ANALYSIS ON SYSTEM DYNAMIC PERFORMANCE OF FLEXIBLE ROTOR SUPPORTED WITH ACTIVE MAGNETIC BEARINGS

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ABSTRACT*
When active magnetic bearing (AMB) is put into use of the industrial fields, there are many problems to be solved before using it, such as system security, reliability and stability when the rotors work on high rotation speed. In China, AMB is going to be used for some applications, for example, turbo expander, and grinding spindle and aircraft engine. This paper deals with the rotor dynamics of the systems, in which rotors are supported with AMBs and have high rotation speed. On which it is focused that the method to solve the problems above is combination between the traditional theories of rotor dynamics and the control theories of AMB system. The analysis and calculation results is presented here and to be used into a 150M^3 oxygen gas expander. Meanwhile, the process and result of experiment are described in detail, in which the rotor speed reaches 95000r/min.

INTRODUCTION
Owing to the significant advantages of active magnetic bearing, AMB has been used into a lot of application field, and also AMB attracts a great deal of attention for scientists and users in many countries. Chinese researchers now begin to work at its applications, such as turbo expander, grinding spindle and aircraft engine. But there are many problems to be solved, when AMB put into use of the machines. The important ones of those problems are security, reliability and stability when the rotors work on high rotation speed[1,2]. This means that the performance of rotor dynamics of the system must be investigated first. This paper focuses on the solution way for these problems.

To solve these problems above, it is the easily and shortcut way for combination the transfer matrix method in the theory of traditional rotor dynamics with the analytical method of transfer function in active magnetic bearing system[3,4]. The key of using the method is to calculate the parameters of the bearings, these are stiffness and damping parameters of AMB. Usually like oil bearing, they are also named as K_{xx}, K_{xy}, K_{yy}, K_{yx}, C_{xx}, C_{xy}, C_{yy}, and C_{yx}.
Of the way for calculating the parameters K_{xx}, K_{yy}, C_{xx}, and C_{yy}, we use the transfer function of one degree of freedom system of AMB. For the others, K_{xy}, K_{yx}, C_{xy} and C_{yx} if there appear or not depend only on the type and set situation of the displacement sensors in the system in this method.
To make sure the method mentioned above, an experiment is designed and put into practice. Some results of theoretic and experiment are presented at the end of this paper.

DYNAMICAL MODEL OF AMB SYSTEM
As an analytical base of rotor dynamic performance of AMB system, the model of one degree of freedom system is very useful. **FIGUER 1** is the schematics of this system. In this system, the transfer function relative between disturbance force F_x and displacement of the rotor X can be described as follows[5]:

$$X(s) = \frac{F_x(s)}{ms^2 - C_1 + C_2 G(s)}$$

Or also

$$X(j\omega) = \frac{F_x(j\omega)}{[C_2 \operatorname{Re}(G(j\omega)) - C_1 - m\omega^2] + j[C_2 \operatorname{Im}(G(j\omega))]}$$

Here, C_1 and C_2 are parameters of displacement stiffness and current stiffness for the system, respectively, ω is angle frequency, here, it equals the rotating frequency

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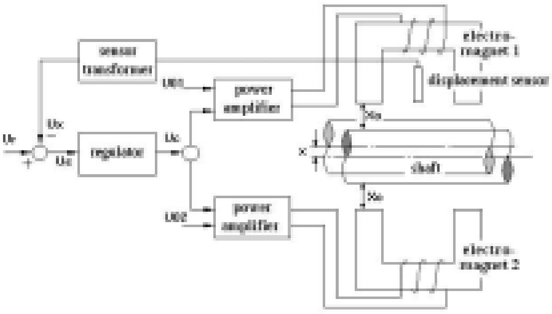


FIGUER 1 Schematics of analysis system

and proportion to speed of the rotor. $\mathrm{Re}(G(j\omega))$ and $\mathrm{Im}(G(j\omega))$ are the real part and imaginary part of transfer function $G(j\omega)$ of system controller respectively.

