NONLINEAR CONTROL OF ZERO POWER MAGNETIC BEARING USING LYAPUNOV'S DIRECT METHOD

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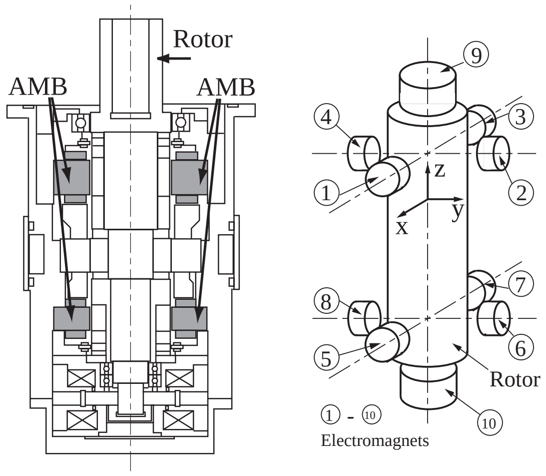
ABSTRACT
This paper proposes a new nonlinear control system to realize a zero power control based magnetic bearings. It is not necessary for the proposed scheme to use not only permanent magnet but also bias current. The behavior of a rotor is controlled by the nonlinear control scheme which is derived based on Lyapunov's direct method.
The control currents to the electromagnets which stand opposite to each other are switched like on and off respectively depending on a displacement, a velocity, a rotation and an angular velocity of the rotor. It is clarified through the simulation that the proposed nonlinear control scheme is a useful approach to realize a zero power control of the magnetic bearing system.

INTRODUCTION
Active magnetic bearings (AMB's) use electromagnetic force to provide noncontact support for rotors in high speed rotating machinery. When it would be applied to a rotor systems, AMB's have advantages of being contactless, of allowing high speed rotation. Industrial applications include flywheels, turbo molecular pumps and milling spindles. In generally, the bias current are supplied to AMB's for linearization of the nonlinear electromagnetic force e.g.[1], however, it is not fit for a energy storage flywheel system supported by AMB's because an energy consumption based on bias current is so much. Therefore it is necessary for AMB's to realize the zero power control which means the control without the bias current.

A zero power magnetic bearing system using permanent magnet has been studied [2]. In this system, it was not necessary to supply the bias current, and a amount of energy consumption became small. However, the actuator had a complicated configuration.
Besides, nonlinear control theories have been applied to AMB's [3] - [8], however, the bias current was used in those studies. Also the nonlinear control system without the bias current have been proposed [9][10], however, it was difficult to design the controller because those nonlinear control systems were complicated. Moreover, Torres *et al.*[11] designed the discontinuous feedback control which was proposed as the sliding mode scheme. This nonlinear control method did not use the bias current, however, the zero power control was not realized perfectly because the control current were supplied to the electromagnet which can be turn off.

This paper proposes a new nonlinear control system to realize a zero power control based magnetic bearings. In this study, it is assumed that the magnetic bearing is composed by only electromagnets, the rotor system is a rigid, and the gyroscopic effect is ignored. The nonlinear control scheme is derived based on Lyapunov's direct method. The derived control scheme switches electromagnets depending on a displacement, a velocity, a rotation and an angular velocity of the rotor. And if the rotor is at the equilibrium point, it is not necessary for the electromagnets to supply their control currents. Therefore the nonlinear control system reduces the energy consumption. It is very easy for the nonlinear control scheme to design, thus the characteristics of closed loop system can be changed easily.

MODELING
The configuration of a vertical type rotor system supported by magnetic bearings is shown in Fig.1(a). The nomenclature in this paper is shown in Table 1. In this study, the fundamental structure is shown in Fig.1(b).



