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ROBUST PID CONTROLLER DESIGN FOR ACTIVE MAGNETIC BEARING

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ABSTRACT
This paper presents a robust PID control scheme for a rigid rotor magnetic bearing system with five-axis-control system of actual test rig. The robust PID design method based on H_∞ closed-loop gain shaping theory is developed and used in the active magnetic bearing. After discussing the design theory of the robust PID control system based on H_∞ control theory, we obtained the PID parameter formulae. Next, we designed the robust PID control system. We tried the simulations of control system for the radial bearing and obtained good performances on simulations. We carried out experiments to verify the robustness based on robust control theory for the actual test rig. The start-up suspension displacement curve and the characteristic of disturbance rejection for lower frequency have been measured. We confirmed that the robust PID control scheme has excellent performances for disturbance rejection and robust performance with parameter variations in this test rig.

1. INTRODUCTION

Robust control is one of the most important topics in recent years and many application studies of robust control theory have been carried out successfully [1,2,6]. Since there are too many uncertainties in magnetic levitation systems, the robust control method must be applicable to these areas. At the present time, there are many robust control theories. In particular, it is well known that H_∞ control theory and μ synthesis are very powerful robust control theory as linear control. H_∞ control theory includes the advantages of classical control and modern control, and is very systematic as loop shaping theory. From a computational point of view, H_∞ control problems requires the solution of two Algebric Riccati Equations with the some rank condition on system matrices in general. An H_∞ controller with excellent disturbance suppression effect, robustness and stability can be designed. However, this theory uses weight function, so designing a controller using enlarged plant allowing for disturbance and error would be time-consuming [2,3]. Classical PID control scheme has the advantages of simple design procedure, but it is not easy to satisfy the robust performance of control systems [3,4,5,6]. In this paper, the PID design method based on H_∞ closed-loop gain shaping theory is developed and used in the active magnetic bearing. The robust PID control scheme not only has excellent performances for disturbance rejection and robust performance with parameter variations in this test rig but also the advantages of simple design procedure.

2. PID PARAMETERS FORMULAE BASED ON H_∞ CLOSED-LOOP GAIN SHAPING THEORY

From the point of view of H_∞ complex sensitivity $S(s)/T(s)$, the design guidelines for the H_∞ controller with good disturbance attenuation and robustness were:

(1) Use frequency-domain weight function relative to sensitivity function to suppress disturbance in the low frequency range.
(2) Use frequency-domain weight function relative to complementary sensitivity function to suppress uncertainty.

Figure 1 shows singular value curves $S(s)/T(s)$ and weighting functions $W1(s)$, $W2(s)$. $T(s)$ is the transfer function of the closed-loop system, and $T(s)$ is under the curve $W2^{-1}$. This means that the upper cut-off frequency and the roll-off rate of the $T(s)$ is defined by $W2^{-1}$. The design of controller needs to select weight functions.

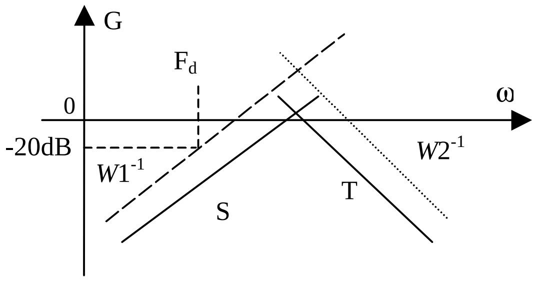


FIGURE 1: Frequency Shaping of S/T

Because S(s) is correlated with T(s) (T(s)=I-S(s)), the shape of S(s) can be fixed when the shape of T(s) has been fixed. Therefore, according to the final shape of

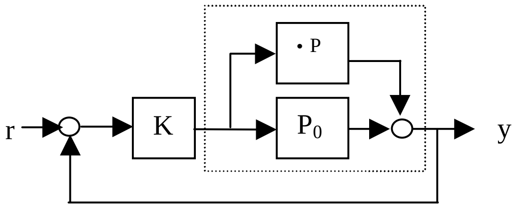


FIGURE 2: Block Diagram of the Closed-loop System with Uncertainty

T(s), the controller K is designed directly without selecting any weighting functions.