Comparing the equation above to the two grade traditional dynamic function like follows:

$$X(j\omega) = \frac{F_x(j\omega)}{(k - m\omega^2) + j\omega c}$$

We can know the following equations:

$$k = K_e = C_2 \,\mathrm{Re}[G(j\omega)] - C_1$$

$$c = C_e = \frac{C_2 \,\mathrm{Im}[G(j\omega)]}{\omega}$$

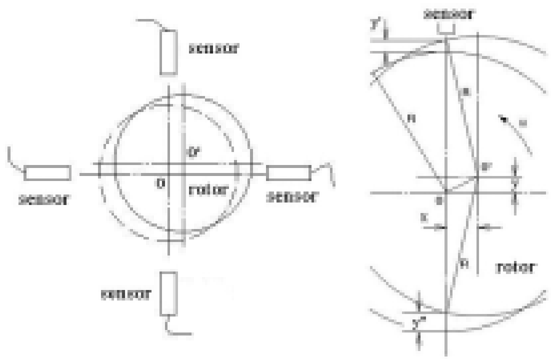
K_e and C_e are named efficient stiffness parameter and efficient damping parameter, respectively. This is the way for calculating the parameters of K_{xx}, K_{yy}, C_{xx} and C_{yy}.

To obtain the parameters of K_{xy}, K_{yx}, C_{xy}, and C_{yx}, please look at the **FIGURE 2**, the scheme of the installation way of the sensors. If you are using single sensor in a direction like the right one in the figure, you can find the following equations,

$$y' = y + \sqrt{R^2 - x^2} - R$$

$$y'' = y + R - \sqrt{R^2 - x^2}$$

Here, R is the radius of the rotor, y and x are the actual displacements in the two directions respectively. So, the single sensor in y direction can see the motion of the rotor like as



a. Differential method b. Single method

FIGURE 2 Scheme of installation way of sensors

$$y' = y + \sqrt{R^2 - x^2} - R$$

This means that the signal measured by the sensor contains not only y but also x. The effect of x on y direction is the reason caused the coefficient of the stiffness and damping K_{xy}, K_{yx}, C_{xy}, and C_{yx}. We can calculate the coefficients according to the equation[2]:

$$K_{xy} = \pm \frac{R}{2x_0} C_2 \,\mathrm{Re}\{G(j\omega)\} \quad \begin{cases} when\ x < 0, & then\ "+" \\ when\ x \ge 0, & then\ "-" \end{cases}$$

$$K_{yx} = \pm \frac{R}{2x_0} C_2 \,\mathrm{Re}\{G(j\omega)\} \quad \begin{cases} when\ x < 0, & then\ "-" \\ when\ x \ge 0, & then\ "+" \end{cases}$$

$$C_{yx} = \pm \frac{R}{2x_0} \frac{C_2 \,\mathrm{Im}\{G(j\omega)\}}{\omega} \quad \begin{cases} when\ x < 0, & then\ "-" \\ when\ x \ge 0, & then\ "+" \end{cases}$$

$$C_{xy} = \pm \frac{R}{2x_0} \frac{C_2 \,\mathrm{Im}\{G(j\omega)\}}{\omega} \quad \begin{cases} when\ x < 0, & then\ "+" \\ when\ x \ge 0, & then\ "-" \end{cases}$$

If using differential method for setting sensors like b in the figure, there is no coefficient K_{xy}, K_{yx}, C_{xy}, and C_{yx}..

BASE OF ROTOR DYNAMICAL THEORETICS

Once obtaining the eight parameters of stiffness and damping of AMB and the structure of the rotor, the properties of the rotor-bearing system, such as natural frequency, critical rotating speed and stability of the system can be explored on the base of an improved traditional transfer matrix method. This method is especially used for a multi-mass model of AMB-rotor system [6], in which the traditional rotor dynamical method is combined with the way to design the controller of the system.

Through the calculation mentioned before, it results like **FIGURE 3**, including the curves of stiffness and damping, as well as the curves of natural frequencies, with their logarithmic rate and mainly the critical speeds of the system.

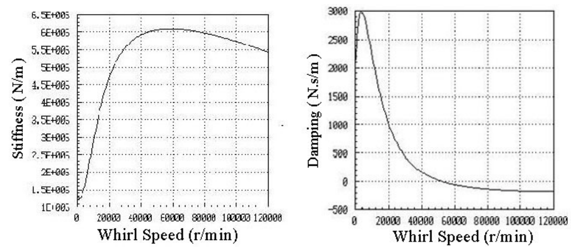


FIGURE 3a Stiffness and damping of system

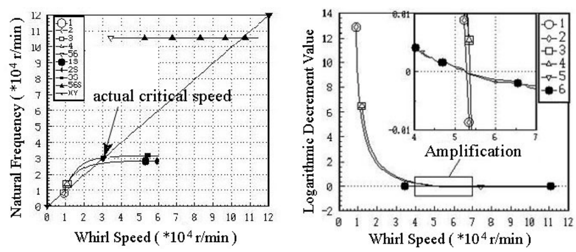


FIGURE 3b Natural Frequency and Logarithmic Decrement Value of system

In **FIGURE 3**, the curves of stiffness and damping of the system are calculated with the transfer function of the system; and it is noticed that on the curve of natural frequency every point is one natural frequency of the system, but must not actual one. **Only those crossing with 45-degree line are the actual ones**. Those actual ones of natural frequencies are also named as critical rotation speeds in the system. With the actual frequencies, only is that unstable, which have negative logarithmic decrement value correspondingly.