(a) Cross sectional view of the AMB system

(b) Fundamental structures of the magnetic bearing system

FIGURE 1: Test rig

TABLE 1: Nomenclature

Mass	M	15.23 [kg]
Moment of inertia about x and y	I_r	0.184 [kgm^2]
Constant of magnetic attractive force	k_h k_l	5.01×10^{-5} [Nm^2/A^2] 3.24×10^{-5} [Nm^2/A^2]
Distance from the center of gravity	L_h L_l	45.77×10^{-3} [m] 113.9×10^{-3} [m]
Constant air gap	X_0, Y_0	0.3×10^{-3} [m]

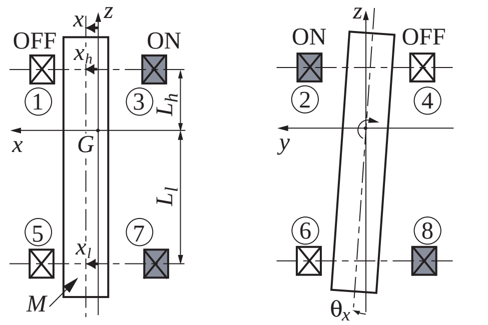
The axial direction is controlled by a conventional controller and the radial direction is controlled by the proposed nonlinear controller.

Here, the displacement in the physical co-ordinate of the rigid rotor from equilibrium line are given as follows:

$$x_h = x + \theta_y L_h\,,\ \ x_l = x - \theta_y L_l$$
$$y_h = y - \theta_x L_h\,,\ \ y_l = y + \theta_x L_l$$

where, the subscript *h* means the higher side, and the subscript *l* means as the lower side.

Furthermore, it is assumed that the gyroscopic effect is ignored. As the results, the coupling between the parallel motion and the rotational motion is not taken into account. Hence the equations of motion are as follows:



(a) Parallel motion (b)Rotational motion

Fig.2 The concept of proposed control using on-off switching

$$M\ddot{x} = k_h \frac{i_1^2}{(X_0 - x_h)^2} - k_h \frac{i_3^2}{(X_0 + x_h)^2} + k_l \frac{i_5^2}{(X_0 - x_l)^2} - k_l \frac{i_7^2}{(X_0 + x_l)^2} \tag{1}$$

$$M\ddot{y} = k_h \frac{i_2^2}{(Y_0 - y_h)^2} - k_h \frac{i_4^2}{(Y_0 + y_h)^2} + k_l \frac{i_6^2}{(Y_0 - y_l)^2} - k_l \frac{i_8^2}{(Y_0 + y_l)^2} \tag{2}$$

$$I_r\ddot{\theta}_x = \left\{ -\frac{i_2^2}{(Y_0 - y_h)^2} + \frac{i_4^2}{(Y_0 + y_h)^2} \right\} k_h L_h + \left\{ \frac{i_6^2}{(Y_0 - y_l)^2} - \frac{i_8^2}{(Y_0 + y_l)^2} \right\} k_l L_l \tag{3}$$

$$I_r\ddot{\theta}_y = \left\{ \frac{i_1^2}{(X_0 - x_h)^2} - \frac{i_3^2}{(X_0 + x_h)^2} \right\} k_h L_h + \left\{ -\frac{i_5^2}{(X_0 - x_l)^2} + \frac{i_7^2}{(X_0 + x_l)^2} \right\} k_l L_l \tag{4}$$

The Eqs. (1) ~ (4) are uncoupled each other, thus we can design the nonlinear controllers for each Eq. (1) ~ (4). Then the control currents are defined as follows:

$$i_j = i_{pj} + i_{rj}\,,\quad (j = 1, \cdots, 8) \tag{5}$$

where i_{pj} is the control current for a parallel motion and i_{rj} is the control current for a rotational motion. In this study, the parallel and rotational motions are controlled independently.

Let's explain about nonlinear control scheme. The electromagnets are switched depending on the parallel and rotational motions of a rotor. For examples, when the rotor approaches electromagnet 1 and 5 as $x > 0$ like Fig.2(a), the currents i_{p1} and i_{p5} should be turn off and the currents i_{p3} and i_{p7} should increase to stabilize.