The feedback system with uncertainty is shown in Figure 2. The transfer function T(s) of the closed-loop system is found as

$$T = \frac{P_0 K}{1 + P_0 K} \quad (1)$$

The frequency characteristic of T(s) is low-pass filter•

- 1• Low-frequency domain:

  $T(s) \approx 1$ and then,

  $P_0 K >> 1$, $S(s) \approx 1/P_0 K$

The system is designed to achieve good tracking properties and disturbance rejection of low-frequency.

(2) High-frequency domain:

  $T(s) \approx 0$ and then,

  $P_0 K << 1$, $S(s) \approx 1$

The system is designed to achieve disturbance rejection of high-frequency and uncertainty rejection.

From the above discussion, the T(s) designed in high frequency may be difference. I.e. the maximum singular value of T(s) is equal to 1, and the roll-off race can be -20dB/decade, -40dB/decade or -60dB/decade. Of course, from the point of view of disturbance rejection of high frequency, the roll-off rate of -60dB/decade is better than -40dB/decade, and the roll-off rate of -40dB/decade is better then -20dB/decade. The order of controller for the roll-off rate of -60dB/decade is higher than -40dB/decade and -20dB/decade. Therefore the design of controller becomes complex. At last analysis, the roll-off rate of -20dB/decade is suitable.

Now, the design guidelines were:

According to the shape of T(s), the controller K is designed directly. The maximum singular value of T(s) is equal to 1, the fix planting. The upper cut-off frequency or bandwidth of T(s) will be determined by the actual system. Which will be discussed in section 4 of this paper. The roll-off rate of T(s) is selected -20dB/decade, the fix planting. When fix on the roll-off slope of T(s) as -20dB/decade, it will be convenient to design PID controller for an actual system with two-order deep strictly proper. The two-order deep strictly proper system is defined as follows:

Definition: Considering two-order strictly proper plant G,

$$G = \frac{b_1 s + b_0}{a_2 s^2 + a_1 s + a_0} \quad (2)$$

if $b_1 = 0$, then the plant is defined as two-order deep strictly proper plant.

If the shaping of T(s) is desired as above, (roll-off rate is -20dB/decade and the maximum singular value is equal to 1), The formula of T(s) will be a one-order inertial system, according to the following equation.

$$T(s) = \frac{1}{T_c s + 1} \quad (3)$$

where T_c is the time constant.

We get

$$\frac{P_0 K}{1 + P_0 K} = \frac{1}{T_c s + 1} \quad (4)$$

From equation (4), the controller K(s) is found as

$$K(s) = \frac{1}{P_0 T_c s} \quad (5)$$

For two-order deep strictly proper controlled plant

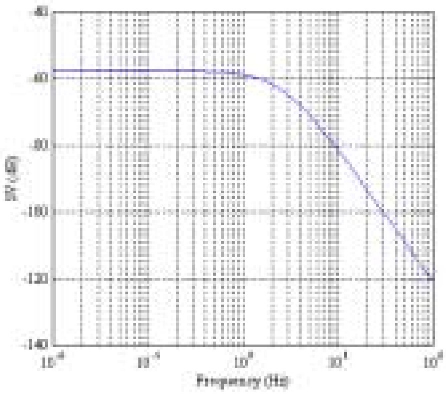
$$P_0 = \frac{b_0}{a_2 s^2 + a_1 s + a_0} \quad (6)$$

The controller (5) becomes

$$K(s) = \frac{a_1}{b_0 T_c} + \frac{a_0}{b_0 T_c s} + \frac{a_2 s}{b_0 T_c} \quad (7)$$

The controller (7) is a standard PID controller, and the corresponding control parameters are

$$K_p = \frac{a_1}{b_0 T_c} \quad (8a)$$



(a)

$$K_i = \frac{a_0}{b_0 T_c} \qquad (8b)$$

$$K_d = \frac{a_2}{b_0 T_c} \qquad (8c)$$

The equation (8) can be used directly for stable controlled plant. The magnetic bearing is unstable controlled plant in essence, so it would be Precompensated to design the PID controller.

3. PRECOMPENSATION

3.1 Question

The transfer function of the radial active magnetic bearing is

$$P_{0m}(s) = k_a \frac{k_i}{s^2 + k_y} \qquad (9)$$

where k_i is the current stiffness coefficient of the magnetic bearing, k_y is the displacement stiffness coefficient of the magnetic bearing, and k_y is negative. k_a is a constant, here k_a =5000.

