for each motor phase as shown in figure 3. This control structure is usually called cascade control. The current control loop may either be implemented as analog control system or it may be implemented as digital control system on a micro-controller or a DSP. In our case we assume that the current controllers R_{i1} to R_{i4} are implemented as analog circuit as part of the power converter. Therefore the voltage source converter together with the analog current controller can be treated like a current source converter.

The task of the position and speed control is to compute an appropriate vector of reference currents to adjust a prescribed operating point. It is apparent that the mathematical model of the bearingless single-phase motor including the voltage converter and the current control is nonlinear. However, this nonlinear system can be linearized with a change of the control inputs from i_{11ref} , ..., i_{14ref} to v_1 , ..., v_3 by means of the decoupling matrix \mathbf{K}_m as shown in figure 3. This nonlinear transformation yields a system featuring a linear behavior between the new inputs v_1 , ..., v_3 and the outputs x_r , y_r and φ . Moreover, the linear system is decomposed into three subsystems which are decoupled. In other words, this means that the new control input v_1 makes it possible to influence the x-direction of the radial rotor position without influencing the y-direction and the rotational speed. By means of the new control input v_2 only the y-direction is influenced, and with v_3 the speed can be controlled. The major advantage of this control scheme is that independent controllers for the x-and y-direction R_x , R_y and a speed controller R_ω can be developed using linear design methods [5].

It should further be noted that the operation mode of the bearingless motor may be altered simply by changing the decoupling matrix \mathbf{K}_m . This can be utilized in the case of failure of one power converter channel to ensure fault tolerant operation without the need of changing the parameters of the linear controllers [8], [11]. Furthermore, a mode of operation is possible where the sum of the phase currents is equal to zero under any load condition, as derived in the previous section.

OPTIMIZED POWER CONVERTER

Without applying a special control scheme the sum of the phase currents of the bearingless single-phase motor may not be zero for any possible load condition. For this reason the bearingless single-phase motor is normally supplied with one full bridge converter per phase. However, by means of the proposed control scheme a mode of operation is feasible that forces the sum of the phase currents to be equal to zero under any possible load condition. Thus, star connection of the motor phases is possible and the number of required power switches can be halved. A block diagram of this simplified power converter including the analog current controllers is shown in figure 4. As can easily be seen



FIGURE 4: Optimized power converter with four current controllers



FIGURE 5: Optimized power converter with three current controllers

this circuit has an immediately apparent problem, namely, it uses four current controllers to adjust only three independent currents. Changes in the parameters of the current controllers or errors in current measurement may cause a current i_0 different from zero and may also lead to undesired changes in the voltages across the capacitors *C*. An improved circuit design can be found in figure 5. To avoid the overdetermined condition the current controller of phase one has been removed and the centerpoint is left unconnected. In theory, the voltage of this phase could be set to any possible potential. However, to achieve a symmetric mode of operation the voltage u_0 is set to ground potential. Therefore the output voltage of phase one can be found as

$$u_{h1} = -u_{h2} - u_{h3} - u_{h4} \,.$$

By means of this optimized power converter a substantial reduction in the overall system costs can be achieved because only four halve bridges are required to drive the bearingless single-phase motor and to actively control three degrees of freedom.

CONCLUSIONS

In this paper a current control scheme for bearingless PM motors has been introduced which allows modification of the operation mode of the motor simply by altering a nonlinear transformation function. Based on this control structure an operation mode of the bearingless single-phase motor has been derived which ensures that the sum of the phase currents is kept zero under any possible load condition. Therefore star connection of the motor phases becomes feasible, and subsequently a simplified power converter with only four half bridges can be used for driving the motor.

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