Similarly, when the rotor rotates as $\theta_x > 0$ like Fig.2(b), the currents i_{r4} and i_{r6} should be shut down and the currents i_{r2} and i_{r8} should increase to stabilize.

Here, we can derive the relation between the higher and the lower currents:

$$\begin{cases} i_{p5} = \dfrac{X_0 - x_l}{X_0 - x_h} K_p i_{p1}\,,\; i_{p7} = \dfrac{X_0 + x_l}{X_0 + x_h} K_p i_{p3} \\ i_{p6} = \dfrac{Y_0 - y_l}{Y_0 - y_h} K_p i_{p2}\,,\; i_{p8} = \dfrac{Y_0 + y_l}{Y_0 + y_h} K_p i_{p4} \end{cases} \qquad (6)$$

$$\begin{cases} i_{r5} = \dfrac{X_0 - x_l}{X_0 + x_h} K_r i_{r3}\,,\; i_{r7} = \dfrac{X_0 - x_l}{X_0 + x_h} K_r i_{r1} \\ i_{r6} = \dfrac{Y_0 - y_l}{Y_0 + y_h} K_r i_{r4}\,,\; i_{r8} = \dfrac{Y_0 + y_l}{Y_0 - y_h} K_r i_{r2} \end{cases} \qquad (7)$$

where $K_p = \sqrt{k_h / k_l}$, $K_r = \sqrt{(k_h L_h)/(k_l L_l)}$.

Using Eqs.(5), (6) and (7), Eqs.(1) ~ (4) are transformed as follows:

$$M\ddot{x} = 2k_h \frac{i_{p1}^2}{(X_0 - x_h)^2} - 2k_h \frac{i_{p3}^2}{(X_0 + x_h)^2} \qquad (8)$$

$$M\ddot{y} = 2k_h \frac{i_{p2}^2}{(Y_0 - y_h)^2} - 2k_h \frac{i_{p4}^2}{(Y_0 + y_h)^2} \qquad (9)$$

$$I_r\ddot{\theta}_x = -2k_h L_h \frac{i_{r2}^2}{(Y_0 - y_h)^2} + 2k_h L_h \frac{i_{r4}^2}{(Y_0 + y_h)^2} \qquad (10)$$

$$I_r\ddot{\theta}_y = 2k_h L_h \frac{i_{r1}^2}{(X_0 - x_h)^2} - 2k_h L_h \frac{i_{r3}^2}{(X_0 + x_h)^2} \qquad (11)$$

As a consequence, the equations of motion are described simply and the nonlinear controller can be designed easily.

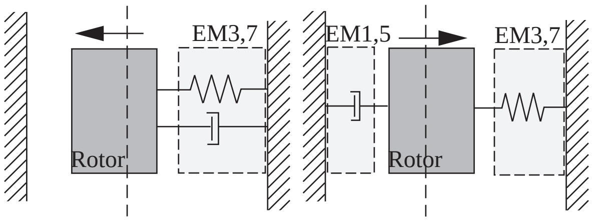
DESIGN OF NONLINEAR CONTROLLER

From Eqs.(8) ~ (11), we derive nonlinear controllers for x, y, θ_x and θ_y independently. In this paper, how to derive the controller for the x axis is particularly explained in detail, because the controllers for another axis are derived similarly.