The transfer function (9) is belong to two-order deep strictly proper controlled plant, and yet, there would be some problems by designing controller directly. Comparing equation (9) with equation (6), the result is a_1 =0, $a_0 = \kappa_y$ <0. So that, we get K_p=0, K_i <0. Such PID control parameters can not stabilify the magnetic bearing system obviously.

3.2 Precompensation

The PID controller is designed based on closed-loop gain shaping theory, so that the open-loop frequency shape of controlled plant is important for designing controller. We compensate the controlled plant by changing its parameters without changing its frequency shape. The parameters of radial magnetic bearing are:

k_i=0.009

k_y =-6.7344

The frequency characteristic (Bode plot) is a two-order low-pass, and is shown in Figure 3 (a). According to electronics theory, the open-loop gain and roll-off rate may remain when denominator is changed into (s^2+6.7344). Which is shown in figure 3 (b). Selecting coefficient a_1 =5 adequately, we can get the frequency characteristic (Bode plot) shown in figure 3 (c) as the same as figure 3 (a). The controlled plant compensated now becomes

$$P_0(s) = k_a \frac{0.009}{s^2 + 5s + 6.7344} \qquad (10)$$

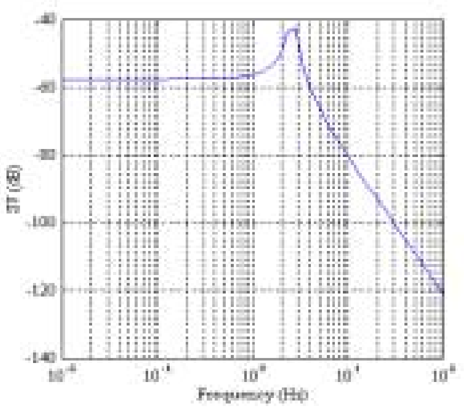
We can obtain these parameters: a_2 =1 a_1 =5 a_0 =6.7344 b_0 =0.009. Substitute the parameters a_2, a_1, a_0 and b_0 into equation (8), we can get the PID parameter formulas

$$K_p = 555.56\omega_c \qquad (11a)$$

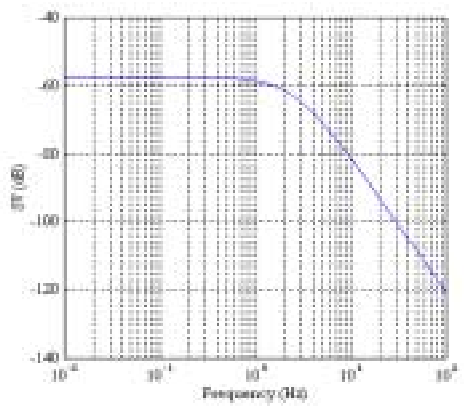
$$K_i = 748.27\omega_c \qquad (11b)$$

$$K_d = 111.11\omega_c \qquad (11c)$$

where $\omega_c = 1/T_c$ is closed-loop bandwidth of the system.



• b•



(c)

FIGURE 3: The Open-loop Gain Shaping for Radial Magnetic Bearing

4. SELECTING OF THE CLOSED-LOOP BANDWIDTH

Usually, the principles to select a closed-loop bandwidth of a system are:

(1) The speed of respond (The speed of respond is proportional to the bandwidth)

(2) The ability of disturbance rejection for high-frequency

The principle (1) is in contradiction with (2). Therefore, the bandwidth is selected eclectically.

In terms of H_∞ theory, the low-frequency disturbance will be rejected by reducing the gain of sensitivity function S(s). Suppose disturbance input, *d*, being low-frequency signal of f_d, the closed-loop bandwidth of a system is required for $f_c > f_d$ (f_c is the upper cut-off frequency of the system). Obviously, the higher the f_c , the stronger the ability of disturbance rejection of system. The ability of disturbance rejection for low frequency vary with f_c. See the *S*(s) singular value curves in Figure 1. In fact, the closed-loop bandwidth ω_c ($\omega_c = 2\pi f_c$) is get in touch with the loop-gain k_a in this paper. Therefore, ω_c is designed according to the request of disturbance rejection and robustness. The ω_c is finally determined according to singular value curves *S*(s)/*T*(s).