Especially to notice that not all of the critical rotation speed position can be changed by the control parameters of system, only those dropped into lower frequency area can be moved their seats according to adjustment of the controller. The both theoretical and experimental results tell us that the critical rotation speeds in higher frequency area can go to other seats only depend on change of the structure of the rotor. And another important thing they may tell us is that they are not stable rotation speeds for the system, because the logarithmic decrement values corresponding to them are negative or become to zero. For both kinds of critical rotation speed described before, they are named as **bearing dominant critical rotation speed** and **rotor dominant critical rotation speed**, respectively.

Here has another important thing comparing with oil film bearings is the unit in X axis, it is just whirl speed and not as the rotation speed as oil film bearings. But, if not pointing out especially, they are equal each other in this paper.

DESIGN EXAMPLE OF THE SYSTEM

Turbo machine is a mainly domain for application of AMB. As an application of using the method above, the rotor dynamic behavior of a 150M^3 turbo oxygen gas expander rotor-AMB system with an analog controller

TABLE 1: Parameters of the bearings

	Radial Bearing	Thrust Bearing
Outer Diameter	72 mm	72 mm
Inner Diameter	30 mm	30 mm
Thickness	15 mm	15 mm
Bias Current	1.0 A	1.2 A
Turns of Coil	42	80
Air Gap	0.17 mm	0.2 mm
DC Resistance	0.2 •	0.3 •
Inductance	1.8 mH	0.8 mH

is investigated. The weight of rotor in the machine is

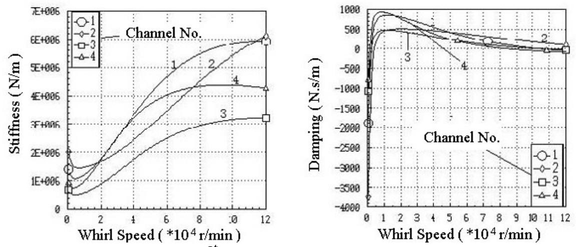


FIGURE 4 the 1st group of stiffness and damping

about 1.16kg, the length and radius at the seat of the bearing of the rotor are about 250mm and 15mm, respectively. The parameters of AMB are listed in the

TABLE 2: the 1st Result of Critical Speeds (r/min) and Logarithmic Decrement Values

	1st	2nd	3rd	4th	5th	6th
n_{cr}	8,050	9,430	12,240	15,380	105,520	105,540
ξ	2.647	4.634	3.540	1.800	-.0046	0.001

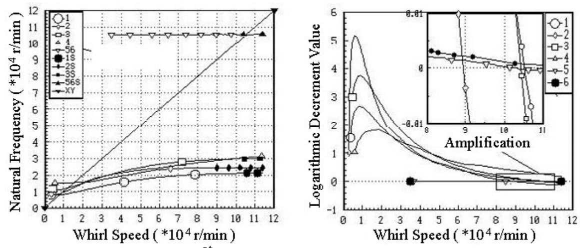


FIGURE 5 the 1st group of natural frequencies and logarithmic decrement values

TABLE 1. There presents three groups of analytical and

TABLE 3: the 2nd Result of Critical Speeds (r/min) and Logarithmic Decrement Values

	1st	2nd	3rd	4th	5th	6th
n_{cr}	8,050	9,480	12,240	15,400	10,5520	105,540
ξ	2.647	4.622	3.539	1.800	-.0048	0.001

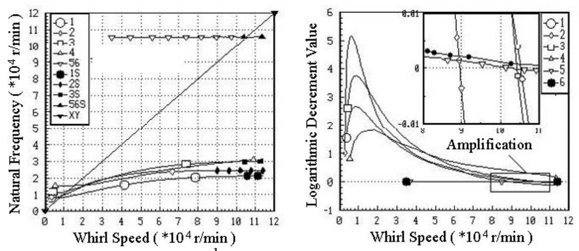


FIGURE 6 the 2nd group of natural frequencies and logarithmic decrement values

calculated result for the system[7].