First, we define the candidate of the Lyapunov function which depends on Eq.(8) as follows:

$$V = \frac{1}{2} a_{px}^2 x^2 + \frac{1}{4} M\dot{x}^2 \qquad (12)$$

Where we define $a_{px} > 1$, and it is explained in detail later. The derivative of Eq.(12) is given by



(a) The rotor leaves the equilibrium point (b) The rotor is returned to the equilibrium point

Fig.3 The modified concept of proposed control using on-off switching

$$\begin{aligned} \dot{V} &= \frac{dV(x,\dot{x})}{dt} \\ &= \dot{x}\left(a_{px}^2 x + \frac{1}{2} M\ddot{x}\right) \end{aligned} \qquad (13)$$

If Eq.(13) is negative, the function V as a candidate will become exact Lyapunov function and it will decrease along the solution of Eq.(8). Consequently, the magnetic bearing system will be asymptotically stable. In this study, the inequality in Eq.(13) is transformed as follows:

$$\dot{V} = \dot{x}\left(a_{px}^2 x + \frac{1}{2} M\ddot{x}\right) < -\gamma_{px}\dot{x}^2 \qquad (14)$$

Where $\gamma_{px} > 0$ is defined. In this paper, the nonlinear controller which satisfies inequality (14) was designed as follows:

$$\begin{cases} i_{p1} = 0 \\ i_{p3} = a_{px}(X_0 + x_h)\sqrt{(x + \gamma_{px}\dot{x})/k_h} \\ \quad \text{when } x > 0 \text{ and } \dot{x} > 0 \end{cases} \qquad (15)$$

$$\begin{cases} i_{p1} = a_{px}(X_0 - x_h)\sqrt{(-\gamma_{px}\dot{x})/k_h} \\ i_{p3} = a_{px}(X_0 + x_h)\sqrt{x/k_h} \\ \quad \text{when } x > 0 \text{ and } \dot{x} < 0 \end{cases} \qquad (16)$$

$$\begin{cases} i_{p1} = a_{px}(X_0 - x_h)\sqrt{(-x - \gamma_{px}\dot{x})/k_h} \\ i_{p3} = 0 \\ \quad \text{when } x < 0 \text{ and } \dot{x} < 0 \end{cases} \qquad (17)$$

$$\begin{cases} i_{p1} = a_{px}(X_0 - x_h)\sqrt{(-x)/k_h} \\ i_{p3} = a_{px}(X_0 + x_h)\sqrt{(\gamma_{px}\dot{x})/k_h} \\ \quad \text{when } x < 0 \text{ and } \dot{x} > 0 \end{cases} \qquad (18)$$

i_{p5} and i_{p7} are calculated from Eq.(6).

The important point to note is that the scheme of on-off switching is modified. This nonlinear scheme uses not only the displacement x but also the velocity $\dot{x}$. If the rotor moves to left hand side from the equilibrium point like Fig.3(a) ($x > 0$, $\dot{x} > 0$), the control currents are supplied to only electromagnets 3 and 7, and the generated forces act on the rotor as a mechanical component which consist of a spring and a damping shown in Fig.3(a). And if the rotor recovers again to the equilibrium point like Fig.3(b) ($x > 0$, $\dot{x} < 0$), the control currents are supplied to four electromagnets. In this case, the forces generated by Electromagnets 3 and 7 act on the rotor as the spring force, and the forces generated by electromagnets 1 and 5 act as the damping force.

In order to verify that the proposed nonlinear controller satisfies the inequality (14), we substitute Eqs.(8) and (15) into inequality (14). As the results we obtain the relation as follows:

$$\begin{aligned}\dot{V} &= \dot{x}\left(a_{px}{}^2 x + \frac{1}{2}M\ddot{x} + \gamma_{px}\dot{x}\right)\\ &= \dot{x}\left[a_{px}{}^2 x + k_h\left\{\frac{i_{p1}{}^2}{(X_0 - x_h)^2} - \frac{i_{p3}{}^2}{(X_0 + x_h)^2}\right\} + \gamma_{px}\dot{x}\right]\\ &= \dot{x}\left[a_{px}{}^2 x - a_{px}{}^2(x + \gamma_{px}\dot{x}) + \gamma_{px}\dot{x}\right]\\ &= -(1 - a_{px}{}^2)\dot{x}^2 < 0\end{aligned} \qquad (19)$$

When we substitute Eqs.(16), (17), or (18) instead of Eq.(15), Eq.(19) can be obtained in the same manner. The expression $a_{px} > 1$ is defined in this paper, therefore, Eq.(19) is negative anytime. For reasons above mentioned, it is proved that the inequality (14) is satisfied by the proposed nonlinear scheme and the magnetic bearing system will be asymptotically stable.