5. SIMULATION

The simulation has been done on a test rig for measurement of inertia moment. For radial bearing the current stiffness coefficient k_i and displacement stiffness coefficient k_y is given: k_i=0.009, k_y =-6.7344. The loop-gain of the control system is designed adequately. If the closed-loop bandwidth f_c is designed 5 times as large as disturbance bandwidth f_d, the PID parameters can be got as

$$K_p = 1.026$$
$$K_i = 1.382$$
$$K_d = 0.205$$

The *S/T* singular value curves is shown in figure 4 corresponding to the PID parameters selected.

The Figure 5 is step response, and the Figure 5(a) is the step response for the reduced order model, the Figure 5 (b) is for the uncertainty model. For comparison, the result from Figure 5(a) and Figure 5(b) is summarized as follows: There is little variance. Thus, we have demonstrated that the PID controller based on H_∞ closed-loop gain shaping theory has excellent robustness and stability.

The comparison of the ability of disturbance rejection is shown in figure 6. The disturbance input for difference frequency have the equal amplitudes, 0.03mm. Note that the ability of disturbance rejection is different for different frequency input. This characteristic is proved from the singular value curves *S*/*T* in Figure 4. In Figure 6, for the disturbance input of ω_1=3.14 rad/s, the

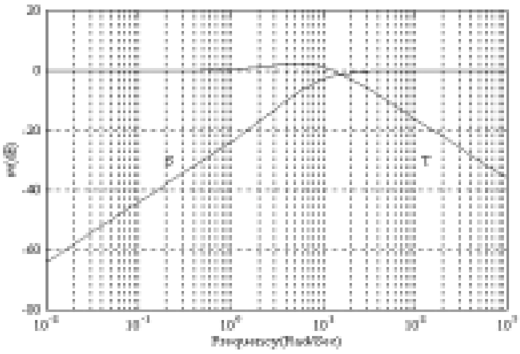
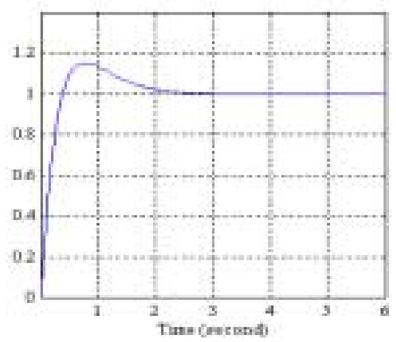
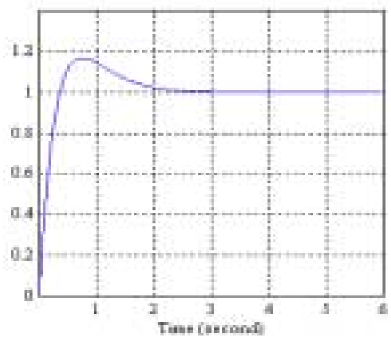


FIGURE 4: S/T Singular Value Curves of the Radial Bearing



(a) Reduced Model



(b) Uncertainty Model

FIGURE 5: The Step Response for Radial Bearing

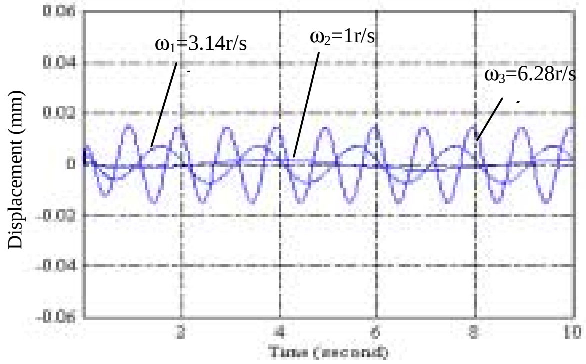


FIGURE 6: The Ability of Disturbance Rejection for Different Frequency

output amplitude is equal to 0.007mm. The amplitude drops by 4.3 times. In Figure 4, the singular value of sensitivity function S at ω_1=3.14 rad/s is -13dB. The singular value of sensitivity function S shown that the lower the frequency of the disturbance input is, the stronger the ability of disturbance rejection of the system.