The first one is showed in FIGURE 4, FIGURE 5 and TABLE 2. The curves in FIGURE 4 give out the coefficients of stiffness and damping against the whirl speed of the rotor. On the base of these coefficients, the curves of natural frequencies and logarithmic decrement value are illustrated in FIGURE 5, the critical speeds those points crossing with 45-degree line and their logarithmic decrement values are listed in TABLE 2.

This result tell us the 1st to 4th critical speeds are bearing dominant ones, the 5th and 6th rotor dominant ones. And if rotation speed reaches 105,000r/min, the system will be unstable. Notice: here not consider the effects of the coefficients of K_{xy}, K_{yx}, C_{xy}, and C_{yx}.

If consider the effects of the coefficients of K_{xy}, K_{yx}, C_{xy}, and C_{yx}, the results of natural frequency and logarithmic value present in FIGURE 6, critical speeds and logarithmic decrement values in TABEL 3. In here, the seats of the critical speed and its stability is same as before one, the effect of the coefficients of K_{xy}, K_{yx}, C_{xy}, and C_{yx} is insignificant.

The third result showed in here is that the parameters of the controller has been changed. The controller is an anisotropic controller and meantime, the effects of K_{xy}, K_{yx}, C_{xy}, and C_{yx} are included. Analyzing the results, that showing and listing in FIGURE 7, FIGURE 8 and TABEL 4 respectively, some phenomena are distinctive:

1. **The stiffness in channel 2 (X-direction) is going to maximum value, about 1.6×10^{7}N/m, and all the others have tiny increasing than before.**
2. **Obviously, there have three curves of natural frequency which cross the 45-degree line two times, this results total nine critical speeds in this system. The number of critical speed has changed from 4 to 7, the region of that has take placed from 8,040 r/min to 15,500 r/min with from 8,070 r/min to 48,300 r/min, so that the reliability of the system is decreased. But, in higher frequency, the critical speeds are not moved any more, this proves that they are rotor dominant critical speeds.**
3. **Anyway, this machine can not work upon to the rotation 105,000 r/min now.**

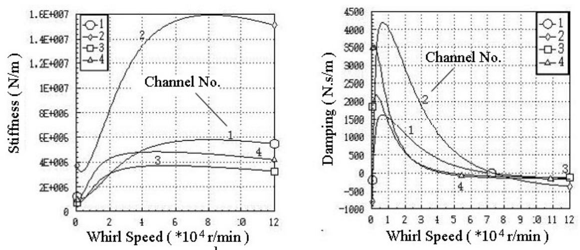


FIGURE 7 the 3rd group of stiffness and damping

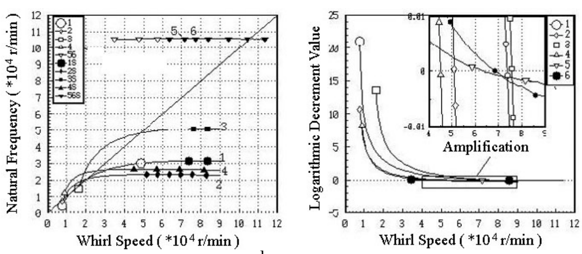


FIGURE 8 the 3rd group of critical speed

EXPERIMENTS AND RESULTS

This machine mentioned above has been put into series of experiment during 1998 ~ 1999. When using the curves of stiffness and damping listing in TABEL 3, the rotating speed of the expander can reach the highest speed 104,000 r/min, and its stabilizing rotation speed is more than 96,000 r/min. If using the curves showing in FGURE 6, only can the machine reach on the top of rotation speed 68,000 r/min. FIGURE 9 is a picture of the oxygen gas turbo expander in experiment running. FIGURE 10 to FIGURE 13 are showing some pictures of rotating track of the rotor center.

These pictures of result have proved the analytical method in theory is correct. In the experiment, four vibrations of rotor can be seen clearly, on these critical speeds the rotor can not stand at all. At the same times, the rotor bumps against the active magnetic bearings. When exceeding the rotating area, the motion of rotor seems to be stable and smooth, and the rotating radius is less than 10 micrometers. As the beginning of 96,000 r/min, the rotating centers of the rotor become extend. When rotation reaches probably 100,000 r/min, the system would not be running.

If structure of the rotor will be changed, the position of critical speeds within higher frequency area can be moved. This means that they are rotor dominant critical speeds indeed.

In the system, five eddy current displacement sensors are set up in five degree of freedom.