Furthermore, it is explained how to decide a_{px} and γ_{px}. Substituting Eq.(15) into Eq.(8), we obtain the equation as follows:

$$M\ddot{x} = -2a_{px}{}^2 x - 2a_{px}{}^2\gamma_{px}\dot{x} \qquad (20)$$

When we substitute Eqs.(16), (17), or (18) instead of Eq.(15), Eq.(20) can be obtained in the same manner. Equation (20) shows that the closed loop system is transformed into a damped free vibration system by the proposed nonlinear control system and its natural frequency is given by

$$\omega_n = \sqrt{\frac{2}{M}}\,a_{px} \qquad (21)$$

and the damping ratio is given by

$$\zeta = \frac{a_{px}\gamma_{px}}{\sqrt{2M}} \qquad (22)$$

This consideration means that the decision of the parameters in the nonlinear control system is equal to the design of the damped free vibration system. By means of this reason, it is not difficult to design the proposed nonlinear control system in this paper.

Here, the nonlinear controllers for Eqs.(16), (17) and (18) are shown as follows:

For the y axis :

$$\begin{cases} i_{p2} = 0 \\ i_{p4} = a_{py}(Y_0 + y_h)\sqrt{(y + \gamma_{py}\dot{y})/k_h} \\ \quad \text{when } y > 0 \text{ and } \dot{y} > 0 \end{cases} \qquad (23)$$

$$\begin{cases} i_{p2} = a_{py}(Y_0 - y_h)\sqrt{(-\gamma_{py}\dot{y})/k_h} \\ i_{p4} = a_{py}(Y_0 + y_h)\sqrt{y/k_h} \\ \quad \text{when } y > 0 \text{ and } \dot{y} < 0 \end{cases} \qquad (24)$$

$$\begin{cases} i_{p2} = a_{py}(Y_0 - y_h)\sqrt{(-y - \gamma_{py}\dot{y})/k_h} \\ i_{p4} = 0 \\ \quad \text{when } y < 0 \text{ and } \dot{y} < 0 \end{cases} \qquad (25)$$

$$\begin{cases} i_{p2} = a_{py}(Y_0 - y_h)\sqrt{(-y)/k_h} \\ i_{p4} = a_{py}(Y_0 + y_h)\sqrt{(\gamma_{py}\dot{y})/k_h} \\ \quad \text{when } y < 0 \text{ and } \dot{y} > 0 \end{cases} \qquad (26)$$

For the θ_x axis :

$$\begin{cases} i_{r2} = a_{rx}(Y_0 - y_h)\sqrt{(\theta_x + \gamma_{rx}\dot{\theta}_x)/(k_h L_h)} \\ i_{r4} = 0 \\ \quad \text{when } \theta_x > 0 \text{ and } \dot{\theta}_x > 0 \end{cases} \qquad (27)$$

$$\begin{cases} i_{r2} = a_{rx}(Y_0 - y_h)\sqrt{\theta_x/(k_h L_h)} \\ i_{r4} = a_{rx}(Y_0 + y_h)\sqrt{(-\gamma_{rx}\dot{\theta}_x)/(k_h L_h)} \\ \quad \text{when } \theta_x > 0 \text{ and } \dot{\theta}_x < 0 \end{cases} \qquad (28)$$

$$\begin{cases} i_{r2} = 0 \\ i_{r4} = a_{rx}(Y_0 + y_h)\sqrt{(-\theta_x - \gamma_{rx}\dot{\theta}_x)/(k_h L_h)} \\ \quad \text{when } \theta_x < 0 \text{ and } \dot{\theta}_x < 0 \end{cases} \qquad (29)$$

$$\begin{cases} i_{r2} = a_{rx}(Y_0 - y_h)\sqrt{(\gamma_{rx}\dot{\theta}_x)/(k_h L_h)} \\ i_{r4} = a_{rx}(Y_0 + y_h)\sqrt{-\theta_x/(k_h L_h)} \\ \quad \text{when } \theta_x < 0 \text{ and } \dot{\theta}_x > 0 \end{cases} \qquad (30)$$