6. EXPERIMENTAL RESULTS

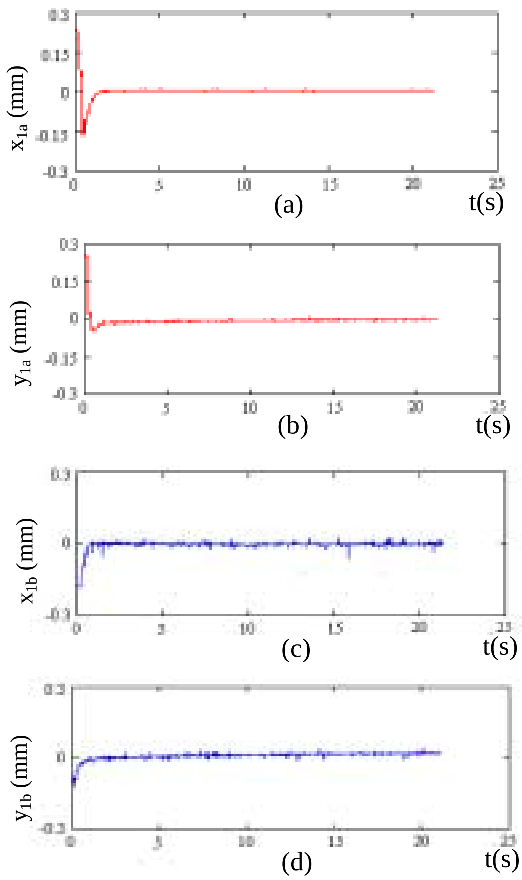


FIGURE 7: The Start-up Displacement for Radial Bearing

The experiment has been done on a test rig for measurement of inertia moment. The rotor is standing. For radial bearing *A* and radial bearing *B*, we have measured the start-up suspension displacement curves. In figure 7, two degrees of freedoms, x and y, are measured.

Figure 8 indicates characteristic of disturbance rejection

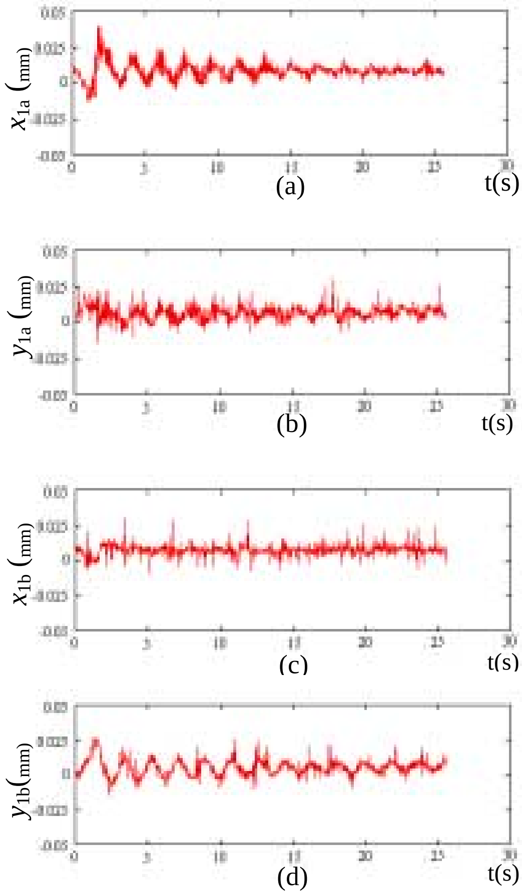


FIGURE 8: Disturbance Rejection for Lower Frequency

for lower frequency. The frequency of disturbance input is 0.5Hz.

7. CONCLUSIONS

The following conclusions can be offered from the above work:

(1) The robust PID controller based on H_∞ closed-loop gain shaping theory is easy for design. This method is handy without selecting weight functions. The PID parameters are calculated only in terms of the formula (8a-8c). (2)Although it is difficult to use formula (8a-8c) directly for unstable controlled plant, the Precompensation method is successful for us to solve the problem. (3) We have realized the magnetic levitation bearing system using the robust PID

controller for robust stability problem, and also suppressed disturbance of low frequency. (4) The robustness and the ability of disturbance rejection for low frequency are relation to bandwidth ω_c. Which is determined according to singular value curves S(s)/T(s).

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