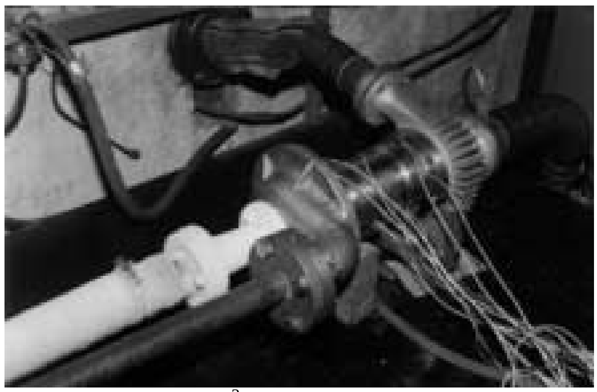


FIGURE 9 150M^3 oxygen gas turbo expander in experiment running.

TABLE 4: the 3rd result of sritical speeds (r/min) and Logarithmic Decrement Values

1st	2nd	3rd	4th	5th	6th	7th	8th	9th
8,070	11,650	16,690	20,580	20,810	25,546	48,300	105,540	105,740
9.742	7.200	11.59	1.369	2.848	0.682	0.784	-.0027	-.0066

CONCLUSIONS

Through analysis, design, calculation and experiment on behavior of rotor bearing dynamics for a 150M3 oxygen gas turbo expander, follow conclusions are clear:

First, there also have eight coefficients named as K_{xx}, K_{xy}, K_{yy}, K_{yx}, C_{xx}, C_{xy}, C_{yy}, and C_{yx} in AMB system like in oil film bearing system. The coefficients relate the control parameters of the system and the setting position of displacement sensor.

Second, for AMB system, also have critical rotation speeds, but these are only the points crossing 45-degree line in the curves of natural frequency. And the logarithmic decrement values which corresponding the speeds can help to analyze the stability of the system. The differences between the curves of AMB system and oil film bearing system are the unit in X-axis, rotation unit is take placed with whirl speed unit.

Next, the critical speeds in AMB system are divided into two types, one is named as bearing dominant critical rotation speed which drop in lower frequency area, another as rotor dominant critical rotation speed in higher frequency area.

The last one is that here mustn't have the highest stiffness but the best suitable stiffness in an AMB system.

In above processing of investigation for rotor bearing dynamics of the system, the effect of thrust bearing on radial ones is not considered.

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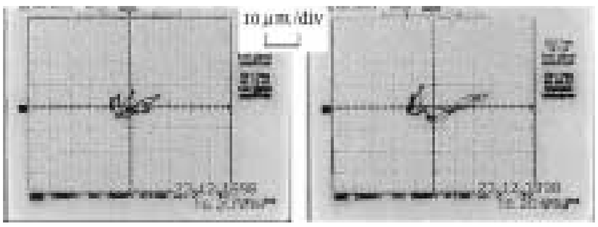
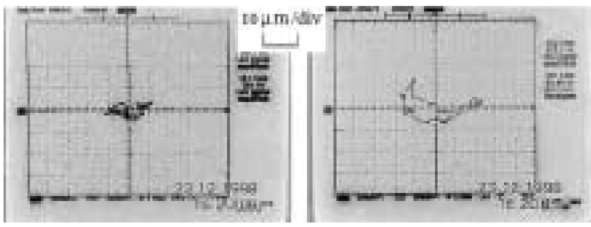


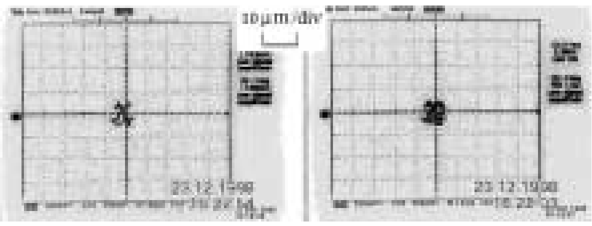
a. Work wheel bearing b. Compress wheel bearing

FIGUR10 rotor center tracks at 7,200 r/min



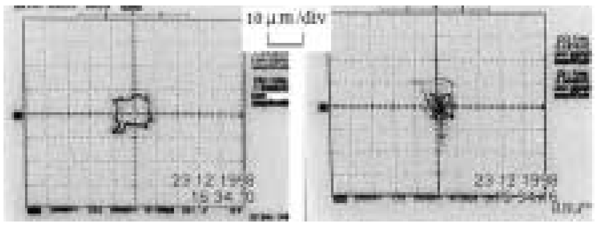
a. Work wheel bearing b. Compress wheel bearing

FIGUR11 rotor center tracks at 14,600 r/min



a. Work wheel bearing b. Compress wheel bearing

FIGUR12 rotor center tracks at 47,100 r/min



a. Work wheel bearing b. Compress wheel bearing

FIGUR13 rotor center tracks at 94,000 r/min