For the θ_y axis :

$$\begin{cases} i_{r1} = 0 \\ i_{r3} = a_{ry}(X_0 + x_h)\sqrt{(\theta_y + \gamma_{ry}\dot{\theta}_y)/(k_h L_h)} \\ \quad \text{when } \theta_y > 0 \text{ and } \dot{\theta}_y > 0 \end{cases} \qquad (31)$$

$$\begin{cases} i_{r1} = a_{ry}(X_0 - x_h)\sqrt{(-\gamma_{ry}\dot{\theta}_y)/(k_h L_h)} \\ i_{r3} = a_{ry}(X_0 + x_h)\sqrt{\theta_y/(k_h L_h)} \\ \quad when\ \theta_y > 0\ and\ \dot{\theta}_y < 0 \end{cases} \quad (32)$$

$$\begin{cases} i_{r1} = a_{ry}(X_0 - x_h)\sqrt{(-\theta_y - \gamma_{ry}\dot{\theta}_y)/(k_h L_h)} \\ i_{r3} = 0 \\ \quad when\ \theta_y < 0\ and\ \dot{\theta}_y < 0 \end{cases} \quad (33)$$

$$\begin{cases} i_{r1} = a_{ry}(X_0 - x_h)\sqrt{(-\theta_y)/(k_h L_h)} \\ i_{r3} = a_{ry}(X_0 + x_h)\sqrt{(\gamma_{ry}\dot{\theta}_y)/(k_h L_h)} \\ \quad when\ \theta_y < 0\ and\ \dot{\theta}_y > 0 \end{cases} \quad (34)$$

As the results the nonlinear control system consists of 16 equations and 8 parameters. The parameters in Eq.(23) ~ (34) are decided based on Eq.(20).

SIMULATION RESULTS

The performance of the proposed nonlinear control system is examined by simulation in this paper. The parameters of nonlinear control system are designed as follows:

$$\begin{cases} a_{px} = a_{py} = 1000 \\ \gamma_{px} = \gamma_{py} = 1.7 \times 10^{-3} \end{cases} \quad (35)$$

$$\begin{cases} a_{rx} = a_{ry} = 150 \\ \gamma_{rx} = \gamma_{ry} = 8.0 \times 10^{-3} \end{cases} \quad (36)$$

The results of step responses are shown in Fig.4. And the control currents for the x axis are shown in Fig.5. These figures show that the closed loop controlled by the nonlinear control scheme is stable and is very similar to the damped free vibration system. Accordingly, it has been clarified that the proposed nonlinear control system can stabilize a rotor system supported by magnetic bearings using only electromagnets and has a enough good performance. And then the controllers switch electromagnets depending on x and $\dot{x}$ perfectly.

When the rotor approaches the equilibrium point, the control currents are not supplied to electromagnets. As the results the nonlinear control system realizes a zero power control and can improve the capacity of energy storage.

Figure 6 shows Lyapunov function which is defined for the x axis. This figure shows that Lyapunov function decreases uniformly. This result suggests that the closed loop system controlled by the proposed controller is globally asymptotically stable.

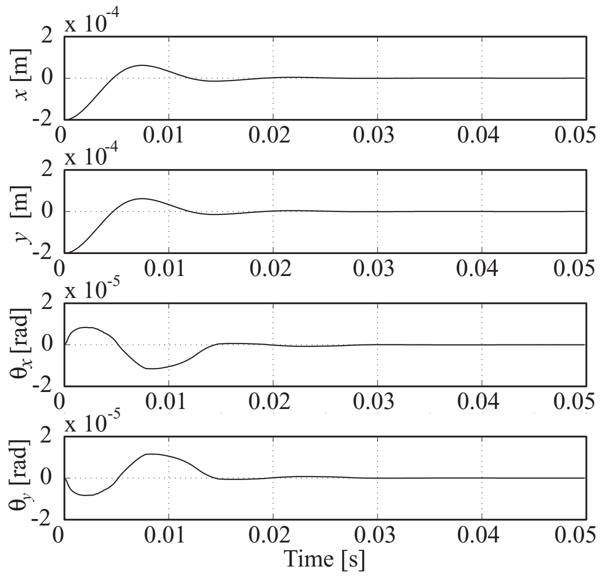


FIGURE 4: Step responses

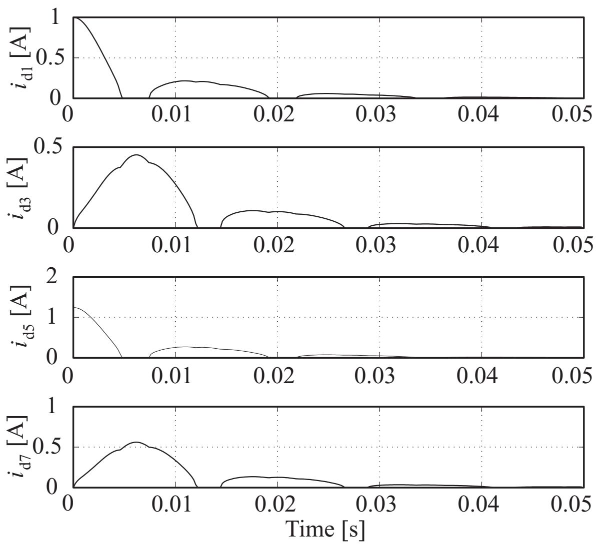


FIGURE 5: Control currents for the x axis

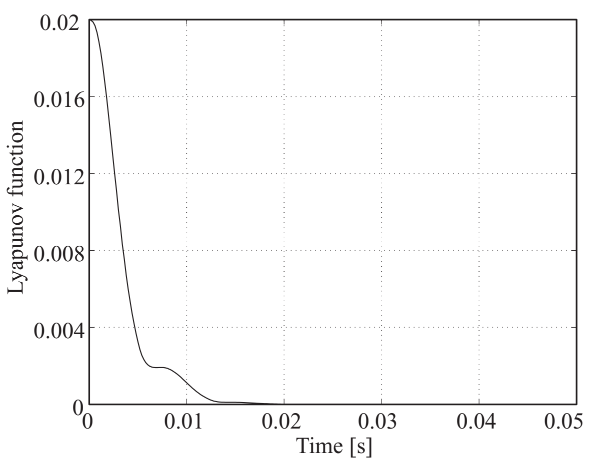


FIGURE 6: Lyapunov function

CONCLUSION

In this paper, we design and evaluate in simulation the nonlinear control system based magnetic bearings only electromagnets. The nonlinear control scheme is derived based on Lyapunov's direct method. The control scheme switches electromagnets depending on a displacement, a velocity, a rotation and an angular velocity of the rotor. The results of simulation clearly shows that the proposed nonlinear control scheme can realize a zero power control of the magnetic bearing system. Therefore, the nonlinear controller can improve the energy consumption significantly. Moreover we should note that this nonlinear control method can easily realize a zero power control of magnetic bearing, because the decision of the parameters of nonlinear control scheme is equal to the design of a damped free vibration system.

For future research, we will evaluate the proposed nonlinear control system in experiment. Moreover we will develop it for the system that gyroscopic effect is taken into consideration.

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